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ELEMENTARY TREATISE

ON

ELECTRICITY AND MAGNETISM

FOUNDED ON

JOUBERT'S

"TRAITÉ ÉLÉMENTAIRE D'ÉLECTRICITÉ"

BY

eorge arey
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PREFACE

In undertaking the present work, our first idea was simply to translate M. Joubert's *Traité élémentaire d'Électricité*, perhaps introducing such small modifications as might seem to make it better adapted to English requirements. But, on further consideration, we decided, while adhering to the general plan and scope of the original, to endeavour to introduce into it that view of the nature of electrical phenomena which was originated by Faraday and developed by Maxwell in his classical Treatise. Although these views have now for some years formed the starting-point of almost all the greatest forward steps in electrical science, they have not yet, so far as we are aware, been adopted as the basis of a systematic exposition of so elementary a kind as the present.

The endeavour to carry out this idea implied the keeping in view, from the beginning, of the dual character of electrification, and the emphasising of the essential part played in the most familiar electrical phenomena by the dielectric medium in which they occur. This led to the early introduction of the idea of lines and tubes of force, and involved the rewriting of several chapters or parts of chapters. In the discussion of Electrostatic Capacity and Energy (see especially Chapter VI.), it seemed to be more in harmony with the general point of view indicated above to speak of charge, capacity, and energy as belonging to the electric field as a whole, rather than to the conductors which form its boundaries.

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PREFACE TO THE FIRST FRENCH EDITION

MY object in writing this book has been to give a simple but fairly complete exposition of the theory of electricity and of the most important practical applications connected with it I have had in view a reader who is wishful, not only to know facts, but to understand their logical interdependence, and to acquire accurate ideas of the essential nature of electrical phenomena. I have not supposed my reader to possess more special knowledge than what goes with an elementary classical education.

I have confined myselt carefully to the region of facts, and have avoided all hypotheses. I have likewise avoided historical details. Interesting and profitable as I conceive these to be for those who know the facts of science, they are, in my opinion, of little use to beginners, and it is to these that my book is addressed.

J. J.

15th June 1888.

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ON

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ERRATA.

Page 31, line 9 from bottom, *before* "clastic" *insert* "stretched."
" 37, line 6, *before* "18" *insert* "§."
" 45, line 3 from bottom, *for* "Q" *read* "Q."
" 52, line 3 from bottom, *for* "f" *read* "If."
" 60, line 7 from bottom, *for* "$2\pi\sigma$" *read* "$2\pi\sigma/K$."
" 61, line 7 from bottom, for "N" *read* "N′."
" 62, line 17 from bottom, *for* "expression" *read* "expressions."
" 67, line 2, *for* "$\int^{e}\frac{de}{s}$" *read* "$\int_{0}^{e}\frac{de}{s}$."
" 80, line 21, *for* "and consists" *read* "and usually consists."
" 84, line 1, *for* "*Lichtemberg*" *read* "*Lichtenberg.*"
" 110, line 17 from bottom, *for* "$\frac{V+V_2}{2}$" *read* "$\frac{V_1+V_2}{2}$."
" 122, line 5, *for* "hydroscopic" *read* "hygroscopic."
" 124, line 3 of § 90, *for* "represents" *read* "represent."
" 156, line 12 from bottom, *after* "as" *insert* "numerically equal to."
" 156, line 9 from bottom, *for* "Where" *read* "When."
" 157, line 7, at the beginning, *insert* "numerically equal to."
" 220, at the middle, *for* "$Cu + SO_4$" *read* "$Cu + ZnSO_4$."
" 237, line 2 from bottom, *for* "ϖ" *read* "ω."
" 239, line 4, *for* "$\frac{\omega}{r^2}$" *read* "$\frac{\varpi}{r^2}$"
" 262, line 17, *after* "ergs" *insert* "per cm^3."
" 262, line 15 from bottom, at the beginning, *insert* "per cm^3."
" 262, line 4 from bottom, at the end, *insert* "per cm^3."

Page 289, lines 11 and 12 from bottom, *for* "$2\frac{MM'}{K^3}(1+f)$" *read* "$2\frac{M}{K^3}(1+f)$, where f is a constant depending on the magnet."

" 293, line 20, *for* "TK^3" *read* "T_eR^3."

" 303, line 19, *for* "*infinite*" *read* "*infinitely long.*"

" 335, last line, *for* "force" *read* "induction."

" 336, line 11, *for* "Kirchoff" *read* "Kirchhoff."

" 339, line 18 from bottom, *for* "infinite" *read* "infinitely long."

" 348, second line of § 292, *for* "phenomenon" *read* "phenomena."

" 360, line 1, *before* "resulting" *insert* "quantity of the."

" 360, line 13, *before* "current" *insert* "quantity of the."

" 391, line 3 from bottom, *for* "$\omega^2 - \frac{f}{m}$" *read* "$\omega^2 = \frac{f}{m}$."

" 435, line 11, in numerator of the second equation, *for* "$\pm$" *read* "$E\frac{s}{g+s}$."

" 436, line 10 from bottom, *before* "in order" *insert* "since."

" 436, line 7 from bottom, *for* "measured; or" *read* "measured,"

" 451, line 7, *for* "1.454" *read* "1.434."

" 495, second line of § 420, *omit* "contrary to what occurs in modern transformers."

" 507, line 4, at the beginning, *omit* "positive and."

" 507, line 4, at the end, *for* "the" *read* "a."

" 507, line 5, *for* "carbon, held in a suitable handle," *read* "carbon rod connected with the positive pole."

ELECTRICITY.

CHAPTER I.

FUNDAMENTAL PHENOMENA.

1. Electrification by Friction.—If a plate of glass and a plate of vulcanised india-rubber, both thoroughly dry, are laid on each other and rubbed together, it is found that some force is required to separate them, and that, after having been separated, they attract each other if not too far apart. If a second piece of glass and a second piece of india-rubber are treated in the same way, it is found that not only does either piece of glass attract either piece of india-rubber, but that the two pieces of glass repel each other, and that the two pieces of india-rubber do the same. Bodies which are in such a condition as to exhibit these properties are said to be *electrified.*

Attraction also occurs when an electrified body is brought near bodies that have not been subjected to friction, and if these are light enough (bits of pith, feathers, shreds of paper, wool, &c.), they may be lifted. Effects of this kind often afford an easier test of the existence of electrification than such experiments as those previously referred to. It is accordingly to be noted that although attraction occurring between two bodies may be evidence of previous electrification, it does not prove that *both* the mutually attracting bodies were previously electrified; and if only one was thus electrified, we cannot say, without other evidence, which it was. On the other hand, repulsion, as between two pieces of glass or two pieces of india-rubber, does not occur unless both the repelling bodies have been previously electrified.

2. Electrification by Contact.—If a pith-ball (A) hung by a fine fibre of raw silk (Fig. 1) is allowed to touch a piece of glass (D) that has been electrified by friction, it is repelled by the glass after contact (Fig. 2). Again, a second pith-ball, similarly hung by a silk fibre, if allowed to touch a piece of electrified india-rubber, is repelled by the india-rubber after contact. The two pith-balls, however, if brought near together, are found to attract each other; in fact, they respectively behave towards the glass and india-rubber, and towards each other, just as a second pair of pieces of glass and india-rubber which had been rubbed together would do. That is, the two pith-balls have acquired properties exactly like

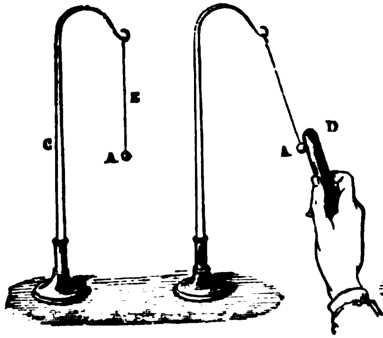

FIG. 1.

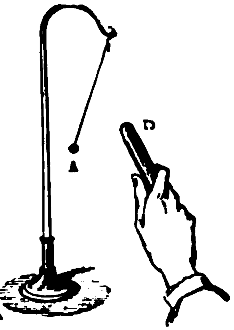

FIG. 2.

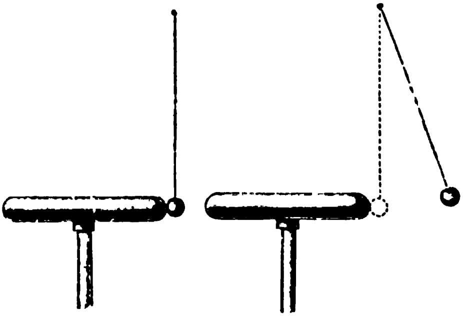

FIG. 3.

those of the glass and india-rubber, and are consequently said to be electrified by contact.

A metal rod or cylinder supported by a dry glass stem, or suspended by dry silk threads, can be electrified by contact with a piece of rubbed glass or of rubbed india-rubber. A pith-ball hanging by a silk fibre so as just to touch the metal before electrification may be repelled in this way to a considerable distance (Fig. 3).

A piece of metal thus electrified exhibits signs of electrification at all parts of its surface, but the evidence of electrification is less marked the larger the piece of metal. At the same time, the electrification of the primarily electrified body is found to be lessened by having imparted its own properties to the metal. In fact, the whole process takes place as though the communication of electrification depended on the sharing of some thing or condition, existing at the surface of the originally electrified body, between it and the body with which it is brought into contact, so that what is gained by one is lost by the other. Facts of this kind have led to the idea of an unknown something capable of being developed on bodies by friction, when two different substances are rubbed together, capable also of spreading from one body to another by contact, and giving rise to the phenomena characteristic of electrification. This supposed cause of electrical phenomena is spoken of as *electricity*.

3. Good and Bad Electrical Conductors.— Bodies which, when electrified at one part, are immediately found to be electrified all over, are called good *conductors* of electricity, and the process is commonly described as depending on the freedom with which electricity can move over them. There are, however, many substances—such as glass, shellac, india-rubber, silk, wool, sulphur, amber—which, when electrified at one part, allow the electrical state to spread only very slowly and gradually to other parts. Such bodies are conceived of as not permitting the passage of electricity through them or along their surface: they are hence called *non-conductors* of electricity. More properly, bodies of this kind are called bad or slow conductors, for they do not absolutely prevent the communication of electrification, and hence their properties differ rather in degree than in kind from those of good conductors. The differences in degree, however, are often enormous, the spread of electrification being in some cases apparently instantaneous, while in others it is so slow as to require very careful observation to detect it at all.

All the metals belong to the class of electrical conductors, as well as graphite and various metallic ores. Water in its ordinary condition is a conductor, and so are aqueous solutions of soluble salts; but absolutely pure water, like, probably, every non-metallic liquid in a pure unmixed state, is a non-conductor. Living plants and animals, being for the most part pervaded throughout by watery fluids, are conductors, though many animal and vegetable tissues, when thoroughly dry, are non-conductors. The air, again, and other gases, as well as vapours, are very perfect non-conductors.

4. The facts stated in the last two paragraphs serve to explain certain points connected with the electrification of bodies by friction. If two bodies, both belonging to the class of electrical non-conductors, are rubbed together, they are both found to be electrified, the degree of electrification varying with the nature of the bodies. But if a piece of metal or other conducting material held in the hand is rubbed against a non-conductor—say a piece of dry flannel—the non-conductor only appears afterwards to be electrified. The reason is that the electrification produced on the metal spreads over the hand, arm, and body of the experimenter to the floor and walls of the room, and is thereby so much diminished in intensity as not to be recognisable without special means. In order that a piece of metal may be electrified either by friction or by contact, it is needful that it should be cut off from electrical communication with the room and surrounding objects by being supported by non-conducting materials. The conductor is then said to be *insulated*, and so long as its insulation remains perfect, the degree of its electrification cannot increase or decrease.

In consequence of their use for preventing the communication of electrification, non-conducting bodies are often spoken of as *insulators*. No more perfect insulator is obtainable than air, if there is a sufficient thickness of it. Of solid insulators, clean and dry flint-glass and ebonite are among the most efficient and convenient for practical purposes. Glass, however, is apt to become covered with a conducting film of moisture deposited from the atmosphere. This can sometimes be prevented by keeping it slightly warm, —not *hot*, as it then conducts appreciably,—or by coating the surface with shellac-varnish, or, better still, by keeping the air in its neighbourhood dry by means of strong sulphuric acid (see Figs. 4, 5).

5. Two Correlative Electrical States.—Let us return to a fact mentioned in § 1. Suppose a piece of glass and a piece of vulcanised india-rubber to have been electrified by being rubbed together, and then that a second piece of each material is treated in the same way. As already stated, the two pieces of glass and the two pieces of india-rubber both repel each other, but either piece of glass attracts either piece of india-rubber. Hence we see that although both the glass and the india-rubber have acquired special properties as the result of the friction, that is, although both are electrified, their properties are not identical. As compared with each other, no difference is apparent: the glass attracts the india-rubber and the india-rubber attracts the glass, and either of them can lift small bits of pith. But towards a third electrified body their action is not only different, it is directly opposed: a body which is repelled by one is attracted by the other. Moreover, if two bodies are rubbed together without being separated, the parts that were subject to mutual friction remaining in contact, and are then presented simultaneously to a third electrified body, no sign of electrification can be detected, since the repulsion that would be caused by one of the rubbed bodies if it were alone is exactly neutralised by the attraction due to the other. Experiments of this kind, which in their details may be varied to an almost endless extent without altering the general result, prove that *the pro-*

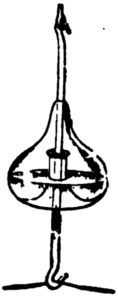
FIG. 4.

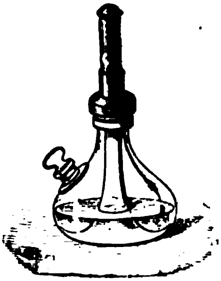
FIG. 5.

cess of electrification consists in the simultaneous development of two electrical states so related that the effects produced by a body in one of these states are opposed to those produced under similar conditions by a body in the other state, and also that *the opposite electrical states are always produced simultaneously to such an extent that they can mutually neutralise each other's action.*

The relation between these two correlative electrical conditions is analogous to that between quantities of opposite sign in algebra; that is, when oppositely electrified bodies are superposed, their combined effect is only equal to the difference of the effects they are capable of producing separately, just as the result of adding quantities of opposite algebraic sign is a quantity of the same sign as the greater of the two, and equal in magnitude to their arithmetical difference. This is expressed by calling one of the two electrical states, that assumed by the glass, *positive*, and the other, that assumed by the india-rubber, *negative.* Again, just as two quantities of opposite sign are said to be equal when their sum = 0, so the electrification of two oppositely electrified bodies is said to be equal in amount, or the bodies are said to be charged with equal opposite quantities of electricity, when the result of their superposition is to do away with all evidence of electrification. Using this mode of expression, we may repeat in a more concise form what was said above, and assert that *the process of electrification consists in the simultaneous and correlative production of equal quantities of positive and negative electricity.*

6. Electrification of Various Substances.—Hitherto, in speaking of electrification by friction, we have almost confined ourselves to a single typical case, that of glass and india-rubber. But in general, any two bodies, after being rubbed together, they having been held, if needful, by insulating supports, are found, if examined with sufficient care, to be more or less electrified. One of them repels a piece of glass that has been rubbed with vulcanised india-rubber and attracts the india-rubber; the other repels the india-rubber and attracts the glass. Accordingly the first is said to be positively electrified, and the second negatively. If a number of different substances are taken and rubbed together two and two, it is possible to arrange them in consecutive order, so that each one becomes positive when rubbed against one of those that follow it in the series, and negative when rubbed against one of those that precede it. For example, the substances in the following list are arranged in this way :—

Fur of a live cat.
Polished glass.
Flannel.
Feathers.
Woody fibre (pure cotton-wool).
Silk.
Shellac.
Resin.
Ground glass.
Metals.

Other things equal, more marked effects are obtained with two substances which occur far apart in such a list than with two that occur nearer together ; the effect appears, however, in many cases to depend on small changes of condition in a way that is not understood. An example is afforded by the difference between polished and ground glass.

7. The Electric Field.—When two bodies have been rubbed together so as to electrify them, the result cannot be detected until the surfaces have been separated (§ 5). In separating them, work has to be done in order to overcome the force with which they mutually attract each other (§ 1). When the rubbed bodies have been separated, any other electrified body brought into their neighbourhood, and especially if put directly between them, is subject to force tending to make it move from one and towards the other (§ 1). If, therefore, the mechanical resistances are so small that motion can actually take place, work is done by the electrical forces. It may be said, then, that the space near the electrified bodies possesses special properties depending on the relative positions of these bodies, and on the degree of their electrification. The existence of the forces already referred to constitutes the most obvious and easily observed of these properties, but, as we shall see later, this space possesses other properties not less characteristic. In order to connote collectively these properties without reference to the special way in which the electrification may have been produced, any space where they exist is called an *electric field.*

An electric field is always bounded by surfaces possessing equal opposite correlative electrifications (§ 5). It may, or may not, have definite geometrical limits. Consider, for example, the field produced by rubbing together and then separating two bodies which are at a very great distance from all other objects as compared with their distance from each other. The boundaries of the field are the surfaces of the electrified bodies ; but as these are entirely external to each other, they do not enclose any definite space, and the field extends on all sides. The only sense in which it can be said to be limited is that, at a greater or less distance, electrical effects can no longer be detected ; but this distance depends essentially on the means of detection employed, and as these are rendered more and more sensitive the apparent extent of the electric field becomes greater and greater. If, on the other hand, one of the bounding surfaces entirely encloses the other, the electric field is definitely limited to the space between them. For example, a case that frequently occurs is when one of the rubbed bodies is a conductor in electrical connection with the inside of the room where the experiment is made. The floor, walls, and ceiling are always composed of materials possessing sufficient conducting power to enable the electrification of the body connected with them to spread over them. In this case the electric field extends between the surface of the rubbed body that remains insulated and the inner surface of the room.

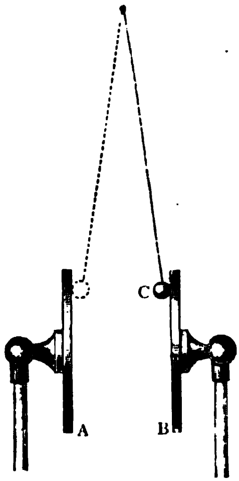

Fig. 6.

8. Energy of an Electric Field.—In whatever way an electric field may be produced, a certain amount of energy must be expended in the process. In such a case as that referred to in § 7 this is evidently equal to the work done against electric force in separating the electrified surfaces, and may be represented by the product fl, if f denotes the average force tending to prevent separation, while two points originally in contact are separated to the distance l. Again, when an electric field exists, the forces that act therein can do work on electrified bodies moving in the field. The total amount of work that can thus be done may be called the *energy of the electric field;* it is equal to the energy expended in establishing the field. How a definite power of doing work, in other words, a definite amount of energy, can be associated with a given electric field may be understood by supposing a very light conducting body to move backwards and forwards from one boundary of the field to the other. For example, let A and B

(Fig. 6) be the electrified bodies forming the boundaries of an electric field, and let C be a gilt pith-ball hanging between them by a fine silk fibre. If C comes into contact with A, which we will suppose positively electrified, it will become electrified, and will then be urged to B by the electric forces of the field, an amount of work being done equal to the product of the distance traversed into the average electric force exerted. When C arrives at B, there will be mutual neutralisation of the positive electricity conveyed by it, and of an equal quantity of negative electricity. This will be accompanied by a proportionate diminution of the energy of the field. Next, C will acquire negative electricity from B, and will be urged back to A, the forces of the field again doing work upon it. On its arriving there, the electricity of C and an equal amount of positive electricity will mutually neutralise each other, and so a further diminution of electric energy will take place. If this process is repeated over and over again, it is evident that the final result will be the entire disappearance of electrification, and that simultaneously the power of doing work by motion of the pith-ball will come to an end.

9. Variation of Electric Force with Distance — Law of Inverse Squares. — The most superficial experiments suffice to show that the force between two electrified bodies becomes less when the distance between them is increased. The law connecting force and distance was discovered experimentally by Coulomb, who compared the forces between two small electrified spheres at different distances by means of the torsion-balance. A thin glass or shellac rod, *b d* (Fig. 7), suspended horizontally by a very fine vertical wire, carries a small ball, *b*, of gilt elder-pith at one end, and a second similar ball, *a*, also carried by a glass rod, is supported in a fixed position. The fine wire is fastened above to a torsion-micrometer, shown separately to the right of the figure, whereby the upper end can be turned about a vertical axis through any required angle. The position of the movable ball can be read off on a graduated scale put round the cage of the instrument, A, B, C, D. The position of equilibrium of the arm, *b d*, is that in which the vertical wire is without twist: if it is displaced from this position through an angle a, the moment of the couple which tends to restore it is proportional to this angle. If now the two balls are electrified in the same way, all other electrified bodies in the neighbourhood being connected with the inside of the room, the movable ball is in a field of force extending between the fixed ball and the floor or table, walls, and ceiling. The ball *b* consequently moves away from *a*, and the suspending wire is thereby twisted. By turning the upper end of the wire in the opposite direction, *b* can be brought back to within any required distance of *a*. Let a be the angular distance between the balls when the top of the wire has been turned through the angle A; then the total twist on the wire is $A + a$, and the moment of torsion, proportional to the angle of twist, evidently measures the repulsion between the balls which is balanced by it. If the equilibrium position of the movable ball (corresponding to no torsion) would coincide with the position actually occupied by the fixed ball, then the lower end of the wire must have been turned through the same angle, a, as that which measures the distance between the balls. If A, A_1, A_2 . . . be the angles through which the top of the wire has to be turned in order that the distance between the balls may be a, a_1, a_2 . . . respectively, the following relation is found to hold good, namely—

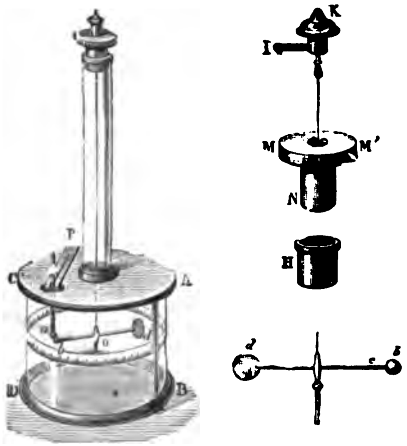

FIG. 7.

$$(A + a)\,a^2 = (A_1 + a_1)\,a_1^2 = (A + a_2)\,a_2^2 = \ldots = \text{constant},$$

provided that the angles α, α_1, . . . are small, and that the balls are small compared with the distance between them.[1]

Since, as we have seen, the angle of twist $A + \alpha$ measures the electric force between the balls, and the distance between them is proportional to α, this result shows that the electric force is obtained by dividing a constant (whose value depends on the degree of electrification and the construction of the instrument) by the square of the distance between the balls. In other words, the *electric force between two small electrified bodies*, of which the electrification remains the same, *is inversely proportional to the square of the distance between them.* In symbols—

$$\frac{f}{f'} = \frac{A + \alpha}{A' + \alpha'} = \frac{\alpha'^2}{\alpha^2}$$

This is known as the "law of inverse squares," and is often referred to as "Coulomb's law." It applies equally to the attraction between oppositely electrified bodies and the repulsion between

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[1] Even when the balls are very small, this result is not quite accurate. Strictly speaking, it is the product fd^2 which is constant, f being the force between the balls at a distance, d (Fig. 8). If l be the common distance of the balls A and B from the centre of the circle, their distance apart is $d = 2l\sin\frac{\alpha}{2}$; the component of the force f, which tends to increase the angle α, is $f\cos\frac{\alpha}{2}$, and its moment, tending to increase the twist of the wire, is $lf\cos\frac{\alpha}{2}$. Hence, putting C for the couple required to keep the wire twisted through unit angle,

$$lf\cos\frac{\alpha}{2} = C(A + \alpha)$$

or

$$fd^2 = 4\,Cl\,(A + \alpha)\sin\frac{\alpha}{2}\tan\frac{\alpha}{2}.$$

As above stated, this quantity is constant for all values of f and d, and for a given instrument C and l are both constant; hence we get $4\,(A + \alpha)\sin\frac{\alpha}{2}\tan\frac{\alpha}{2} =$ constant. If α is a small angle, $2\sin\frac{\alpha}{2}$ and $2\tan\frac{\alpha}{2}$ are both nearly equal to α, and so we have, in this case, the result given in the text. For example, if $\alpha = 30°$, α^2 differs by little more than 1 per cent. from $4\sin\frac{\alpha}{2}\tan\frac{\alpha}{2}$. As the errors of experiment are commonly as great as this, it is not generally worth while to use the more complicated formula.

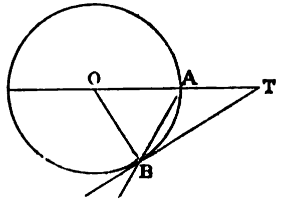

FIG. 8.

bodies electrified in the same way, and is found experimentally to be more and more accurately verified the smaller the bodies experimented on, so long as the force between them is great enough to admit of accurate measurement. Consequently we are justified in supposing that the law of inverse squares is the fundamental law of electric force, and that it expresses accurately the mutual action between the smallest parts of two electrified bodies. This being admitted, it follows that the experimentally measurable forces between electrified bodies must be the resultants of the forces acting according to the law of inverse squares between their smallest parts. In all cases in which it is possible to test practically the results of this assumption it is found to be exactly verified, and in this way more conclusive experimental proof of the law of inverse squares has been obtained than can be derived from the form of experiment employed by Coulomb.

10. Quantity of Electricity—Unit of Electricity.—We have no means of judging of the degree of electrification of bodies except by the mechanical forces they exert by reason of it. If, while the distance between the balls in a Coulomb's balance, and the electrification of one of them, are kept constant, the electrification of the other is varied in such a way as to make the force 2, 3, 4 . . . times what it was before, we say that the *quantity of electricity*, or the *electric charge* of this ball, has become 2, 3, 4 . . . times as great. And if two equal balls placed successively in the same position relatively to a third electrified ball, all other conditions remaining the same, exert the same force, we say that they have equal charges or quantities of electricity.

We shall adopt as the *unit quantity of electricity*, or *unit electric charge*, the quantity that must be imparted to each of two small spheres, so that, when placed unit distance apart in air, they may repel each other with unit force.

The unit of force is the force that produces unit acceleration in unit mass, or, using, as we shall do throughout this work, 1 *centimetre* as unit of length, 1 *gramme* as unit of mass, and 1 *second* as unit of time,—that is, the so-called C.G.S. (centimetre-gramme-second) system of units,—the unit of force is 1 *dyne.* We may accordingly define the unit of electricity in concrete terms as follows:—*The C.G.S. electrostatic unit of electric quantity is that quantity which repels an equal quantity at a distance of* 1 *centimetre in air with a force of* 1 *dyne.* To give some idea of the magnitude of this unit, we may say that if two pith-balls, each weighing 0.5 gramme, are hung from the same point by silk fibres 1 metre long without appreciable weight, and are equally electrified so as to make them separate 10 cm. from each other, the charge of each ball must be about 22 units.

This unit being very small in comparison with the quantities that come into account in many electrical phenomena, another unit, known as the *coulomb*, equal to three thousand million, 3×10^9, times the unit just defined, is often employed, especially in connection with practical applications. The reasons why it is convenient to adopt this particular quantity as the unit for practical purposes will appear afterwards.

It is found by experiment that, if an electrified conducting sphere is allowed to share its electrification with another equal sphere previously unelectrified, the charges of the two spheres, which are necessarily equal by reason of symmetry, are each equal to one-half of that which the first sphere had to begin with. Thus, as stated in § 2, electricity is shared between the two spheres as though it were a material substance, no increase or decrease taking place on the whole.

A point of considerable importance in connection with the sharing of electrification between two or more conductors is, that the ratio in which a given quantity divides itself among them does not depend upon any physical characters of the conductors, as whether they are solid or hollow, or composed of one substance or another, but only on their geometrical properties, size, shape, and relative position. On the other hand, the attraction or repulsion of two electrified bodies depends, not only on their electrification and geometry, but also on the nature of the insulating material between them; so that two balls, electrified to a given degree and placed at a given distance apart, attract or repel each other with a different force in air from what they do if immersed in a non-conducting liquid.

It follows from Coulomb's law that the force between two small bodies possessing electric charges q and q' respectively, separated by the distance r, may be represented by the formula—

$$f = \frac{1}{K} \cdot \frac{qq'}{r^2},$$

where K is a coefficient depending on the properties of the insulating medium in which the bodies are placed, and known as its "specific inductive capacity," or "dielectric coefficient," and also on the units adopted for the measurement of q and f. With the units adopted above, if q and q' are each unity, and r is one centimetre, we have

$$f = \frac{1}{K};$$

but, under these conditions, if the electrified bodies are in air, the force f is one dyne. Hence it follows that the coefficient K must have the value 1 for air. It should be noted that this value is not obtained as the result of an investigation of the electrical properties of air, but simply as a consequence of an arbitrary definition which includes a reference to these properties. The quantity K, however, is not a mere number: it has a physical meaning as well, and expresses the fact that the force due to a given quantity of electricity is influenced in some way, not yet understood, by the nature of the medium within which it is exerted.

In the electrostatic system of electrical measurements, the physical significance of K is not taken into account in the case of air. It is treated as having the merely numerical value unity; and therefore, as not affecting the value of any expression into which it enters as a factor, it is very commonly omitted altogether. For other substances it is recognised as having a value different from unity, but still merely numerical.

If q and q' in the above formula are both of the same sign, their product is positive. Consequently the force acting between two similarly electrified bodies, that is to say, repulsion, is to be considered positive, while a negative value of f is to be interpreted as indicating attraction.

11. Seat of Electric Energy.—It has been pointed out already (§ 8) that the space between two correlative oppositely electrified surfaces, a so-called electric field, may be said to possess energy. It is a matter of great importance, in connection with any inquiry into the ultimate nature of electrification, to ascertain the seat of the condition to which this energy is due,—whether it is a property of the electrified surfaces by which the field is bounded, or whether it is to be sought in the non-conducting medium existing throughout the field itself.

The facts that the operations whereby an electric field is established are carried on with the bodies forming the bounding surfaces of the field, and that the question whether electric force acts at any given point or not depends upon the position of the point relatively to these bodies, long caused attention to be fixed almost exclusively upon them, and the accepted terminology of electric science has been framed almost entirely from this point of view. On the other hand, such facts as those mentioned in § 10,—that the conducting boundaries of a field affect it only by virtue of the geometrical relations of their surfaces, whereas the forces acting anywhere in the field, and therefore its total energy, depend essentially on the physical properties of the intervening non-conducting medium,—seem to indicate the latter as that in which the electrical state is really inherent. There is, in fact, evidence, to be discussed later, tending to show that the medium occupying an electric field is in a state of stress, consisting in a tension at every point, in a direction parallel to that in which an electrified particle at that point would be urged, and of a pressure in all directions at right angles thereto. The forces tending to make the oppositely electrified conductors that bound the field approach each other are supposed to result from this stress, which, however, is not made evident by direct mechanical effects produced in the homogeneous field itself. The case is to some extent comparable with that of an ordinary fluid under pressure: the stress existing in the fluid tends to make the bounding surfaces of the containing vessel move away from each other, but any portion of the fluid itself is in equilibrium under the action of balancing forces.

On this view, what is commonly called the quantity of electricity on a conductor, or the amount of its charge, is to be looked on as an indication of what total force is applied to its surface; and the sharing of the charge of a conductor with another previously unelectrified, as the transfer of the surface of application of part of the total force from the first conductor to the second.

Although it may not be possible as yet to give a complete account of the part played by non-conducting media in the production of electrical phenomena, there can be no doubt that any satisfactory explanation of these phenomena must take account of the functions of non-conductors as well as of conductors.

CHAPTER II.

DISTRIBUTION OF ELECTRICITY.

12. Electric Density.—When the boundaries of an electric field are formed by conductors, the electric forces may cause motion of electricity on or in these without causing motion of the conductors themselves, and it is found that the resulting distribution of electricity is in conformity with the same general law, the law of inverse squares, as that which determines the motion of electrified bodies.

If a quantity, q, of electricity exist within a space of volume, v, the quotient $\rho = q/v$ gives the average quantity in each unit of volume, or the mean *electric density*, of the space. If the space is very small, the density at any point within it cannot differ appreciably from the mean density. Hence we may define the electric density at a given point as being that to which the value of the above quotient approximates when the volume v is taken smaller and smaller, but always so as to contain the point in question. This is expressed by writing for the density at a point $\rho = dq/dv$.

In the same way, if a quantity q of electricity exists on a surface of area s, the average quantity per unit of area is $\sigma = q/s$, and the surface-density of the charge at a given point on an electrified surface is expressed by $\sigma = dq/ds$, the limiting value of the quotient q/s when s diminishes without limit, but so as always to include the given point. When it is necessary to distinguish, ρ is called volume-density, and σ surface-density.

The electric density at each point of a conductor is determined by the force of the electric field in accordance with the general tendency of the field to contract longitudinally, that is, along the direction of force, and to spread transversely,—sometimes one of these tendencies and sometimes the other producing the predominating effect.

13. Gold-leaf Electroscope—Proof-Plane.—The experimental study of the relative electric densities at different parts of a conductor requires the use of apparatus that we have not yet described.

It can be carried out to a considerable extent by means of the two instruments named at the head of this paragraph.

1. *The gold-leaf electroscope* serves for the detection, and to some extent for the comparison, of smaller degrees of electrification than could be recognised by any of the means yet referred to. It consists essentially (Fig. 9) of two strips of gold-leaf suspended close together from an insulated metal stem. To protect the gold-leaves from air-currents and from mechanical injury, they are surrounded by a glass bell-jar, through the top of which the metal stem projects, being terminated by a ball or sometimes by a metal plate. The bell-jar is closed below by a piece of wood, covered at least on the upper side with tin-foil and uninsulated. Two vertical uninsulated strips of tin-foil, or else two metal rods, are carried up inside the bell-jar so that their upper ends are a little above the bottom of the strips of gold-leaf.

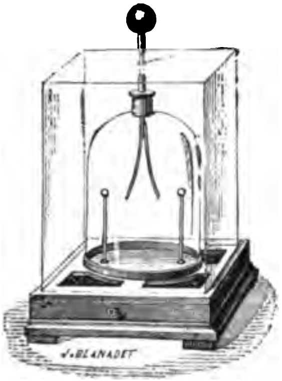

FIG. 9.

When the electroscope is entirely unelectrified, the two gold-leaves hang vertically face to face ; but a very small degree of electrification causes them to stand apart at the bottom, the amount of divergence increasing with the electrification. When the electroscope is charged, it becomes *pro tanto* one of the boundaries of an electric field, and we may say that the divergence of the gold-leaves results from the tendency of the field to contract along the lines of force.

2. The *proof-plane* consists of a small disc of copper-foil or of gilt cardboard fastened at the end of an insulating rod of shellac or glass. If the disc is applied tangentially to any part of the surface of a conductor, it coincides sensibly with the surface at that part and becomes charged with the quantity of electricity previously existing thereon. If it is then removed at right angles to the surface, it carries this charge with it. The relative amounts of the charges acquired by the proof-plane in different experiments can be judged of by their effects on the gold-leaf electroscope. A *proof-ball* may be used instead of a proof-plane, but the relation between the charge received by a ball when put in contact with a

conductor is related to the surface-density at the part touched in a much more complicated way than is the case with a proof-plane. It is evident that the charge taken up by the latter is directly proportional to the surface-density.

14. The Charge of a Conductor in Electrical Equilibrium is purely Superficial: in other words, **An Electric Field does not penetrate the Surfaces of the Conductors which form its Boundaries.**—The following are some of the most striking experiments whereby this important principle may be verified.

1. An insulated metal sphere, A (Fig. 10), is electrified and then enclosed in two metal hemispheres, B and C, of somewhat larger diameter than itself, and held by insulating handles. The hemispheres while kept closed are raised or lowered so as to touch the inner sphere for a moment and are then separated and removed simultaneously. On now testing the ball with the electroscope, it is found entirely unelectrified, while the hemispherical cups are found to be charged.

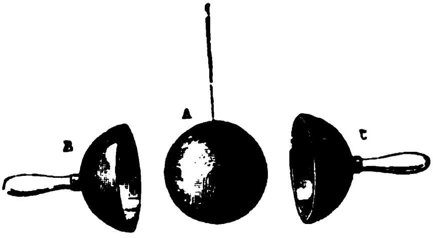

FIG. 10.

This experiment, which is due to Cavendish, will be interpreted somewhat differently according as we fix our attention only on the charge of the ball, or take account of the whole electric field of which the surface of the ball forms one boundary.

In the former case, we may say that at the moment of contact the ball and the enclosing cups form a single conductor, and that the whole of the electricity passes to the outer surface.

In the latter case, we have to consider that the electric field extends from the surface of the ball to the surface (usually the inner surface of the room where the experiment is made) on which the correlative opposite charge to that of the ball is situated, and that the charge passes from the ball to the outside of the cups in accordance with the general tendency to longitudinal contraction of the field.

2. A hollow insulated conductor of any shape (Fig. 11) is electrified. If now a proof-plane is made to touch any part of the outer surface, it is found when removed to be electrified. But if the proof-plane is inserted through a hole, so as to touch any part of the inner surface, it is found on removal to be entirely unelectrified.

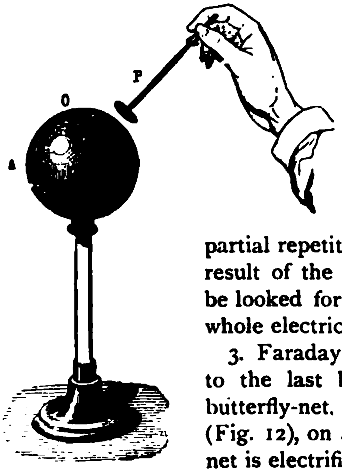

FIG. 11.

The first part of this experiment may be regarded as a partial repetition of that just described. The result of the second part is just what might be looked for when it is considered that the whole electric field is outside the conductor.

3. Faraday made an experiment analogous to the last by means of a sort of muslin butterfly-net, mounted, as shown in the figure (Fig. 12), on an insulating stand. When the net is electrified, it can be shown by the proof-plane that the charge is entirely on the outside. But if the net is turned inside out by means of a silk thread fastened to the point, the charge passes to the other side of the muslin, so as again to be on the outer surface, and this happens every time the net is reversed.

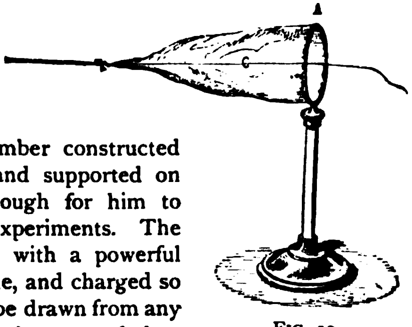

FIG. 12.

4. Faraday had a chamber constructed with conducting walls, and supported on insulating feet, large enough for him to get inside it and make experiments. The chamber was connected with a powerful electrical machine outside, and charged so that strong sparks could be drawn from any part of the outer surface; but nevertheless not the smallest trace of electricity could be detected inside, nor the least sign of electric force, even by the most delicate instruments.

There is no need in this experiment that the surface of the conductor should be perfectly continuous. It may, in fact, be easily performed by means of a parrot-cage formed of a simple network of wire. A gold-leaf electroscope placed anywhere inside the cage remains unaffected even when the cage is strongly electrified.

5. An important experiment correlative to that of Faraday is due to Mr. F. C. Webb. Taking an electrical machine into an insulated chamber, and connecting the rubbers of the machine with the floor of the chamber, he was able to electrify insulated conductors inside the chamber, exactly as he might have done in an ordinary room. When this was done, all parts of the inner surface of the chamber were found to be electrified, but no sign of electrification could be detected outside.

In this form of the experiment, the electric field extends from the surface of the enclosed conductors to the inner surface of the chamber, and is complete inside the chamber. In Faraday's form of the experiment the field extends from the outer surface of the chamber to the inner surface of the room in which it is placed, and is complete outside the chamber.

All these experiments go to establish the fundamental principle of electric distribution, that there is *neither electricity nor electric force* within a conductor: the charge is entirely superficial, being, in the case of a hollow conductor, on the outside or the inside surface, according as the correlative opposite charge is external or internal respectively.

15. Faraday's Ice-Pail Experiment.—A hollow conductor provided with an opening, such as the sphere shown in Fig. 11, or the cylinder shown in Fig. 13 (Faraday used a pewter "ice-pail"), is placed on an insulating support, and an electrified proof-ball is put well inside it and made to touch the side. On withdrawing the ball, it is found entirely unelectrified, but a charge equal to that originally possessed by the ball is found on the outside of the conductor. This process may be repeated any number of times: every time the ball comes out unelectrified, and the charge of the outside of the conductor is increased by as much as was on the ball when put in.

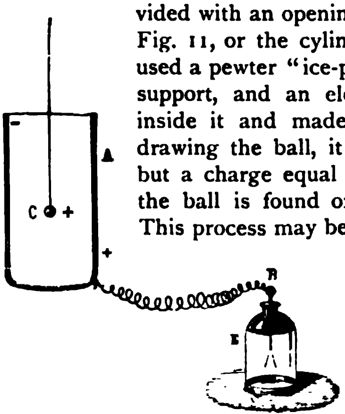

FIG. 13.

In this way an electroscope connected with the hollow conductor can be graduated, if a scale is placed behind the gold-leaves, and the divergence produced by inserting the ball,—always electrified to the same degree,—once, twice, three, four . . . times is noted once for all.

By means of a metal jar and an electroscope thus graduated the rule as to the algebraic addition of electric charges (§ 5) may be verified.

16. In order to understand the ice-pail experiment, it is perhaps simplest to think of the electrified ball as being at rest, and to imagine the metal jar, or other hollow conductor, to be put round it. At first there is an electric field extending between the ball and the surface of the room. When the jar is put round the ball, the field is cut into two independent portions, one extending from the surface of the ball to the inner surface of the jar, the other from the outer surface of the jar to the surface of the room. The two boundaries of the original, undivided field possessed equal, opposite charges; and as each of the two parts into which the original field is divided by the interposition of the jar has for one of its boundaries a boundary of the original field, it follows that the two new boundaries, formed by the inner and outer surfaces of the jar, must each possess a charge equal to either of the charges of the original field. When the ball is allowed to touch the inside of the jar, the opposite boundaries of the internal field come together and this field consequently disappears, but the field outside the jar remains entirely unaffected by this occurrence. These statements are confirmed by the fact that if the gold-leaves of an electroscope connected with the jar are carefully observed, they are seen to diverge more and more as the ball is inserted at the mouth of the jar, but that no increase of divergence is caused by making the ball touch the jar when once it is well inside.

This experiment is often described as depending on the tendency of an electric charge placed inside a hollow conductor to pass to the outside surface; but the fact that the outer surface of the jar appears charged as soon as the ball is put inside, and that the external charge is in no way altered by letting the ball touch the inside of the jar, proves that such a statement of the case is incomplete.

17. Determination of the Electric Density at Different Parts of an Electrified Conductor.—It is proved by experiments such as those described in § 14 that there is no electricity within an electrified conductor, or in other words, that the volume-density of electrification is zero. The surface-densities at different parts may be compared by help of a proof-plane (Fig. 14), which, when applied tangentially at any part and then removed normally, carries with it the charge existing on that part of the surface to which it was applied (§ 13). It only remains to compare the charges of the proof-plane in different cases. This may be done by means of the torsion-balance if a constant charge is given to the movable ball and the proof-plane is introduced in place of the fixed ball. In this case the torsion required to bring the movable ball to a definite position is evidently a measure of the charge of the proof-plane.

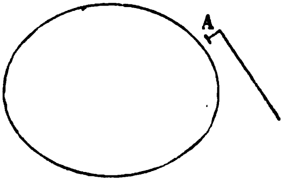

FIG. 14.

But a similar comparison can be made much more easily and quickly by putting the proof-plane inside a metal jar connected with an electrometer graduated as indicated in § 15. If the proof-plane is allowed to touch the jar, the jar and electroscope must be discharged before making a new experiment.

Whatever method is adopted, it is needful to guard against errors that may be caused by the gradual loss of electrification due to imperfect insulation. This may be done by the method of *alternate trial.* To compare the densities at two points, A and B, three experiments are made, first at A, then at B, then at A again, equal intervals of time being allowed to elapse between the first and second, and between the second and third. The charge obtained from B is compared with the mean of the two charges obtained from A.

18. General Conditions determining Surface-Density.—For purposes of calculation, as we have already seen (§§ 9, 10), we may treat electricity as though it were an actual substance, each smallest portion of which exerts force upon every other according to Coulomb's law, and may regard the force exerted upon a quantity, q, of electricity by any electrified bodies whatever as the resultant of the forces exerted by all the elementary portions of the electric charges, each of them being considered as acting independently of all the rest, whether they belong to different bodies or make up the charge of a single body. We shall speak of the value of this resultant for the case of a unit quantity of positive electricity, supposed to be placed at a given point without modifying the distribution of electricity in the neighbourhood, as the *intensity of electric force at the* *point*, or sometimes, more shortly, as the *electric intensity at the point.*

Starting from this point of view, it is merely a question of calculation to determine *à priori* the law of the distribution of electricity on a specified system of conducting surfaces, though it is only in a comparatively small number of cases that the required calculations can be put into a sufficiently simple form to admit of being carried out accurately. From what has been said (§ 14), it will be seen that the problem to be solved may be stated as follows:—What must be the electric density at each point of the conducting surface, or system of surfaces considered, in order that the electric force may be zero at every point within the conductor or conductors?

Another mode of stating the same problem, which, though different in form, is equivalent in effect, is this: What must be the electric density at each point of the surface, or surfaces, in order that the direction of electric force at every such point may be along the normal to the surface? It follows in fact from the properties of conductors that the resultant electric force must be everywhere normal to the surface; otherwise, there would be a component tangential to the surface which would cause movement of electricity along the conducting surface, and consequently there could not be electrical equilibrium.

19. Electrification of an Isolated Sphere—Electric Force due to a Sphere.—Consider an electrified conducting sphere at a great distance from other objects, and therefore at a great distance from the opposite boundary of the corresponding electric field. In such a case the only forces of appreciable magnitude, tending to produce a definite distribution of the charge upon the sphere, are those arising from the mutual repulsion between the several elementary portions of the charge itself. Considerations of geometrical symmetry require us to assume that, in such a case, the charge distributes itself uniformly over the surface of the sphere, or, in other words, that the electric surface-density is uniform. It is easy to prove that such a distribution satisfies the general condition laid down in § 18, in either of the forms in which it is there given.

To determine the action of a uniform spherical charge at an internal point, P, suppose AB and A′B′ (Fig. 15) to be two small areas intercepted at the surface of the sphere by a cone of infinitesimal solid angle, $d\omega$, and with vertex at P. If σ is the surface-density of the charge, the forces at P due to the electricity on these two portions of the surface are $\sigma\,\frac{\text{AB}}{r^2}\,\frac{1}{K}$ and $\sigma\,\frac{\text{A}'\text{B}'}{r'^2}\,\frac{1}{K}$ respectively, and act in opposite directions, r and r' being the distances from P to AB and A′B′ respectively. If we put α for the angle, OAP = OA′P, we have evidently $\text{AB} = \frac{r^2 d\omega}{\cos\alpha}$ and $\text{A}'\text{B}' = \frac{r'^2 d\omega}{\cos\alpha}$, and consequently the forces exerted at P are equal as well as opposite, and so cancel each other. It is obvious that the whole surface of the sphere may be divided up into similar pairs of mutually compensating elements, and therefore that a uniform spherical charge exerts no force at any point within the sphere.

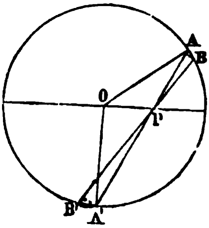

FIG. 15.

To determine the effect of a uniformly electrified sphere at an external point P (Fig. 16), take P′ on OP so that

$$\text{OP}'\cdot\text{OP} = R^2,$$

R being the radius, and let P′ be the vertex of a cone of very small solid angle, $d\omega$, which intercepts infinitesimal areas, A and A', at the surface of the sphere. Putting r for P′A and r' for P′A′, and also α for the angle OAP′ = OA′P′ = OPA = OPA′ (from the similarity, each to each, of the triangles OAP′, OA′P′, and OPA, OPA′ which follows from the common angle at O in each pair of triangles respectively, and from the relation, $\frac{\text{OP}'}{R} = \frac{R}{\text{OP}}$), we get, for the areas A and A'

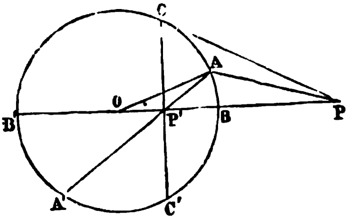

FIG. 16.

$$\frac{r^2 d\omega}{\cos\alpha} \text{ and } \frac{r'^2 d\omega}{\cos\alpha}.$$

Put s = PA and s' = PA′: then the electric forces at P due to the charges on A and A' are

$$\sigma\,\frac{r^2 d\omega}{s^2\cos\alpha}\,\frac{1}{K} \text{ and } \sigma\,\frac{r'^2 d\omega}{s'^2\cos\alpha}\,\frac{1}{K};$$

but, by the similarity of triangles above referred to, we have both $\frac{r}{s}$ and $\frac{r'}{s'} = \frac{R}{\text{OP}}$, and therefore the forces exerted by the elementary areas at A and A′ are equal, and their resultant bisects the angle between AP and A′P, that is, it acts along OP. Obviously the whole surface may be divided into similar pairs of elements, one lying on one side and the other on the other of the plane, CPC′, perpendicular to OP: the resultant effect of each such pair, and therefore of all the pairs, or of the whole spherical surface, must be along OP, as might have been assumed without other proof than the consideration of geometrical symmetry.

We shall get the effective component of the force due to an element of the surface, that is, the component along OP, by multiplying the total force due to the element by cos α. Hence the effective component due to any element may be written

$$\sigma \frac{R^2}{\text{OP}^2} \frac{1}{K} \cdot d\omega,$$

and, to get the effect of the whole sphere, we have only to multiply the constant factor $\sigma \frac{R^2}{\text{OP}^2} \frac{1}{K}$ by the sum of all the values of $d\omega$. This evidently is 4π.

Hence the force at the point P due to the sphere charged to the uniform surface-density σ is

$$f = \frac{4\pi R^2 \sigma}{\text{OP}^2} \frac{1}{K} = \frac{Q}{D^2} \frac{1}{K},$$

if Q be put for $4\pi R^2 \sigma$, the whole charge of the sphere, and D for the distance OP. Hence it appears that the force at an external point due to a charge distributed uniformly over the surface of a sphere is the same as if the whole charge were concentrated in one point at the centre of the sphere. This result is of very great importance on account of its frequent application.

If the point P is close to the surface of the sphere, we have $D = R$, and therefore, in this case

$$f = 4\pi\sigma \frac{1}{K},$$

and the force acts along the normal to the surface, outwards if σ is positive, inwards if σ is negative.

It may be noted that, for a given value of the charge, the force is independent of the radius of the sphere.

It follows that a homogeneous sphere, or a sphere that might be subdivided into concentric homogeneous layers, acts at any external point as though its whole mass were concentrated at the centre. To find the action of a homogeneous sphere at an internal point distant r from the centre, imagine a spherical surface, concentric with the sphere, drawn through the point: then, the force due to the part outside this surface vanishes, and the part inside acts as though it were concentrated at the centre. The action of the whole sphere is therefore represented by

$$f = \frac{\frac{4}{3}\pi r^3 \rho}{r^2} \frac{1}{K} = \tfrac{4}{3}\pi\rho \cdot r \frac{1}{K},$$

if ρ is the volume-density of the sphere. The force is consequently proportional to r.

The case here considered could only occur electrically if the sphere were composed of some non-conducting material, for with a conductor we should have $\rho = 0$. K in the last formula must be taken as the dielectric coefficient of the material of the sphere.

20. Power of Points.—The observed facts as to the electric density in conductors, as has been already stated (§ 18), are such as might be deduced mathematically from the idea of two *electricities*, conceived of as capable of moving easily through conducting substances, but not able to pass through non-conductors, and such that each smallest portion of one repels every other portion of the same kind and attracts every portion of the second kind, according to the law of the inverse square of the distance. The same facts, however, might also be deduced from the conception of stresses in the dielectric field of the nature of a tension between the opposite boundaries and a pressure in directions transverse to the lines of tension. When the opposite boundaries of the field are near together,—as, for example, in the case of the field between two metal plates facing each other at a short distance,—the transverse force tending to widen the field cannot produce much effect, because of the small length of the field, and the resulting form of the field (or, as it is expressed in the other order of ideas, the electric distribution) is mainly determined by the tension. On the other hand, if the boundaries of the field are very far apart, as in the cases considered in § 19, the effect of the transverse pressure becomes so great that that of the tension is inappreciable in comparison.

If an electric field is bounded on the one hand by a flat conducting surface of considerable size, say the wall of a room, and on the other by a metal point presented towards the flat surface at a moderate distance, then both the tension and the pressure in the electric field tend to cause a concentration of it about the point, so that the electric force in the air just outside the point becomes very great. But, as will be shown more fully later, the air cannot sustain more than a limited electric force. When the limit is exceeded, the air gives way to the electric stresses, and all signs of electrification cease, almost or quite as completely as if the bounding conductors of the field had been brought into contact.

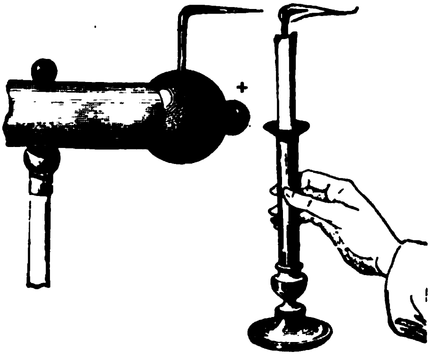

FIG. 17.

This sudden breaking down of a dielectric medium, known as a *disruptive discharge*, is accompanied by luminous phenomena and sound. If the medium is gaseous or liquid, its continuity is of course immediately restored after the discharge ; in the case of a solid, permanent rupture is the result.

When an electric field is formed between a sharp point and an opposing plate, the stress in the field close to the point may be sufficient to cause disruptive discharge although at other parts of the field the stress may be far less. On this account the arrangement mentioned is often used in the construction of electrical apparatus when it is desired to facilitate discharge through a greater or less thickness of air. On the other hand, points or projections of any kind are avoided as far as possible when it is wished to prevent such discharge. By connecting a point and a plate with the two sides of an electric machine, the electric field between them is constantly reproduced, and discharge takes place continuously. The air close to the point having given way, the stress in the air farther off has nothing to counterbalance it, and the air consequently moves in a continuous stream away from the point, constituting what is known as the *electric wind.* This effect is easily made evident by means of a candle-flame (Fig. 17). The breaking down of tension in front of the point may also be illustrated by the *electric wind-mill* (Fig. 18), the points of which move backwards for the same kind of reason that a man pulling hard at a rope tumbles backwards if the rope breaks.

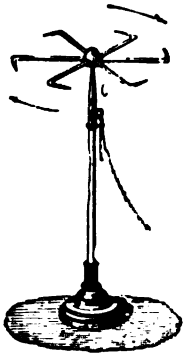

FIG. 18.

If the air near the point is charged with solid or liquid particles, such as exist in smoke, these are rapidly deposited on the opposing surface and the smoke disappears. This process has been practically employed to hasten the deposition of dust or fumes from the air.

The production of the electric wind is accompanied by a sort of hissing sound, and in the dark the point exhibits a projecting tuft of violet light if it is positive, and a small brilliant star if it is negative.

CHAPTER III.

ELECTRIC INDUCTION.

21. Lines of Force.—We have already (§ 18) defined the *intensity of electric force at a given point;* it is the resultant of the forces exerted by every portion of electricity upon a *unit* of positive electricity imagined as being at that point.[1]

The intensity of electric force has a definite magnitude and direction at every point in an electric field. If a line is drawn in the field so that it is, at every point in its course, tangential to the direction of electric force, it is called a *line of force.* It represents the path which would be followed by an electrified particle without mass, entirely free and unacted on by any other force. It is reckoned as running in the direction in which a positively electrified particle could move. Lines of force, therefore, are to be thought of as traversing the whole electric field from the positive to the negative boundary.

One line and one only passes through every point of an electric field. If it were possible for two lines of force to pass through the same point, the resultant electric force at this point would have two directions at the same time, which is absurd.

As we have seen (§§ 14, 18), there is no electric force inside a conductor, and, just outside, the direction of force is everywhere normal to the surface. Consequently, lines of force terminate at conducting surfaces, and always meet them normally. A surface from which lines of force start is a positively electrified surface; one at which they end is negatively electrified.

If it were possible to draw all the lines of force of an electric field, the resulting diagram, being completely full of lines, would be unintelligible; but by drawing representative lines selected according to some easily recognisable system, the general course of the whole set may be indicated, and so the character of all parts of the electric field, as to direction and intensity, can be represented. The rule generally followed is to make the number of lines such that the number traversing the unit area of a surface, supposed perpendicular to their direction at any part of the field, shall represent the electric intensity at that part. Hence, if the electric intensity diminishes as we follow a line of force, this is represented by divergence of the lines, and increase of intensity by convergence. A uniform field, that is, one in which the intensity and direction are everywhere the same, is represented by equidistant parallel straight lines, and reciprocally.

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[1] As it is only in imagination that a unit of electricity is put at the point in question, there is no need to be concerned about possible disturbances of equilibrium which it might cause if put there in reality.

In many cases it facilitates thinking and reasoning about the properties of the electric field to refer the stresses of which it is the seat to the lines of force, and to think of these as having not merely a geometrical significance, but as possessing certain physical properties, namely, a tendency for each line to shorten, and a tendency for separate lines to repel each other.

22. Electrification by Induction.—There being no electric force within a conductor, there can be no lines of force within it. Suppose, then, an unelectrified piece of metal placed in an electric field: the space outside the surface of the metal is to be thought of as full of lines of force, and the space inside the surface as free from such lines. The conductor thus represents a gap in the field, or an interruption of the lines of force. The lines which previously traversed the space occupied by it are each cut into two branches, one branch ending on one part of the surface, and the other, originally the continuation of the same line, starting from another part. But (§ 21) a surface at which lines of force end is negatively electrified, and one from which such lines start is positively electrified. Hence we should expect an insulated unelectrified conductor immersed in an electric field to acquire equal opposite electrifications at different parts of its surface, that part becoming positive which is presented towards the negative boundary of the field, and *vice versâ*.

Such electrification is found experimentally to occur, and is known as *electrification by induction*.

Fig. 19 represents the distribution of lines of force for the case of a metal ball placed in an originally uniform electric field. It will be seen that in this case there is not only an interruption, but also a deflexion of the lines by the ball. This is a necessary consequence of the fact that the lines meet a conducting surface at right angles. Again, a deflexion of the lines that actually encounter the conductor results, on account of the transverse stress in the field or the mutual repulsion of the lines of force, in an appreciable deflexion of some of those which do not encounter the surface.

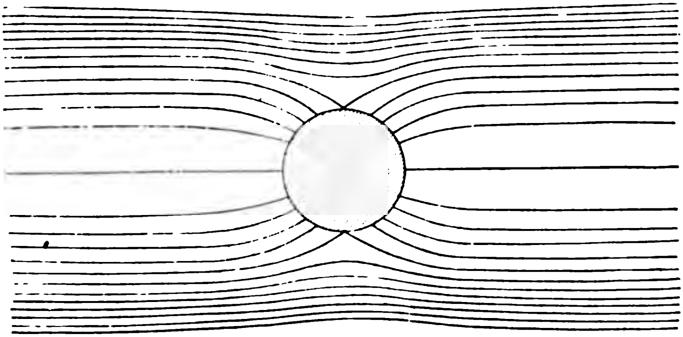

FIG. 19.

23. Laws of Electrostatic Induction.—In order to give a more complete account of electrostatic induction we will consider a special typical case of a kind that frequently occurs in actual experiments.

a. Consider an insulated metal ball or other conductor that has been positively electrified by an ordinary electrical machine, the rubber of which is in electrical connection with the floor of the room. We have, then, an electric field extending from the ball on all sides to the floor, walls, and ceiling, which together constitute the second boundary. To assist the mind in picturing the distribution of force in the field, we may think of the lines of force as if they were elastic threads extending between the two boundaries of the field, each one repelling every other, and their extremities capable of moving freely about on the conducting surfaces to which they are attached, but not able to leave these surfaces, except when, under extreme conditions, by the crowding together of lines, the force in any part of the field becomes so great that the medium breaks down and disruptive discharge takes place.

Mechanical consequences of the suppositions made as to the properties of the lines of force are that they would all meet the boundaries of the field at right angles, and that there would be the greatest concentration of lines at the parts of the field where the opposite boundaries are nearest together, and where, therefore, the lines are shortest. These results correspond to the facts already pointed out (§§ 18, 20), that the direction of electric force close to the surface of a conductor is normal to the surface, and that the electric force is relatively great at any part of an electric field where the opposite boundaries come near together. Again, since the beginning of a line of force corresponds to a positive charge, and the termination of a line to a negative charge, and since the number of beginnings and endings are necessarily equal, the conception of lines of force includes the essential equality of the opposite charges on the two boundaries of an electric field. It was, moreover, pointed out in § 19 that the electric force, just outside a uniformly electrified sphere, is equal to the surface-density of the charge multiplied by 4π and divided by the dielectric co-efficient of the surrounding medium; and we shall see subsequently (§ 43) that this relation between electric force and surface density of electrification is quite general. Hence, a statement of the number of lines of force starting from or ending upon a given area, and a statement of the surface-density of the charge of the same area, are only different modes of expressing the same facts.

b. Let the conductor, A, be completely surrounded by a hollow conductor, B, of any shape whatever. This conductor divides the electric field into two parts, that between A and the inner surface of B, and that between the outer surface of B and the inside of the room. All the lines of force issuing from A are cut by B, so that the number starting from A and ending on the inner surface of B is identical, and so that also the continuation of every line ending inside B is represented by a line issuing from its outer surface and ending on the surface of the room. In other words, the surface of the charged body A, the inner and outer surfaces of the surrounding conductor B, and the surface of the room, are all charged with equal quantities of electricity, the charges of the two external surfaces (that of A and the outer surface of B) being of one kind, and those of the two internal surfaces (the inner surface of B and that of the room) being of the opposite kind.

Since no force acts inside the conductor B, no force can be transmitted through it, and the electric fields inside it and outside it are entirely independent of each other so far as concerns the distribution of force in each, or (what comes to the same thing) the density of charge at any part of the boundary of each. In fact, surface-density and distribution of lines of force in each field are entirely determined by the forces of that field.

c. The two fields—for convenience we may call them *internal* and *external*—are indeed so entirely independent of each other, that either of them may be destroyed without in any way affecting the other. For example, if the electrified body A be displaced so as to come into contact with the inside of B, the internal field ceases to exist, but no electrical effect of any kind is thereby produced outside B. Again, if the conductor B be put into contact with the inside of the room, either directly or by means of a wire, the external field is destroyed, but nothing whatever is thereby altered inside B. To say that an electric field anywhere ceases to exist is the same thing as to say that there all electric force ceases to act; hence, from the point of view which refers electric force to the action at a distance of electric charges, we may express the facts last stated, by saying that the two internal charges mutually neutralise each other so far as regards all action outside the conductor B, and that the two external charges mutually neutralise each other in regard to action inside B.

d. If, after the external field has been destroyed, the conductor B is opened and the electrified body A is withdrawn, what was previously the internal field remains still unchanged as to the number of lines of force in it—that is, as to the quantity of electricity on its bounding surfaces; but the form of the field is altered by the changed relative positions of its boundaries. In accordance with their tendency to shorten and to separate from each other, the lines of force which previously all ended on the inner surface of B come, more and more of them, to end on the outer surface as A is removed to a greater distance. If A is put in contact with the inside of the room, it becomes electrically part of a conducting enclosure surrounding B, and the result is that we have now an electric field whose positive boundary is the surface of the room, and whose negative boundary is the outside of the conductor B. We have then, in a sense, reversed the original field extending between A and the room: the charges remain throughout of the same magnitude as at first (always assuming perfect insulation), but the charge of the room is positive instead of negative, and that of the enclosed conductor is negative instead of positive.

e. If an insulated conductor is placed in the electric field existing between any charged body and an enclosing surface, as that between the electrified body A and the room, it becomes charged, as pointed out in § 22, differently at different parts of its surface, the parts presented towards each boundary of the field acquiring a charge opposite to that of the corresponding boundary. If the conductor is withdrawn, it still remaining insulated, or if the electric field is destroyed, all signs of electrification of the conductor cease, thus proving that the positive and negative charges possessed by different parts were equal in amount. This might also be inferred *à priori* from the consideration that the conductor, having remained insulated, cannot have received positive or negative electricity from without, and therefore the production of one kind in or upon it implies an equal production of the other. The same conclusion also follows very simply from the consideration of lines of force. The negative charge of one part of the surface corresponds to the termination on that part of a certain number of interrupted lines of force, and the continuation of each of these lines must start from some other part of the surface.

f. If the insulated conductor be brought nearer and nearer to the charged body, which for distinctness we will still call A and suppose to be positively electrified, it will interrupt more and more lines of force—that is, its induced charges will become greater. When it gets very near to A, the parts of the lines of force extending between A and it, being very short, cannot repel each other much (the total repulsion between two lines being supposed proportional to their lengths); consequently they crowd more and more about the parts of the two conductors which are nearest together, until at last disruptive discharge takes place between A and the conductor brought near it. This is accompanied by the disappearance of all the lines in question, but the continuations of all these lines still extend between the insulated conductor and the surface of the room or other enclosure. This is the process of sharing the charge of one conductor with another, previously unelectrified, which is brought into contact with it. The total charge is not altered thereby, but merely its distribution.

g. If the insulated conductor, instead of being moved into contact with the conductor A, is made to approach and finally touch the surface of the enclosure, the resulting changes are essentially the same as in the last case, with only such differences as arise from the fact that the conductor is in general passing from a stronger to a weaker part of the field. When it comes sufficiently near to the surface of the enclosure, mutual discharge takes place between the induced positive charge on the conductor and an equal quantity of the negative charge on the enclosure, with the result that the conductor retains only a negative charge, which, together with what remains on the enclosure, is equal to the total original charge of the enclosure. In effect, the enclosure shares its charge with the insulated conductor, just as in the previous case the body A shared its charge. In the one case, the insulated conductor virtually becomes part of the positive boundary of the field; in the other, it virtually becomes part of the negative boundary.

h. If the insulated conductor, after coming into contact with the surface of the enclosure, be separated from it again, the lines of force from A end, some of them on the insulated conductor, and the rest on the surface of the enclosure. Let now A be brought into contact with the enclosure: the lines of force between it and the enclosure shorten up to nothing and the electrification represented by them disappears; but the lines of force which just before ran from A to the insulated conductor remain, but their positive ends, instead of being confined to A, are now free to spread over the whole surface of the enclosure. We thus get, as in Case *d* above, a reversed field, which, however, contains a smaller number of lines of force than the original field, instead of the same number, as in the previous case.

It is easy to see that the results we have mentioned as being produced by displacing a conductor in an electric field may also be produced by connecting the conductor with one or other boundary of the field by means of a wire. In some cases this method would be experimentally more effectual than moving the conductor.

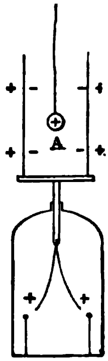

FIG. 20.

24. Experimental Verifications.—Case *a*.—Suspend a good-sized metal ball, or conductor of any shape, by a silk cord, and electrify it as described, and place it near one of the walls of the room. The surface-density of the charge on different parts of the ball and of the walls can be easily compared by means of the gold-leaf electrometer and proof-plane (§ 17). Cases *b*, *c*, *d*.—The various results stated under these heads can be verified by means of a metal jar (or "ice-pail") connected with a gold-leaf electroscope (Fig. 20), together with a second electroscope and a proof-plane. The conductor A may be a small metal ball at the end of a silk thread.

Case *e*.—For this experiment an insulated metal cylinder with rounded ends is convenient. It should be put lengthwise between a good-sized electrified body and the wall of the room, not very far from either. For this purpose any large, flat, conducting surface, electrically connected with the room, may represent the wall. The induced electrification of the cylinder can be tested with a proof-plane and electrometer, and the increase of induction with decrease of distance can be similarly proved. Another way of making the experiment is to hang at intervals along the cylinder pairs of pith-balls attached to very fine wires (Fig. 21): the greater or less divergence of the pith-balls indicates the greater or less density of charge at the different parts of the cylinder.

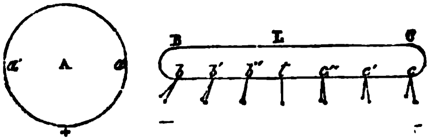

FIG. 21.

Cases *f*, *g*, *h*.—An electrified ball hung inside a metal jar may represent the conductor A, and a proof-plane also placed inside the jar may represent the insulated conductor. Or, what is better, the inside of the room may represent the enclosure, and the apparatus employed in the verification of Case *e* may be used.

25. Inductive Action on Bad Conductors.—The effects of induction are less distinct with badly conducting substances. Such a body placed in an electric field seems to behave as though it were composed of a multitude of minute conductors separated by an insulating medium, each of which is acted on inductively in the way already described, the side nearest the positive boundary of the field acquiring a negative charge and *vice versa*. The perceptible effects are as though the body became electrified in the same manner as a conductor occupying the same space would do, but to a smaller extent. A non-conductor in an electric field is often said to be *polarised*.

If the inductive action last for only a very short time, as soon as it ceases the polarisation disappears, and the body returns to its natural state. But if it lasts for a good while, the body may exhibit for some time afterwards two oppositely electrified regions. This effect may be supposed to result from a more or less complete mutual neutralisation of the opposite charges of neighbouring conducting particles taking place through the intervening imperfectly insulating medium.

26. Conducting Sphere in a Uniform Field.—In the case of a conducting sphere in a uniform electric field, it is not difficult to calculate the surface-density of electrification required to satisfy the general condition, pointed out in 18, that the electric force at any point within the conductor shall be zero. The force at any such point must evidently be the resultant of the forces which would be exerted there by the field and by the induced electrification of the sphere if these acted separately. And as the force of the field is, by supposition, everywhere the same in magnitude and direction, the problem comes to be to find such a surface-distribution of electricity on the sphere as would, if it existed independently, produce at all internal points a uniform force equal and opposite to that of the inducing field. We get this by supposing two spheres, at first coincident, of equal opposite electric densities, $+\rho$ and $-\rho$, uniform throughout, to undergo a small relative displacement AA′ (Fig. 22), the positive sphere moving in the direction of the force of the field, and the negative one in the opposite direction. The result of this is to give, as shown in the figure, a central portion, throughout which the electric densities $+\rho$ and $-\rho$ are superposed, and where therefore the resultant density is zero; while there is a layer of density $+\rho$ at B symmetrically distributed with respect to the diameter parallel to the force of the field, and a similar layer of density $-\rho$ on the opposite side of the sphere. The diameter just mentioned may be called the axis of electrification of the sphere, or simply the axis; the points where it cuts the surface may be called the poles; and a great circle perpendicular to the axis and half way between the poles (passing through C and D in the figure), the equator of the sphere. The two electric layers, called *displacement-layers*, which respectively cover the two hemispheres, have a maximum thickness equal to AA′ at the poles. The equator separates the positive from the negative layer, and the thickness there is nothing.

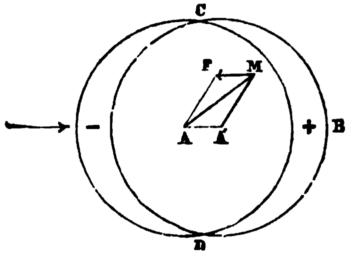

Fig. 22.

The required electrification of the conducting sphere in the case we are considering is given by supposing the two halves of its surface oppositely electrified to a surface-density proportional to the thickness of these displacement-layers.

To prove this, let us investigate the electric force at a point, M, within the two nearly coincident spheres. The force due to the positive sphere, with centre A′, is $\frac{4}{3}\pi\rho \, . \, \text{A}'\text{M}. \, \frac{1}{\text{K}}$ (§ 19), acting in the direction A′M, and that of the negative sphere, with centre A is $\frac{4}{3}\pi\rho \, . \, \text{MA}. \, \frac{1}{\text{K}}$, in the direction MA. In the triangle A′MA, the sides A′M and MA are respectively parallel and proportional to the two forces acting at M, and consequently the third side, A′A, is parallel and proportional to their resultant. The resultant, $\frac{4}{3}\pi\rho \, . \, \text{A}'\text{A}. \, \frac{1}{\text{K}}$, is therefore independent of the position of the point M—that is, the force due to the given distribution is uniform in magnitude and direction at all internal points. Further, it is evident that the force at any point, internal or external, would be unaltered by removing the coincident portions of the two spheres and leaving only the displacement-layers; for as the density throughout the region of coincidence $=0$, the force must be what the displacement-layers would exert if they existed alone.

If we put σ_0 for the maximum surface-density, that at the pole B, we have

$$\sigma_0 = \rho . \, \text{AA}',$$

and for the force at an internal point due to the electrification of the sphere $-\frac{4}{3}\pi\sigma_0 \frac{1}{\text{K}}$ (acting in the direction A′A), opposite to the force of the field. Putting f for the force due to the field alone, and considering that the resultant force inside the sphere is nothing, we get

$$f - \frac{4}{3}\pi\sigma_0 \frac{1}{\text{K}} = 0,$$

which determines σ_0 when f is given.

To get the density elsewhere than at the poles, we must remember that the thickness of the displacement-layers is everywhere AA′ if measured parallel to the axis; hence, as is easily seen on drawing a figure, the thickness measured along the radius at any point is AA′ cos α, if α is the angle which the radius through the point makes with the axis. The surface-density being proportional to the radial thickness of the displacement layer, is then given by

$$\sigma = \sigma_0 \cos a = \frac{3fK}{4\pi} \cos a.$$

The value $a = 90°$ corresponds to points on the equator, where, as we have already seen, the density is nothing.

To get the action of the displacement-layers at external points, we may replace them by the two overlapping spheres with centres at A and A′. Each of these would act as though its whole mass were concentrated at the centre (§ 19); hence, the effect is the same as would be produced by two equal masses of opposite signs concentrated in the two points A and A′. The lines of force for this case are shown in Fig. 19 (§ 22), which must be considered as a section of the field of force by a plane through the axis. The surface of the sphere is supposed to be divided into zones having equal charges.

27. Attraction of Light Bodies.—Let us return to the experiment mentioned in § 1, of the attraction of bodies that have not been previously electrified, when a body that has been electrified by friction or otherwise is brought near. It is evident that in such a case electrification by induction precedes attraction, and that what is really observed is attraction between opposite electric charges. It may indeed be taken as a fact proved experimentally that ordinary matter as such is not acted upon by electric force, which acts only between different quantities of electricity.

If a pith-ball pendulum is used to illustrate electric attraction, the effect is somewhat different according as the suspending thread is of linen or cotton, which are pretty good conductors, or of silk, which is a non-conductor. In the former case, the pith-ball is in electrical connection through the thread with the inside of the room, which commonly forms one boundary of the electric field; the pith-ball, therefore, forms a part of this boundary, and is naturally strongly attracted when the electrified body which forms the other boundary is brought near. When the suspending thread is an insulator, we have the pith-ball merely subjected to the electric field extending between the electrified body and the room. It is electrified inductively, as described above (§§ 22, 23 *e*), and it moves in obedience to the resultant force, which acts towards the stronger part of the field. As the means of testing whether a body brought near to it is electrified or not, a pith-ball suspended by a conducting thread is accordingly more sensitive; but if we want to find out *how* a body is electrified, we must use a pith-ball hung by a non-conducting thread: we electrify the ball in a known manner, positively or negatively, and observe whether repulsion or attraction takes place when the body to be tested is brought near.

28. Inductive Action on the Gold-Leaf Electroscope.—A very common example of electric induction, and one sufficiently important to make it desirable to refer to it specially, is presented by the gold-leaf electroscope (Fig. 9). Suppose the instrument, as usual, to be standing on the table, and that an electrified body is held over it at a distance of several inches, or even a foot or two. An electric field extends between the electrified body and the table, and envelops the electroscope. Lines of force run between the electrified body and the cap, and the continuation of the same lines between the leaves and the table, and the opening of the leaves is to be attributed to the tension in these latter lines, together with the repulsion between the lines proceeding from one leaf and those proceeding from the other.

If the cap is now touched with the finger, and thereby put electrically into connection with the table, the lines of force between the latter and the gold-leaves disappear and the divergence ceases, but at the same time the field between the electrified body and the cap of the electroscope becomes stronger than before. After removing the finger, let the electrified body be removed; the electric field is thereby distorted and we may think of the body as dragging after it the lines of force that are attached to it and stretching them out. As they become longer, their mutual repulsion asserts itself more strongly; they spread from the cap in greater or less numbers over the rest of the electroscope, some of them pass to the gold-leaves, and these again open. This action is carried to its limit if the electrified body is put into electrical contact with the room, but it comes to sensibly the same thing if it is removed in any direction to a much greater distance from the electroscope than the surface of the table or other parts of the second boundary of the electric field. It is important to note that the electrification which is now indicated by the electroscope is *opposite* to that of the electrified body brought near it at the beginning of the experiment. For definiteness, assume the body to have been positive, and therefore the table negative: in the first stage of the experiment, the divergence of the gold-leaves indicated lines of force running from the leaves to the table; the final divergence indicated lines running from the table to the leaves.

When an electric field of a known kind has been established between the electroscope and the table—that is, when the electroscope has been charged with a known kind of electricity—it serves as a delicate test to distinguish the nature of the electrification of any object brought near it. If a body, slowly moved up towards the cap of the electroscope from a considerable distance, causes the leaves to diverge continuously more and more, we may conclude that it is electrified in the same way as the electroscope. If, on the other hand, it makes the leaves gradually fall together, collapse completely, and then open again, we may conclude that it is electrified oppositely to the electroscope. In the first case, the electric field due to the body to be tested is superposed upon the field due to the electroscope and strengthens it. In the second case, contrary fields are superposed: as the second field is gradually strengthened, there is first weakening, then neutralisation, and finally inversion of the original field of the electroscope. If an oppositely electrified body is quickly brought near the electroscope, the inversion of the field may occur before the leaves have had time, by falling together, to indicate the previous weakening of the original field. In such a case we may mistake the sign of the electrification. It is important, therefore, to bring up the body slowly towards the electroscope and to note the first visible effect. A mere diminution of divergence when a body is brought near the cap of the electroscope is not trustworthy evidence of contrary electrification. If the hand, or any uninsulated conductor, be held a little way above the cap, it alters the electric field, and so causes a diminution of the divergence of the leaves, in a way that is not difficult to follow out.

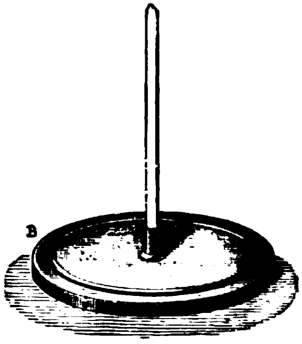

FIG. 23.

29. The Electrophorus.—The action of the electrophorus (Fig. 23) depends essentially on electrical induction. The instrument consists of a flat, smooth plate of resin or ebonite (called the *cake*), resting on a metal plate (the *sole*), and of a second metal plate (the *cover*), of rather less diameter than the cake, provided with an insulating handle.

To use the electrophorus, the cake is electrified by beating it with a catskin, the cover is put on, connected with the sole, insulated from it again, and lifted by the insulating handle. The cover is now found positively electrified (if the cake is negative as usual), and its charge may be transferred to any other conductor as desired. To renew the charge, it is only necessary to replace the cover, connect it for an instant with the sole, and raise it again by the insulating handle, and this can be repeated a great number of times in succession without its being needful to renew the electrification of the cake, which only undergoes a very gradual loss due to imperfect insulation.

The electrical changes that take place may be described as follows:—By friction with the catskin, an electric field is set up, originally between the catskin, which forms the positive boundary, and the resin, which forms the negative boundary. When the catskin has been removed to a distance, the lines of force no longer extend all the way from it to the cake, but are interrupted by neighbouring conducting objects, usually by the table on which the electrophorus stands. They then shorten, and the electric field contracts so as to be confined almost entirely to the space occupied by the cake; the upper surface of the sole becomes the positive boundary, and the upper layer of the resin the negative boundary. The lines of force are thus short lines, practically straight, running up through the cake. When now the cover is laid on and connected with the sole, the positive ends of the lines are to a great extent transferred from the sole to the cover, as they thereby become still shorter than before. A great part of the field is thus transferred to the upper side of the cake, where it occupies the thin stratum of air between the face of the cover and the cake, and also probably a thin layer of resin itself near the upper surface. When the cover is raised, the field is drawn out and soon divides into two, one extending from the cover to the surface of surrounding objects, and the other, as before, from the sole to the upper part of the resin.

CHAPTER IV.

ELECTRICAL POTENTIAL.

30. Conditions of Electrical Equilibrium.—If two electrified conductors are brought into contact, or, remaining at a distance, are connected by a third conductor, as, for instance, by a long fine wire, there is in general a change in the electrical state of both; whatever signs of electrification were previously shown by one of them become weaker, while those exhibited by the other become stronger. The effect is what is expressed by saying that electricity passes from the first to the second. In special cases, however, it may happen that no electrical change is produced—that no electricity passes either way. We then say that the connected conductors were in *electric equilibrium.* In any case, electric equilibrium is established in a very short time after connection is made.

Electric equilibrium, when attained, evidently consists in such a distribution of electricity that the resultant electric force at every point of the conductor, or system of conductors in question, is equal to nothing; for, if there were electric force at a point of a conductor, there would be displacement of electricity, and consequently equilibrium would not exist.

The question then arises, On what does electric equilibrium depend? What is the condition which, when possessed in the same degree by two conductors, makes it possible to connect them together without any alteration of their electric state? It is not equality of total charge, nor yet equality of surface-density, for two unequal spheres are not in equilibrium when they have either equal charges or the same surface-density.

31. Electric Potential.—The required condition, which is distinct from any that we have so far considered, is what is known as *electric potential.* This term expresses a condition in relation to electrified bodies which is analogous to that denoted by temperature in the case of heat. Just as bodies are said to have the same temperature if no transfer of heat from one to the other occurs when they are put into contact, so electric conductors between which no transfer of electricity takes place when they are connected are said to have (or to be at) the same potential. On the other hand, if two conductors which have different potentials are connected, a passage of (positive) electricity takes place from the one which has the higher to that which has the lower potential, until equality of potential, and therewith electric equilibrium, is reached.

To speak of any system of bodies as being in a state of electric equilibrium, is equivalent to saying that there is no electric force tending to cause movement of electricity in any way among them. But all the sensible phenomena by which we become aware of the electrical state of bodies—such as the motion of an electroscope, or the passage of an electric spark—are evidences of electric force. Hence it follows, from what we have just said, that if all the objects about us are at the same potential, they will not exhibit any electrical phenomena, for, being at the same potential, they are all in electrical equilibrium. This would still be the case, however high, or however low, the common potential might be. It consequently appears that we have no means of recognising the absolute potential of bodies, but only the *differences* between the potentials of different bodies.

The strict definition of electrical potential is founded upon the fact, just alluded to, that the existence of electric force implies a difference of electric potentials. We may define electric potential as a property of space whose value at any given point, above or below that of some point or body chosen as an arbitrary zero of potential, is susceptible of numerical statement, and such that the line through that point along which it decreases most rapidly is the direction of electric force, and that the rate of its decrease in any direction is equal to the intensity of force in that direction.

This definition, being necessarily expressed in general terms, may perhaps fail to convey any definite meaning on a first reading. If so, the following comparisons may help to make the matter clearer. Adopting a similar form, we might say that temperature is a property of space whose value at any given point above or below an arbitrary zero (say the temperature of melting ice) is susceptible of numerical statement, and such that the direction through that point in which heat would flow (if the point were within a conducting material) is the direction in which temperature decreases most rapidly, and that its rate of decrease in this direction is proportional to the intensity of the flow of heat. [The fact that we are not in the habit of speaking of *thermal force* as determining the transfer of heat between bodies of different temperatures makes it impossible to keep the wording of the definition of temperature parallel throughout to the definition of potential.]

Potential may also be usefully compared with surface-level in the case of liquids ; for the passage of electricity, from a conductor of higher to one of lower electrical potential when they are connected by a wire, is very closely analogous to the flow of water from a cistern at a high level to one at a lower level when a connecting pipe is opened between them. In fact, the analogy is so complete, that the expression "electrical level" is not infrequently used instead of electric potential.

In connection with the comparison between potential and temperature, it is to be noted that whereas the temperature of a body is not altered by mere change of position, so long as it does not lose or gain heat, the electric potential of a body is in general altered whenever it is moved in an electric field. Another point of difference is that, whereas important changes in the physical properties of matter are produced by changes of temperature, nothing of the same kind results from rise or fall of potential. Great differences of potential between neighbouring parts of a dielectric medium produce various physical effects, such as the mechanical shattering that accompanies discharge ; but any portion of matter throughout which potential is uniform remains unaffected, however high or low the potential may be.

32. Electric Force and Potential.—In order to see what is involved in the definition of potential given in § 31, we will apply it to a special case where the electric force is known, that of an electric field bounded by two concentric spherical conducting surfaces. Let P (Fig. 24) be a point in the field at a distance, r, from the common centre, O, of the spherical boundaries. If Q be the charge on the inner surface, and therefore $-Q$ the charge on the outer surface, the electric force at P will be $\frac{1}{K} \cdot \frac{Q}{r^2}$ acting in the direction OP (because the charge of the inner sphere will act at P as though it were concentrated at O, and the charge $-Q$ on the outer surface will exert no force at P). Hence the potential at P must be a quantity such that its rate of decrease is most rapid in the direction OP (or r), and is equal to $\frac{1}{K} \cdot \frac{Q}{r^2}$. Putting V for the potential at P, and V' for the potential at a point P′, distant r' from the centre, the change of potential as we pass from P to P′ is $V' - V$, and the average rate of change is this divided by the distance PP′, or $\frac{V' - V}{r' - r}$. If the point P′ be taken nearer and nearer to P, the distance PP′, or $r' - r$, becomes smaller and smaller, and may finally be denoted by dr, the difference of potentials, $V' - V$, simultaneously becoming infinitely small so as to be represented by dV. Hence we see that the rate of variation of the potential *at* the point P is expressible by the ratio dV/dr. The two factors of this ratio, however, represent increments of the corresponding quantities; hence, to express the rate of *decrease* of potential, which by the definition is the electric force at P, we must prefix a negative sign. We thus get finally for the electric force at P

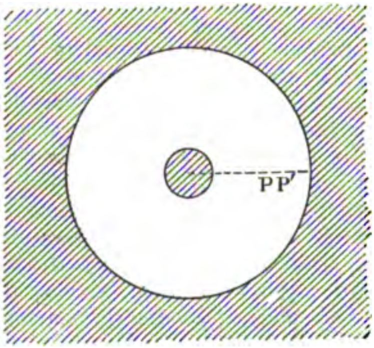

FIG. 24.

$$f = -\frac{dV}{dr} = \frac{1}{K} \cdot \frac{Q}{r^2}.$$

By the definition of § 31, the rate of decrease of potential in any direction must give the electric force in that direction. Therefore, if dV be again the change of potential that takes place in the distance ds, measured from P in any direction, s, making an angle θ with r, and if f_s be the component of electric force in this direction we must have

$$f_s = -\frac{dV}{ds}.$$

But resolving the resultant force f along s and at right angles thereto, we have also

$$f_s = f \cos \theta.$$

Consequently,

$$\frac{dV}{ds} = \frac{dV}{dr} \cos \theta,$$

and since $\cos \theta$ is always less than unity when θ is greater than 0, dV/ds is less than dV/dr, or the rate of change of potential along s is less rapid than along r.

33. Equipotential Surfaces.—Consider the case where θ is a right angle, that is, the case where ds is perpendicular to r, and therefore lies along the surface of a sphere passing through P, and with centre at O. We know that, in this case, the component force $f_s = 0$, and consistently with this, the last equation gives $\frac{dV}{ds} = 0$ when θ is a right angle.

Put into words, this gives the result that there is no variation of potential along the surface of a sphere with centre O, or that the potential has the same value at all points of such a surface; or again, more shortly, that a spherical surface with centre O is an *equipotential* surface. And similar reasoning would show that in all cases a surface that is everywhere at right angles to the direction of electric force is an equipotential surface.

Conversely, an equipotential surface must be such that no electric force acts along it from one point to another of the surface. Hence electric force (if any) acts everywhere normally to an equipotential surface.

It follows that the surface of any conductor or system of conductors in electrical equilibrium is an equipotential surface, and that electric force (if any) is everywhere normal to such a surface.

34. Work Done by Electric Force.—The formula $f_s = -dV/ds$ may be said to embody the definition of potential. If we multiply both sides by ds and write it

$$f_s \, ds = -dV,$$

it admits of a simple physical interpretation. The product of the force f_s into the infinitesimal displacement ds in the line of action of the force, would evidently be the work done by electric force on a unit of positive electricity moved from P through the distance ds. We may accordingly write

$$dW = f_s\, ds = - dV;$$

whence it appears that the rate of decrease of potential in any direction is identical with the rate at which work is done by electric force upon a unit of positive electricity displaced in that direction. It follows that the total work done upon a unit of positive electricity transferred from a given point, A, to another point, B (it being understood that the electric force is not altered in any way by the transfer), is equal to the whole decrease of potential from A to B, or, in symbols

$$W_{AB} = \int_A^B dW = \int_A^B - dV = V_A - V_B.$$

If the quantity of electricity transferred is any quantity Q, the work done is, of course, equal to Q times that which corresponds to the transfer of a positive unit. Thus, if we put W for the electric work done, when any quantity, Q, of electricity is transferred from a point where the potential is V to a point where the potential is V', we have

$$W = Q\,(V - V').$$

From this it appears that the electrical work done when electricity passes from any point of one equipotential surface to any point of another, depends only on the difference of potential of the surfaces, and not on the particular points between which the passage takes place, nor upon the path by which it occurs.

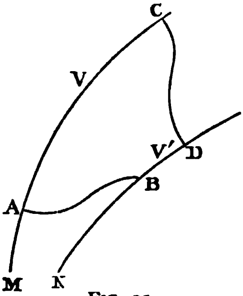

FIG. 25.

The proof of this very important proposition is contained in what has been said already, but it may be well to consider it from another point of view. Let M and N (Fig. 25) be portions of two equipotential surfaces having respectively the potentials V and V', then what we have to prove is that, if a quantity of electricity be transferred from any point, A, of the surface M to any point, B, on the surface N, the work done by electric forces is the same as if an identical quantity of electricity were carried from any other point, C, of the first surface to any point, D, of the second. That this is necessarily the case follows easily from the principle of the conservation of energy, combined with the conception of an equipotential surface. For if it be possible for these two amounts of work to differ, let the electric work along AB, per unit of electricity, exceed that along CD by any finite amount, say, 10 ergs: then it will require an expenditure of energy less by 10 ergs to carry a unit of electricity back from N to M by the path DC than to carry it by the path BA. Consequently we may carry a unit of electricity from M to N along AB, during which process an amount of work, say W ergs, is done by electric force, then transfer it from B to D along the equipotential surface N—during this process, since no force acts between any points on the same equipotential surface, no work will be done—next carry it to the surface M along the path DC, doing $W - 10$ ergs of work against electric force, and lastly restore it to the starting-point A, by carrying it along the equipotential surface M. As the result of this series of operations, no permanent electrical change whatever will have been produced, since all the electricity taken from A has been brought back to it, but 10 ergs more work will have been done by electric force than has been done against it, or there has been a gain of energy to the extent of 10 ergs, without any corresponding expenditure; and by repeating the same operation again and again, it would be possible to gain any amount of energy, a result inconsistent with the principle of conservation of energy. Hence we conclude that it is not possible for the amount of electric work done during the passage of electricity from one equipotential surface to another to vary with the path followed.

35. Difference of Potentials defined in terms of Electrical Work.—The proposition arrived at in § 34 furnishes a new mode of defining difference of potentials which may be put in the following form:—*The electric potential at any point,* A, *exceeds the potential at any other point,* B, *by an amount equal to the work that would be done by electric force upon a unit of positive electricity transferred from* A *to* B, it being understood that no change of force is caused by the transfer. If we put W for the work done when a quantity of electricity, Q, is transferred, the above statement is equivalent to the equation

$$V_A - V_B = \frac{W}{Q}.$$

33. Difference of Potentials as depending on Electric Charge. —Returning to the consideration of an electric field contained between two concentric conducting surfaces (Fig. 26), such as that referred to in § 32, it is easy now to calculate the difference of their potentials when the charge Q and the radii of the surfaces, say a and b, are given. As we have seen, the force at any point, P, distant r from the centre, O, is $\frac{1}{K} \cdot \frac{Q}{r^2}$, and is directed along OP. If we take a point, P′, very near P in OP produced, the force there is $\frac{1}{K} \cdot \frac{Q}{r'^2}$, and is nearly equal to the force at P. The average electric force along PP′ is therefore very near to either of these values, and still more nearly equal to the intermediate value $\frac{1}{K} \cdot \frac{Q}{rr'}$. Adopting this expression, we have for the electrical work corresponding to the passage of a unit of positive electricity from P to P′

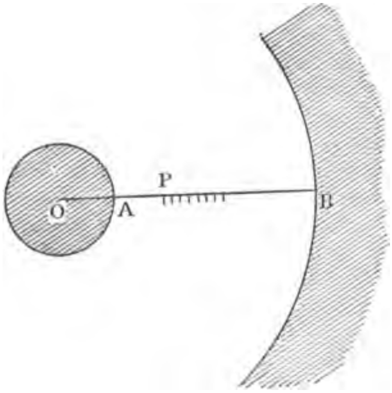

FIG. 26.

$$\frac{1}{K} \cdot \frac{Q}{rr'}(r' - r) = \frac{Q}{K}\left(\frac{1}{r} - \frac{1}{r'}\right).$$

Similarly, for the work done between P′ and P″, a point still farther from O at the distance r'', we have

$$\frac{Q}{K}\left(\frac{1}{r'} - \frac{1}{r''}\right),$$

with precisely similar expressions for each of the very small segments into which the line AB, extending from one boundary of the field to the other, may be subdivided. The total work done by electric force when the unit of electricity is carried from A to B, or the amount by which the potential V at A exceeds the potential V' at B, would be found by adding together all such expressions as the above formed by giving to r in succession every value from $r = a$ to $r = b$. Accordingly we get

$$V - V' = \frac{Q}{K}\left[\left(\frac{1}{a} - \frac{1}{r_1}\right) + \left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \ldots + \left(\frac{1}{r_n} - \frac{1}{b}\right)\right].$$

In this sum, every term, except the first and last, enters once negatively and once positively, and therefore disappears from the result, which becomes

$$V - V' = \frac{Q}{K}\left(\frac{1}{a} - \frac{1}{b}\right).$$

Similarly, for the difference of potentials between a point at an intermediate distance, r, from the centre O, and the outer surface, we should have

$$V_r - V' = \frac{Q}{K}\left(\frac{1}{r} - \frac{1}{b}\right).$$

If the radius b of the outer boundary of the field is very great, its reciprocal $1/b$ is very small, and may be neglected in comparison with $1/r$; hence, in such a case, the last expression becomes

$$V_r - V' = \frac{Q}{Kr}.$$

And if the outer boundary of the field, without being spherical, is of great extent and everywhere very distant from the inner spherical surface,—as, for example, the walls, floor, and ceiling of a large room near the middle of which a small electrified ball is suspended,—it is evident that the force within a moderate distance of the sphere will be sensibly the same as in the last case, and that the force at great distances from it will be very small; consequently the difference of potentials between a point at distance, r, and the outer boundary of the field will still be $\frac{Q}{Kr}$.

The potential of the earth or of any conductor connected with it is usually taken as the conventional zero of potential. Accordingly if V' is the potential of the walls of a large room, as in the case supposed above, we may put $V' = 0$, and the potential at any point whose distance, r, from the centre of an isolated electrified sphere is small compared with its distance from the nearest point of the surrounding surface becomes

$$V_r = \frac{Q}{Kr}.$$

37. Potential due to Several Charges.—Suppose now that, in the neighbourhood of a given point, P, there are several electrified bodies, each so small compared with its distance from P that its charge acts as though it were concentrated at a single point, and suppose also that the point P is very distant from any other electrified bodies ; then the potential at P will be the algebraic sum of the potentials due to the several electrified bodies, each supposed to act by itself. Thus the expression for the potential at the point P becomes

$$V = V_1 + V_2 + \ldots = \frac{1}{K}\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \ldots.\right) = \frac{1}{K}\Sigma\left(\frac{Q}{r}\right),$$

where the symbol $\Sigma\left(\frac{Q}{r}\right)$ means that each charge is to be divided by its distance from the given point, and that the sum of the quotients is to be taken.

In general, the distribution of electricity in the neighbourhood of a given point cannot be taken account of by treating it as being concentrated at a finite number of separate points. The process of calculating the potential at a point is then equivalent to considering the whole quantity of electricity as subdivided into an infinite number of infinitesimal portions, dQ, each at its own distance, r, from the point, and determining the value of the integral $\int\frac{dQ}{r}$ for the whole.

As a special case, suppose a quantity of positive electricity, Q, concentrated at a point A (Fig. 27), and a quantity Q' at A′ ; the potential at any point, P, whose distances from A and A′ are r and r' respectively, is then given by

$$V = \frac{1}{K}\left(\frac{Q}{r} + \frac{Q'}{r'}\right).$$

If we make V constant, this equation characterises an equipotential surface. All such surfaces, in the case supposed, are symmetrical about the line AA′ as an axis, and Fig. 27 represents a section by a plane through the axis of a system of equipotential surfaces corresponding to the values $Q = +20$ and $Q' = -5$, the potentials of consecutive surfaces differing by unity from each other. The surface given by the equation

$$\frac{Q}{r} + \frac{Q'}{r'} = 0,$$

or the surface of zero-potential, is a sphere S surrounding the point A′. At all points inside this surface the potential is negative; and at all points outside it is positive.

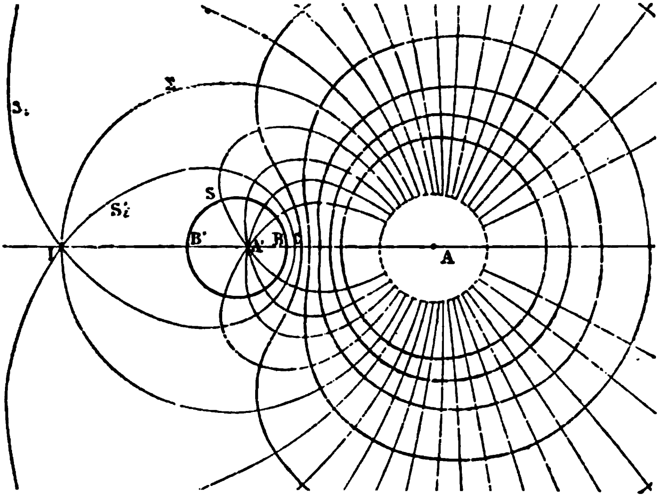

FIG. 27.

One of the equipotential surfaces, marked S_i in the figure, consists of two sheets which cut each other at the point I, where the electric force is nothing, that is to say, the point for which

$$\frac{Q}{r^2} + \frac{Q'}{r'^2} = 0, \text{ or } \frac{r^2}{r'^2} = -\frac{Q}{Q'}.$$

For the point I, we have $r = r' + \text{AA}'$, and therefore, with the values adopted for Q and Q', $(r' + \text{AA}')^2/r'^2 = 4$, whence $r' = \text{AA}'$, and $r = 2\,AA'$. This gives, for the potential at I, and therefore for the whole of the equipotential surface we are considering

$$V = \frac{20}{2AA'} - \frac{5}{AA'} = \frac{5}{AA'}$$

and since in the figure AA′ is taken equal to 2.5 cm., the numerical value of V is 2. The surfaces with potentials greater than 2 are closed surfaces enclosing the point A only. The surfaces having positive potentials less than 2 consist of two separate parts, one being a closed surface lying between the inner loop of the surface S_i and the sphere BB′ of potential zero, and the other, also a closed surface, lying outside the outer loop of the surface S_i and enclosing both the points A and A′. These are not shown in the figure.

The curves which are shown cutting the equipotential surfaces at right angles represent lines of force. It will be seen that 40 lines diverge from A, and that 10 converge to A′, thus indicating the relative magnitude of the charges supposed to exist at these points respectively. The remaining 30 lines from A go off to infinity.

38. Potential of an Isolated Sphere.—In § 36 the difference of potentials between a conducting sphere of radius, a, and a concentric spherical surface of radius, b, which form the boundaries of an electric field, with charge, Q, occupied by a dielectric with specific inductive capacity, K, was shown to be

$$V - V' = \frac{Q}{K}\left(\frac{1}{a} - \frac{1}{b}\right).$$

If the radius of the outer boundary becomes indefinitely great, and if we assume its potential $V' = 0$, we get for the potential of the isolated sphere

$$V = \frac{Q}{Ka}.$$

The equipotential surfaces outside the sphere are spheres concentric with it, the difference of potentials between any two of them being proportional to the difference of the reciprocals of their radii.

If a conductor is introduced into an electric field, it causes in general a change of potentials throughout which may be expressed as a distortion of the equipotential surfaces. The only case in which no such distortion is produced is when the surface of the conductor coincides with one of the pre-existing equipotential surfaces. For example, if a metal cylinder is introduced into the field referred to above, where the equipotential surfaces are concentric spheres, the spheres will be distorted. The surface of the cylinder will form one sheet of an equipotential surface of some definite value, say V_1, and will be intersected at right angles along the line of zero surface-density by a deformed surface of the same potential derived from the original field. The other equipotential surfaces will be bent so as to follow the form of the cylinder more and more closely as their potentials are nearer to the special value V_1, those of higher value than V_1 passing between the cylinder and the sphere, and enclosing the latter only, while those of lower value pass outside the cylinder and enclose both it and the sphere.

39. Units of Measurement.—If we employ, in the preceding formulæ, the C.G.S. electrostatic system of units, work is expressed in *ergs*, and the unit difference of potentials exists between two points when the transfer of an electrostatic unit of electricity from one point to the other requires work equal to one erg.

For practical purposes the unit of quantity generally adopted is the *coulomb*, which corresponds to 3×10^9 C.G.S. electrostatic units (§ 10), and differences of potential are expressed in terms of the *volt*, corresponding to $\frac{1}{300}$ of a C.G.S. electrostatic unit of difference of potential.

The work represented by the product of a volt into a coulomb is thus equivalent to $\frac{1}{300} \times 3 \times 10^9 = 10^7$ ergs. As this quantity recurs constantly in calculations, it has been adopted as the *practical unit of work*, and has been named a *joule*.

The joule being 10^7 ergs, and a kilogrammetre being $10^5 \times g$ ergs, it follows that a joule is equal to $100/g$ kilogrammetres, or to $1/10$ kilogrammetre nearly, since g is about 981.

Thus in the fundamental formula—

$$W = Q(V - V'),$$

if Q and $V - V'$ are expressed in C.G.S. *electrostatic units*, W is a number of *ergs;* whereas if Q is expressed in *coulombs* and $V - V'$ in *volts*, W is expressed in *joules.*

CHAPTER V.

GENERAL THEOREMS.

40. Tubes of Force—Flux of Force.—Let A be an equipotential surface, and dA an element of this surface at the point P (Fig. 28). If lines of force be drawn through every point in the contour of this element, they form a kind of tube everywhere perpendicular to the equipotential surfaces which it intersects, and which we shall call a *tube of force.*

If F is the intensity of the field at the point P, the product, FdA, of the force into the perpendicular section of the tube at the point P is what we shall speak of as the *flux of force* within this tube at the point in question.

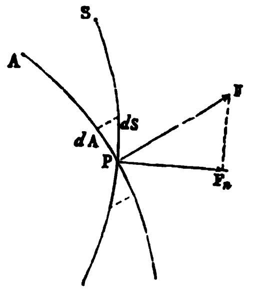

FIG. 28.

This tube cuts out an element, dS, from a surface, S, drawn in any manner through the point P. The flux can be easily expressed as a function of the surface dS, for if α is the angle between the normals to the two elements, we have $dA = dS\cos\alpha$, and therefore

$$FdA = FdS\cos\alpha = F\cos\alpha\, dS = F_n\, dS.$$

F_n being the projection of the force on the perpendicular to the element dS, or the component of the force perpendicular to this element. Consequently, *the flux of force* across any element of surface whatever, dS, is equal to the product $F_n dS$ of *the surface of the element into the component of force normal to this element.*

The total flux of force which traverses any given surface is the sum of the fluxes across the various elements.

41. Gauss's Theorem.—*Given a closed surface drawn in any way in an electric field, the total flux of force across this surface is equal to the quantity of electricity enclosed by the surface multiplied by 4π.*

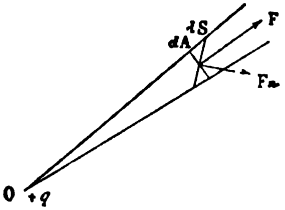

FIG. 29.

We may observe, in the first place, that if a charge, q, is placed at the point O (Fig. 29), and if a cone of infinitely small solid angle $d\omega$ be supposed to start from this point, the flux within the cone is constant.

For consider the sections of the cone at any point, A, at the distance r, one a right section, dA, and the other, dS, in any direction whatever, we have

$$F_n dS = F dA;$$

but, further

$$F = \frac{1}{K}\frac{q}{r^2}, \text{ and } dA = r^2 d\omega.$$

The flux is therefore

$$F_n dS = \frac{1}{K} q d\omega$$

and is independent of the distance r—that is to say, of the position of the point A.

Let us now draw in the field an imaginary, wholly convex surface (that is to say, a purely geometrical surface, and not the

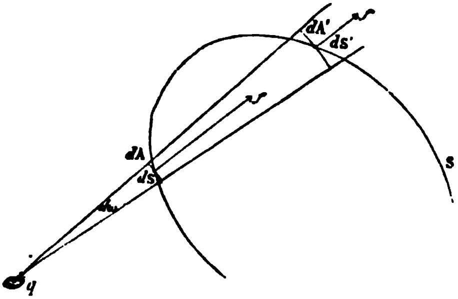

FIG. 30.

surface of a conductor), and consider an electrical charge, q, placed at the point O (Fig. 30). The cone of infinitely small solid angle

$d\omega$ intercepts two elements, dS and dS', for which the fluxes $F_n dS$ and $F_n' dS'$ are both equal to $q d\omega$; but, as regards the surface, they should be taken as of contrary signs, because *one enters the surface and the other leaves it;* we shall take outward flux as positive—that is to say, that for which the normal component is directed towards the outside. We have, then

$$F_n dS + F_n' dS' = 0.$$

All the surface is thus cut into pairs of elements, the fluxes of which mutually compensate each other. If the surface has concavities, and if the cone in question cuts it more than twice, it cuts it an even number of times; the sum of the elementary fluxes, which are always equal to each other and alternately of opposite signs, is still equal to nothing.

Thus, *the flux which traverses a closed surface is nothing when this flux proceeds from a charge outside the surface.*

Consider now a charge, q (Fig. 31), situate inside the surface. The two elements, dS and dS', intercepted by an infinitely small cone, give as before

$$F_n dS = F_n dS' = \frac{1}{K} q d\omega;$$

but the two elementary fluxes are of the same sign, positive if the charge is positive, and negative if the charge is negative.

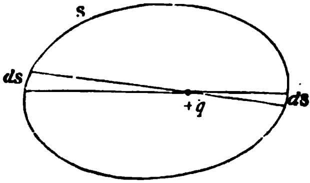

FIG. 31.

To obtain the total flux, the charge q must be multiplied by 4π, the sum of the solid angles of all the cones, or by the solid angle subtended by the surface of a complete sphere described about the point q. That is: *The flux through a closed surface proceeding from a charge, q, inside the surface is $4\pi q$.*

Next consider any charges whatever, q, q', q'', ... q_1, q_2, q_3, ... (Fig. 32), some inside and some outside the surface. On an element, dS, each charge, whether internal or external, has a normal

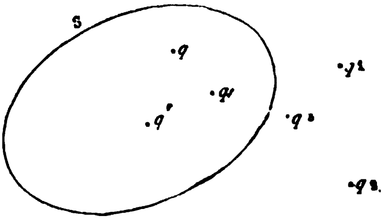

FIG. 32.

component, and in order to get the total flux across the surface, we must take the sum of the products obtained by multiplying each element by the normal component due to each of the charges. If in this sum all the terms corresponding to the same charge are added together, the sum is nothing if the charge is outside, and is equal to $4\pi q$ if the charge is inside the surface. The total sum is thus equal to the product of 4π into the algebraical sum of all the internal charges; which is Gauss's theorem.

Conversely, in order to find the electric charge contained within a closed surface, it is sufficient to determine the normal component F_n at each point, to add all the products, $F_n dS$, and divide by 4π.

42. Properties of Tubes of Force.—Let us consider a tube of force (Fig. 33), bounded by any two surfaces, S and S′, and let us apply Gauss's theorem to the volume thus defined. If it contains no electricity, the total flux through its surface is nothing; but as the lateral surface has no influence on the sum since the normal component of force is everywhere nothing, the sum reduces to the fluxes across the two ends, which must, therefore, be equal and of opposite signs.

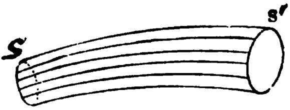

FIG. 33.

From this there results the following important consequence:—*At all points of the same tube of force the value of the flux is constant.*

If the tube is infinitely narrow, and the terminal surfaces are perpendicular to the axis, we have

$$FdS = F'dS',$$

and therefore, *the force at each point of the tube is inversely as the cross-section.*

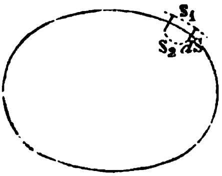

FIG. 34.

43. Coulomb's Theorem.—Let us take an element of surface, dS, on an electrified conductor in equilibrium in the presence of any given electric charges (Fig. 34); and let us suppose that the tube of force corresponding to this element is terminated on the outside of the conductor by an equipotential surface, S_1, infinitely near to dS, and on the inside by an arbitrary surface, S_2. The force is nothing at every point of the surface S_2, and the normal component is nothing on the lateral surface of the tube; there is consequently no flux, except on the outer surface, dS_1. If F is the electric force just outside the element, and σ is the density on the surface dS, Gauss's theorem gives

$$FdS_1 = 4\pi\sigma dS \frac{1}{K},$$

and as in the limit $dS_1 = dS$

$$F = 4\pi\sigma\frac{1}{K}.$$

Thus *the electric force at a point infinitely near the surface of a conductor in equilibrium, whatever be the acting charges, is equal to the product by $4\pi/K$ of the electric density near this point.*

Its direction is, moreover, perpendicular to the surface.

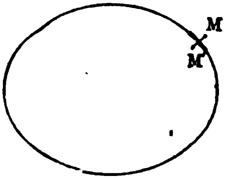

FIG. 35.

44. Electrostatic Pressure.—The total force F at a point, M, infinitely near the element dS (Fig. 35), is made up of the force, f, due to the element itself, and of the force, f', due to all the other charges, and we have (§ 43)

$$f + f' = F = 4\pi\sigma/K.$$

The total force is nothing at a point, M′, symmetrical with the first, but just inside the conductor; further, the force f' due to all the electricity external to the element is the same at M′ as at M, and the action f of the element dS has only changed its sign; we have then

$$f' - f = 0$$

and therefore

$$f' = 2\pi\sigma/K.$$

Hence the force due to all the electricity outside the element is equal to $2\pi\sigma$. The force on the quantity of electricity contained in the element dS, is $2\pi\sigma \,.\, \sigma dS/K$ and for unit surface $2\pi\sigma^2/K$. This force is always directed outwards, whatever be the sign of σ. It is what is called the *electrostatic pressure*.

45. Corresponding Elements.—Consider a tube of force extending from one conducting boundary of an electric field to the other (Fig. 36), and terminated at its two ends by the areas dS

and dS' respectively; these two areas are called *corresponding* areas. Since there is no force inside the surface of either of the conductors by which the field is bounded, and there is no force normal to the walls of the tube, the total flux of force outwards from the tube is nothing. Consequently (§ 41), the algebraic sum of all the electricity within the tube must be nothing. But the only electricity within the tube is the quantity σdS upon one end and $\sigma' dS'$ on the other end, whence it follows that

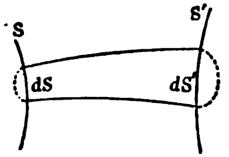

FIG. 36.

$$\sigma dS = -\sigma' dS'$$

and therefore that *corresponding elements of the bounding surfaces of an electric field possess equal and opposite charges.*

46. Complete Representation of the Field by Lines of Force.—The lines of force define at each point of the field the direction of the force, and we may further require that they represent the intensity. Let us consider an infinitely narrow tube of force (Fig. 37), and in this tube let us take two normal sections, dS and dS'. Suppose N points per unit of area uniformly distributed over the surface dS, and draw the lines of force which pass through these points. These lines of force to the number NdS are comprised within the tube; if N' is their number per unit area of the surface dS', we have evidently

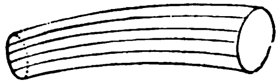

FIG. 37.

$$N' = \frac{NdS}{dS'}.$$

But from the fundamental property of the tube (§ 42) we have

$$FdS = F'dS'$$

and therefore

$$\frac{N}{N} = \frac{F'}{F}.$$

If $N = F$, we shall have $N' = F'$, that is to say, at each point of the tube the numerical value of the force will be given by the number of lines of force cut by unit area of a surface drawn through this point normally to the lines. If the unit surface is oblique to the lines of force, the number of lines of force cut is equal to the projection of the force on the perpendicular to the surface, in other words, to the normal component of the force.

The condition $N = F$ amounts to saying that the number of lines of force and the flux within the tube have the same numerical value.

In order that this may be the case for the whole extent of the field, it is necessary and sufficient that the number of lines of force which start from each quantity of electricity be equal to the flux from it. From an isolated charge, q, a number of lines of force will start equal to $4\pi q$; from the surface of a conductor at a point where the density is σ, a number, $4\pi\sigma$, of lines per unit area will proceed (§ 43).

These lines may be drawn, bit by bit, by the condition of being tangents at each point, to the resultant of all the forces acting through that point. The lines of force of Fig. 27 have been drawn on this principle. The lines of force may be considered as starting from positive electricity. Some go off to infinity, others terminate on negatively charged surfaces. A line of force which goes from one conductor to another always has contrary electricities at its two ends. Hence there can be no lines of force between two points charged with the same electricity. Further, no line of force can terminate at any point where there is no electricity.

With these conventions relative to the lines of force, the expression *flux of force* and number of lines of force are equivalent. The number of lines of force cut by any given surface is the numerical value of the flux across this surface.

47. Application to Induction.—Let q, q', q'', be quantities of electricity situate inside a closed hollow conductor, A (Fig. 38), and M the quantity developed by induction on the internal surface. If Gauss's theorem (§ 41) be applied to any surface, S_1, drawn inside the mass of the conductor, where the force is nothing at every point, we have

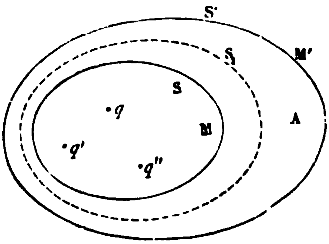

Fig. 38.

$$M + \Sigma q = 0$$

The layer M developed by induction on the inner surface is thus equal and of opposite sign to the algebraical sum of all the charges within the cavity (§ 23*b*).

All the lines of force proceeding from charges inside the cavity are then absorbed by the inner surface, S, of the conductor. The combined effect of all the internal electricity, and of the charge which covers the surface S, is thus nothing for all points outside this surface. In other words, so far as regards internal charges, the surface S forms an absolute *screen* for all points on the outside.

Hence, any charge given to the insulated conductor, A, will be distributed on the outer surface in a manner independent of the internal charge, and only depending on the form of the surface S′, and on the distribution of the external charges. Reciprocally, the combined effect, at any point inside the conductor, of the electrification of the outer surface together with that of all electricity outside the conductor, is nothing.

This result agrees with the mutual independence of the electric fields inside and outside a hollow conductor that we have already pointed out (§ 23).

In the case in which an insulated body, C, is in the presence of an electrified body inside a closed conductor, A (§§ 22, 23 *c*) part of the flux of force from A terminates at the negative region of C; but an equal flux starts from the positive region, and ends finally at the closed conductor, so that there is always the same quantity of negative electricity on the interior surface, which follows, moreover, from the preceding equation.

48. Poisson's Theorem.—*The force which given electric charges contained within a closed surface exert at any point outside that surface, is the same as that of a layer of the same quantity distributed on this surface according to a certain law.*

In fact, let q, q', q'' or Σq be the internal charges, which we shall suppose fixed as if they belonged to non-conducting bodies, and let S be an ideal surface which encloses them. If this surface were a conductor, it would be covered on the inside with a quantity $-Q = -\Sigma q$, whose action at all outside points would be equal and opposite to that of the given charges. A quantity $+Q$ of electricity, distributed over the surface S according to the same law, will therefore produce at all external points an effect equal and similar to that of the enclosed charges Σq, and may therefore be substituted for them.

If the imaginary surface S were replaced by an actual conducting shell of the same size and shape, the inside of the shell would (as already pointed out in § 23 *b*) have a charge $-Q$, distributed so as to counteract, for all external points, the effect of the enclosed charges, and the outside would have a charge $+Q$, but this would not be distributed according to the same law unless the surface S were an equipotential surface of the original system. If, however, such a shell were coincident with a pre-existing equipotential surface, the electric conditions at all external points would be in no way altered by introducing it into the field. It follows that the force at any point in an electric field would be unaltered if, the charge remaining constant, the shape and position of one, or of both, boundaries of the field were to change in such a manner as always to coincide with one of the original equipotential surfaces. We have already (§ 19) pointed out a special case of this kind: the force at any point due to an electrified sphere at a great distance from all other conductors is independent of the radius, so long as the charge of the sphere and the position of the centre remain unchanged.

CHAPTER VI.

ELECTRICAL CAPACITY—CONDENSERS.

49. Capacity of an Electric Field.—In nearly all cases in which we require to deal practically with electric fields, the bounding surfaces of the fields are formed of conducting materials, and we shall assume this to be the case throughout what follows. Under these circumstances the potential of each boundary is uniform, and the difference of potentials between the boundaries is, by definition (§ 36),

$$V - V' = \bar{F}e$$

if e is the distance from one surface to the other measured along a line of force, and $\bar{F}$ is the mean electric intensity along this line. The intensity at any point is proportional, other things remaining the same, to the charge of the field (meaning thereby the total quantity of positive electricity upon one boundary, or of negative electricity on the other), and therefore $\bar{F}$ must be proportional to the charge. Consequently, for a given field, charge and difference of potentials are proportional to each other; and putting C to stand for a constant coefficient characteristic of the field, we may write

$$Q = C(V - V').$$

This coefficient, whose definition is implied in the above equation, is called the *capacity* of the field. In words, we may define it as follows:—

The capacity of an electric field is the ratio of the charge of the field to the difference of potentials between its boundaries, and is numerically equal to the charge required to establish unit *difference of potentials between them.*

The term "capacity" is borrowed from the theory of heat, where the thermal capacity of a body is defined as the ratio of a quantity of heat given to a body to the change of temperature produced by it, and as being numerically equal to the quantity of heat needed to produce unit change of temperature. Thermal capacity depends on mass and material; the capacity of an electric field, on the size and shape—that is, on the geometry of the field—and on the material by which it is filled.

50. Capacity of a Tube of Force.—An electric field may be thought of as made up of tubes of force running from one boundary to the other, and each charged at its ends with a definite proportion of the whole charge of the field. We may call this the charge of the tube, and the ratio of the charge to the difference of potentials between the ends of a tube may be called the capacity of the tube, just in the same sense as the ratio of corresponding quantities for the whole field is called the capacity of the field. For a given field, the difference of potentials is the same for all the tubes, because the boundaries of the field are equipotential surfaces. If, then, q be the charge of a tube, we may write for its capacity

$$c = \frac{q}{V - V'},$$

or for different tubes

$$c_1 = \frac{q_1}{V - V'},\ c_2 = \frac{q_2}{V - V'},\ c_3 = \ \ldots$$

$V - V'$ being a common denominator.

Adding these values, we get

$$c_1 + c_2 + c_3 + \ldots = \frac{q_1 + q_2 + q_3 + \ldots}{V - V'} = \frac{Q}{V - V'} = C.$$

Whence we see that the capacity of an electric field is equal to the sum of the capacities of the tubes of force into which it may be subdivided.

As for a complete field, so for a tube of force, its capacity depends on its geometrical relations and on the material medium that occupies it. The difference of potentials between the ends is $= \bar{F}e$, as above, or what comes to the same thing $\int_0^e F\,de$, where F is the intensity at any point in the tube, and de an element of length. If s is the area of the tube at the part where the intensity is F, and if s_0 and F_0 are corresponding values for the positive end, we have

$$Fs = F_0 s_0$$

and therefore

$$V - V' = \int_0^e F de = F_0 s_0 \int_0^e \frac{de}{s} = \frac{4\pi\sigma s_0}{K} \int_0^e \frac{de}{s}$$

since, if K be the dielectric coefficient for the medium, $F_0 = \frac{4\pi\sigma}{K}$.

The charge of the tube is $q = \sigma s_0$, and therefore the capacity becomes

$$c = \frac{q}{V - V'} = \frac{K}{4\pi} \cdot \frac{1}{\int_0^e \frac{de}{s}} \cdot$$

The value of the integral $\int \frac{de}{s}$ which occurs in this expression depends on the size and shape of the tube, and cannot be determined till these are known.

51. Calculation of Capacity in Special Cases.—In certain cases, where the tubes of force are of simple shape, it is easy to calculate the capacity of the field of force. We shall consider here the three simplest cases.

1. Field bounded by *plane parallel surfaces*, whose distance apart is small in comparison with their linear dimensions. Here, leaving out of account a small space near the margin of the terminal surfaces of the field, the intensity is uniform and normal to the boundaries; hence, in virtue of the relation (§ 42)

$$Fs = F's'$$

the tubes of force are cylindrical, and s has the same value from end to end of a given tube. Consequently

$$\int_0^e \frac{de}{s} = \frac{1}{s} \int_0^e de = \frac{e}{s}$$

and therefore the capacity of a single tube of force is

$$c = \frac{Ks}{4\pi e}$$

and the capacity of total field is

$$C = \Sigma c = \frac{K}{4\pi e} \Sigma s = \frac{KS}{4\pi e} \cdot$$

The larger the transverse dimensions of the field as compared with the distance between the boundaries, the smaller the relative error committed in neglecting, as we have done, the marginal portions of the field in which the cross-section of the tubes of force is not uniform throughout.

2. Field bounded by *co-axal cylinders* whose length is great in comparison with the radius of the outer cylinder. Again neglecting a small space near the ends of the cylinders, the tubes of force may be taken as wedge-shaped spaces bounded by two planes passing through the axis of the cylinder and making a small angle, say θ, with each other, and by two planes perpendicular to the axis at distance, say l, from each other. Then if r be the distance from the axis at which the cross-section of a given tube is s, we have

$$s = lr\theta$$

l and θ being both constant. Putting a for the radius of the inner cylinder, and b or $a + e$ for that of the outer cylinder, we may write

$$\int_o^e \frac{de}{s} = \frac{1}{l\theta}\int_a^b \frac{dr}{r} = \frac{1}{l\theta} \log \frac{b}{a}.$$

Consequently the capacity of a tube of force is

$$c = \frac{Kl\theta}{4\pi} \cdot \frac{1}{\log \frac{b}{a}}.$$

To get the capacity of the field, we have only to write L, the length of the cylinders, instead of l, and 2π, the angle corresponding to a complete circumference, instead of θ. This gives

$$C = \frac{KL}{2 \log \frac{b}{a}}.$$

For purposes of numerical calculation it is important to bear in mind that the logarithm in the denominator of this expression is a so-called natural logarithm having the base 2.718 . . . and is equal to the corresponding common logarithm (base 10) divided by 0.4343. The above formula may therefore be written

$$C = \frac{0.4343}{2} \frac{KL}{\log_{10} \frac{b}{a}} = 0.2171 \frac{KL}{\log_{10} \frac{b}{a}}.$$

3. Field bounded by *concentric spheres.* The tubes of force in this case are cones having the centre of the spherical surfaces as their common vertex. Take a and b $(= a + e)$ respectively for the radii of the spheres, let ω be the solid angle of one of the conical tubes of force, and r the distance from the vertex at which the tube has cross-section s. Then

$$s = r^2\omega$$

and

$$\int_0^e \frac{de}{s} = \frac{1}{\omega}\int_a^b \frac{dr}{r^2} = \frac{1}{\omega}\left(\frac{1}{a} - \frac{1}{b}\right).$$

The capacity of a tube is

$$c = \frac{K\omega}{4\pi} \cdot \frac{1}{\frac{1}{a} - \frac{1}{b}} = \frac{K\omega}{4\pi} \cdot \frac{ab}{b - a},$$

and the capacity of the field is

$$C = K\frac{ab}{b - a}$$

obtained by putting 4π for the sum of the solid angles of all the tubes of force.

52. Remarks.—The method of calculation employed above is applicable to any field for which we are able to determine the value of the integral $\int \frac{de}{s}$ for one tube of force. The three cases to which we have applied it are specially simple, because the tubes of force in each case are all identical, and we might therefore have dealt at once with the whole field. Indeed, the first and third cases, the parallel planes and concentric spheres, can be treated very easily by other methods as follows.

With parallel planes, the intensity being uniform from one boundary to the other, we have at once

$$V - V' = Fe = \frac{4\pi\sigma}{K} e$$

and therefore

$$C = \frac{QK}{4\pi\sigma e} = \frac{SK}{4\pi e}$$

as above.

In the case of the concentric spheres, we have to remember that at any point in the field the intensity due to the charge of the outer surface is nothing, and therefore that the whole intensity is that due to the charge on the inner sphere, which acts as though it were concentrated at the centre. The intensity at the outer surface is, therefore, $\frac{Q}{b^2K}$, at the inner surface $\frac{Q}{a^2K}$, and the mean intensity (comp. § 36) is $\frac{Q}{abK}$; consequently the difference of potentials is

$$V - V' = \frac{Q}{abK}(b - a)$$

and the capacity

$$C = K\frac{ab}{b - a}.$$

In a field of force of any shape, if the distance between the boundaries is uniform and small compared with their linear dimensions, it is evident that the electric intensity must be nearly uniform throughout the field, and therefore that the formula $\frac{SK}{4\pi e}$, found for two parallel planes, must apply to any case which satisfies the conditions named. It is easy to verify that the formulæ for cylinders and spheres reduce to this form when e is very small.

53. Isolated Sphere.—If the outer surface, in the case of a field bounded by concentric spheres, becomes very great, the corresponding radius, b, becomes very great, or, what comes to the same thing, the radius a becomes very small in comparison, and may be neglected in the difference $b - a$. Consequently the capacity of the field becomes

$$C = K\frac{ab}{b} = Ka$$

and depends only on the radius of the smaller (convex) sphere and on the nature of the dielectric medium.

When the outer boundary of the field is very distant, a change of its shape would not alter the potential at a point on the inner boundary. Consequently the expression

$$C = Ka$$

gives the capacity of any field having for one boundary a conducting sphere of radius a, and for the other boundary any very distant surface. For example, if a conducting sphere is suspended in a large room at a great distance from the walls or other conductors, the capacity of the electric field extending between the sphere and the walls, floor, and ceiling of the room is equal to the radius of the sphere multiplied by the dielectric coefficient of air.

54. Successive Electric Fields.—It happens not unfrequently that one electric field is itself completely surrounded by another, or, what comes to the same thing, that a field is divided into two parts by a closed conducting shell, and that it is required to express the charge in terms of the potentials of the first and last bounding surfaces. The arrangement indicated in Fig. 39 may be taken as a typical case of this kind. The shaded portions being understood to represent conducting material and the unshaded to represent dielectrics, we may consider that we have here a field of capacity, C_1, with bounding surfaces at potentials V and V_1 respectively, and that this is surrounded by a second field of capacity, C_2, with surfaces at potentials V_1 and V_2. Or we may look upon the figure as representing a single field, with surfaces at potentials V and V_2, divided into two parts by a conductor at potential V_1. It is obvious that the charge of each partial field is the same: representing it by Q, we have

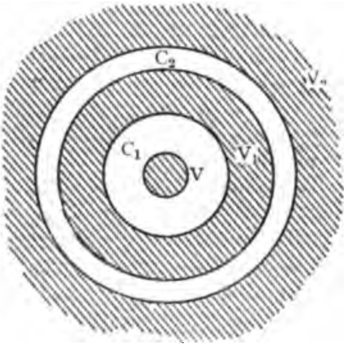

FIG. 39.

$$Q = C_1(V - V_1) = C_2(V_1 - V_2)$$

and eliminating V_1

$$Q = \frac{C_1 C_2}{C_1 + C_2}(V - V_2).$$

If we put C for the capacity of the simple field, which, with surfaces at potentials V and V_2, would have the same charge as the two consecutive fields, we must write

$$C = \frac{C_1 C_2}{C_1 + C_2}.$$

By extending the same method of calculation, we may find the capacity of the single field equivalent to any number of separate fields arranged in succession. For example, if there are three fields of separate capacities C_1, C_2, C_3, the capacity of the single equivalent field is

$$C = \frac{C_1C_2C_3}{C_2C_3 + C_1C_3 + C_1C_2}.$$

55. Energy of an Electric Field.—As already pointed out (§§ 8, 11), an electric field constitutes a store of energy, the amount of which is measured by the work which the electric forces of the field can do during exhaustion of the field; or, what comes to the same thing, by the work that must be done against electric forces to establish the field.

We can estimate this amount of work as follows. We may suppose the total charge of the field to have been produced by the successive transfer of elementary portions of positive electricity, dQ, from the negative boundary of the field to the positive boundary, until the total charge, Q, had been transferred; or we may suppose an equal amount of negative electricity transferred in successive quantities $-dQ$ in the opposite direction; or lastly, we suppose $\frac{1}{2}Q$ transferred from the negative to the positive boundary, and $-\frac{1}{2}Q$ from the positive to the negative boundary. These three operations would be physically identical and differ only in the mode of expression; we shall, therefore, for the sake of definiteness, adopt language appropriate to the first mode. If then v and v' are, at any stage of the process, the respective potentials of the positive and negative boundaries, the amount of work done in transferring an elementary quantity of electricity at this particular stage is

$$dW = (v - v')\,dQ.$$

If C is the capacity of the field and q the quantity already transferred

$$v - v' = \frac{q}{C}$$

therefore

$$dW = \frac{q}{C}\,dQ$$

and the whole amount of work, W, done in producing the final electrification is the sum of all the values of the last quantity as q

varies from 0 up to its final value Q, or

$$W = \frac{1}{C}\int_{0}^{Q} q\, dQ = \frac{1}{2}\cdot\frac{Q^2}{C}.$$

The same conclusion can be easily established geometrically. In Fig. 40, let the straight line OA be drawn so that the ratio of the ordinate of any point to the abscissa of the same point may be equal to the ratio of the difference of potentials of the boundaries of the field to be considered to the charge of the field. Then if OQ represents the charge q at any stage of the electrification, QM will represent the corresponding difference of potentials, and the work dW due to an increment of charge, dQ, represented in the figure by QQ′, will be expressed by the area of the trapezium QMM′Q′. The sum of all such areas standing upon the base OP is equal to the area of the triangle OAP = $\frac{1}{2}$ OP · PA ; hence if OP represents the total charge Q, and PA the resulting difference of potentials $V - V'$, we have for the work spent in electrification

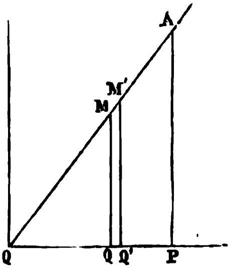

FIG. 40.

$$W = \tfrac{1}{2}\,\text{OP}\cdot\text{PA} = \tfrac{1}{2}\,Q\,(V - V').$$

By combining this with the fundamental expression (§ 49) $Q = C\,(V - V')$ connecting charge, capacity and difference of potentials, we see that it is equivalent to the formula already obtained. In fact, the energy of an electric field may be expressed in terms of any two of the three quantities named, as follows :

$$W = \tfrac{1}{2}\,Q\,(V - V') = \tfrac{1}{2}\,\frac{Q^2}{C} = \tfrac{1}{2}\,C\,(V - V')^2.$$

56. Geometrical Representation of the Energy of an Electric Field.—If, as we suppose, the forces that act on an electrified body in an electric field are the result of stresses in the field itself, we may expect the energy of the field to be definitely related to the distribution of force within it as expressed by tubes of force and equipotential surfaces. This is, in fact, the case, and the energy of a field and the mode in which it is distributed admit of very simple representation. Suppose the positive boundary of a field, whose charge we will, for simplicity, assume to be some whole number of units, to be divided into areas such that each is charged with a unit of positive electricity, and let each of these areas be taken as the base of a tube of force extending to the negative boundary. In this way the whole field will be exhaustively divided into tubes of force whose number gives the numerical value of the charge of the field. If we now suppose a series of equipotential surfaces to be drawn in the field, the potential of each differing by unity from that of the next, every tube of force will be divided into a set of compartments or cells, the number of which gives the numerical value of the difference of potentials between the boundaries of the field. That is, the number of tubes of force gives the value of Q, and the number of cells into which each tube is divided by the equipotential surfaces gives the value of $V - V'$; consequently the whole number of cells gives the value of $Q\,(V - V')$, or *half* the number of cells into which the whole field is divided by tubes of force and equipotential surfaces drawn in the way described gives the electric energy, $\frac{1}{2}\,Q\,(V - V')$, of the field.

If we imagine one or both of the boundaries of the field to be capable of yielding to the electric forces which tend to draw them together, the field would ultimately collapse, its energy being exhausted by the work done on the movable boundaries. During this process the difference of potentials between the boundaries would gradually fall from the initial value $V - V'$ to nothing when the surfaces were in contact, the mean difference of potentials being $\frac{1}{2}\,(V - V')$, and the work done being the product of this mean value into the charge of the field.

57. Condensers.—When two electric conductors are so placed that their surfaces form the boundaries of a field of relatively great capacity, the arrangement is very frequently spoken of as a *condenser*. For example, we have seen that a conducting sphere of radius a centimetres, at a great distance from other conductors, forms one boundary of a field of capacity Ka; whereas, if the sphere be surrounded by a concentric conducting shell of radius b, the resulting field has capacity $K\frac{ab}{b - a}$. Thus, if a = 10 cm., and b = 11 cm., the capacity in the second case is 11 times that in the first; or, a being still 10 cm., if b = 10.1, the capacity of the field bounded by the two spheres becomes 1010 K, instead of 10 K.

58. Various Forms of Condensers.—One of the simplest forms of condenser is that known as *Æpinus's condenser* (Fig. 41), consisting of two equal insulated metal plates, supported so that they can be brought into contact with the opposite faces of a plate of glass larger than themselves, or simply placed so as to face each other at a short distance in air. In either case the capacity can be diminished by drawing the plates apart, or increased by moving them nearer together. To charge the condenser, it is only necessary to connect the two plates with opposite sides of an electric machine or other contrivance for producing a difference of potentials.

Another equivalent arrangement, except that the distance between the conductors cannot be varied, is that known as *Franklin's pane*, consisting of two equal pieces of tin-foil pasted opposite each other on the two faces of a sheet of glass large enough to project a good way beyond them on all sides. For better insulation, the uncoated parts of the glass are usually covered with shellac varnish.

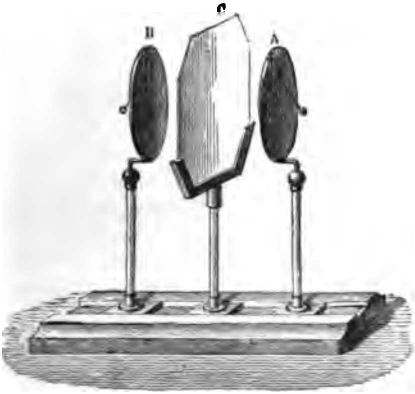

FIG. 41.

When condensers of very great capacity are required, but are not to be exposed to great differences of potential, they are often made by superposing alternately sheets of tin-foil and thin plates of mica, or sheets of paraffined paper. The first, third, fifth . . . . sheets of tin-foil are connected together, and form one boundary of a field of great area but very small thickness, and the second, fourth, sixth . . . . sheets are connected to form the other boundary.

Another form of condenser that is very frequently employed in connection with high differences of potential is the *Leyden jar* (Fig. 42). This consists of a glass jar or bottle, coated internally and externally with tinfoil to within a moderate distance of the opening, the rest of the surface being varnished with shellac. A brass rod, terminated by a ball, connected with the inner coating, and passing out through the neck without touching it, enables electrical connection to be made with the inner coating when required. Sometimes the jar is closed at the mouth by a non-conducting lid; in this case the rod is usually supported by being put tight through the lid, and makes contact with the inner coating by means of a flexible spring or a few inches of chain. Electrically, the Leyden jar is exactly equivalent to the coated pane, but it is more compact and convenient to handle. The tin-foil coatings may be replaced by conductors of any other material; the reason why tin-foil is commonly used being chiefly that its flexibility allows it to be readily adapted to the surface of the glass. Where very high insulation is required, it answers well to use strong sulphuric acid (oil of vitriol) to make contact with the inner surface of the glass (which, in this case, is not varnished), and to connect the acid with the other apparatus by a platinum wire dipping into it.

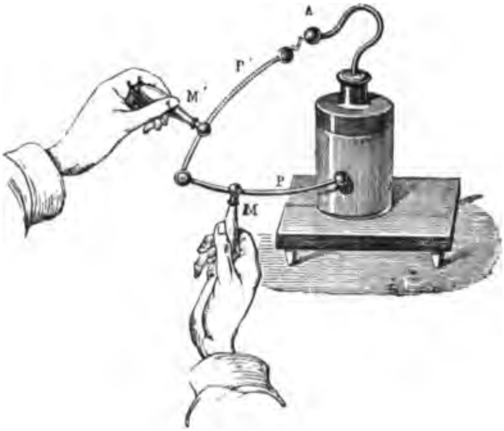

FIG. 42.

59. Quantity of Electricity in a Condenser.—The charge of a condenser is, of course, the charge of the electric field which is bounded by its conducting surfaces ; it is therefore represented by

$$Q = C\,(V - V')$$

and depends partly on the condenser itself and partly on the difference of potentials established between its surfaces.

The formula $C = \dfrac{SK}{4\pi e}$ (§ 52) applies with sufficient accuracy to the capacity of any of the forms of condenser described. From this we see that, with a given insulating material, the capacity can be increased by increasing the area S of the opposed conducting surfaces, for these determine the area of cross-section of the field, and also by decreasing the distance e between the surfaces, or, in other words, the length of the field. But if the charge is to be great, not only the capacity, but the difference of potentials, $V - V'$, must have a considerable value. This is usually determined by the electric machine or other instrument used for producing the charge, and under fixed conditions in this respect may be taken as constant. The value of the difference of potentials that is to be employed puts a practical limit to the extent to which capacity can be increased by diminishing the thickness, e, of the dielectric. A small thickness and great difference of potentials implies a high

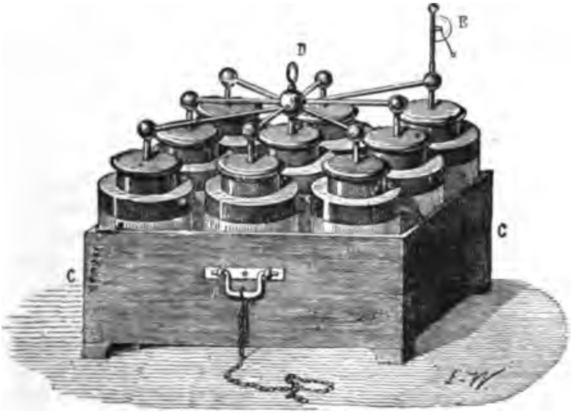

FIG. 43.

value of the ratio $\frac{V - V'}{e}$, which measures the mean electric intensity in the field, and, as already mentioned (§ 20), when the stress in the field reaches a limit depending on the nature of the material, the dielectric gives way and is burst through by a disruptive discharge. The maximum stress that a given material can support without allowing discharge to take place through it is called its *electric strength*. Little is known about the exact value of this property for different substances, but it is certainly many times greater for glass and various other solid dielectrics than it is for air. For this reason, in condensers intended to support great differences of potential, as well as to have a large capacity, glass is generally used as the dielectric, and the commonest form is that of the Leyden jar. To obtain sufficient surface without using jars of unwieldy size, several jars are often combined, as shown in Fig. 43, all the inner coatings being electrically connected by insulated metal rods, and the outer coatings by placing the jars on a tray, or in a box lined with tinfoil. The capacity of such a combination of jars, sometimes called a *Leyden battery*, is equal to the sum of the capacities of the separate jars (comp. § 50).

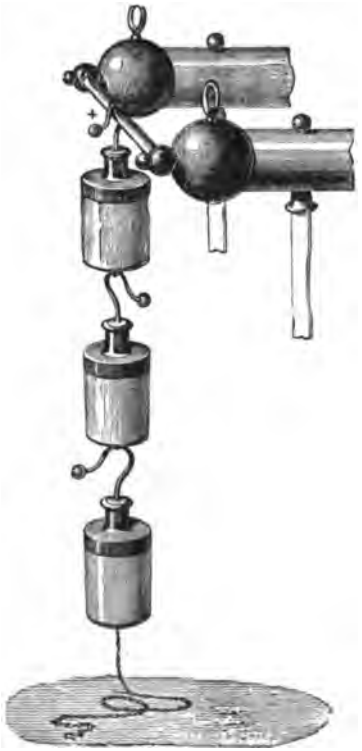

FIG. 44.

When it is not desirable to expose a condenser to the full difference of potentials that may be available, it is sometimes advantageous to connect two or more jars in series, or *in cascade* as it is called, in the way shown in Fig. 44. The inner coating of the jar at one end of the series is connected with one electrode of the electrical machine, the outer coating of the jar at the other end of the series to the other electrode, and the outer coating of each of the rest to the inner coating of the jar that comes next in the series. If, in such a case, we may neglect the small capacity of the field existing between the outer surface of each jar and the walls of the room, by comparison with the much greater capacity of the field between the tin-foil coatings, the charge of each jar is the same, and writing V_0 and V_n for the potentials of the first and last coatings respectively, and V_1, V_2, V_3 . . . for the potentials of the intermediate coatings taken in order, we have for the charge of any one jar

$$Q = C_1 (V_0 - V_1) = C_2 (V_1 - V_2) = \dots = C_n (V_{n-1} - V_n).$$

But (§ 54), we have also

$$Q = \frac{C_1 C_2 \dots C_n}{C_2 C_3 \dots C_n + C_1 C_3 \dots C_n + \dots + C_1 C_2 \dots C_{n-1}} (V_0 - V_n)$$

where C_1, C_2, . . . are the capacities of the several jars, and the multiplier of $V_0 - V_n$ has a numerator formed by multiplying together all the Cs, and a denominator formed by adding together the series of terms obtained by striking out from the numerator each of its factors taken one at a time from C_1 to C_n. From this we can express the difference of potentials acting on any one jar when we know the whole difference of potentials and the separate capacities. Suppose, for example, that there are four jars, then the difference of potentials acting on the second is

$$V_1 - V_2 = \frac{C_1 C_3 C_4}{C_2 C_3 C_4 + C_1 C_3 C_4 + C_1 C_2 C_4 + C_1 C_2 C_3} (V_0 - V_4).$$

If a number of jars combined in cascade are all of the same capacity, it follows that the difference of potentials acting on each jar is the same, and is equal to the whole difference divided by the number of jars. In this case also the sum of the separate charges is the same as any one jar would have received if it had been charged to the full difference of potentials $V_0 - V_n$.

60. Charging and Discharging a Condenser.—Since the process of charging a condenser consists in establishing a difference of potentials between the boundaries of the corresponding electric field, a charge is produced whenever the opposed conducting surfaces of the condenser are respectively connected with other conductors at different potentials. In general the surfaces are connected with the positive and negative conductors of an electric machine. If a common friction machine, with the rubber uninsulated from the room, is used, a sufficient connection between the outer surface of a Leyden jar and the rubber is usually afforded by the table on which both jar and machine stand ; a special conducting connection need be arranged in such a case only between the inner coating and the insulated conductor of the machine.

To discharge a condenser is to bring its two surfaces to the same potential. This may be done suddenly by connecting them by means of a *discharger* (Fig. 42), consisting of two metal rods hinged together like a pair of compasses and provided with glass handles, M, M′, and terminated by brass knobs. If one knob is put into contact with the outer coating of a charged Leyden jar, and the other is gradually brought near the knob A connected with the inner coating, a bright loud spark passes before the knobs come quite into contact, and the jar is afterwards found to be discharged. In this case the whole process occupies only a very small fraction of a second, but it is, nevertheless, a somewhat complicated one, and consists of a succession of discharges of gradually decreasing magnitude, and alternately in opposite directions. The phenomenon is, in its general character, closely analogous to what happens when a bent spring is suddenly released, and, instead of simply returning to its position of equilibrium and remaining at rest there, is carried beyond it by its momentum and is bent in the opposite direction, then passes again through its position of equilibrium towards the side of the original bending, and so backwards and forwards through a gradually decreasing range until the whole of the energy due to the original strain has been converted into heat. In the electrical case we must imagine some kind of strain as existing in the charged dielectric, and that the sudden relaxation of this strain at the instant of discharge gives rise to a series of electric oscillations of progressively decreasing extent until the whole energy of the charge has here also been converted into heat.

If the two surfaces of a charged condenser are connected through a very long thin wire or through a damp thread, the resulting discharge consists simply of a comparatively gradual equalisation of potentials without reversal or oscillation. In this case the process may be compared with the result of releasing a bent spring which is immersed in a liquid so viscous that the elastic forces, urging the spring through it towards its position of equilibrium, are not able to make it arrive there with any velocity.

61. Discharge by Successive Contacts.—Discharge takes place in a different way if a condenser is charged, and its surfaces, which are both insulated from other conductors, instead of being connected with each other, are alternately connected, one at a time, with a third conductor, which may be the room containing the condenser. In order to follow out what happens in such a case, we will suppose the condenser to be a coated pane, and that its surfaces are alternately connected with the inside of the room. There is here a complex field, or rather there are three separate fields. Distinguishing the coatings as A and B (Fig. 45), we have a field of capacity, say C_A, bounded by the coating A and the inside of the room; next, a field of capacity, C, extending through the glass between the coatings A and B; and, lastly, there is a field of capacity, C_B, bounded by the coating B and by the inside of the room.

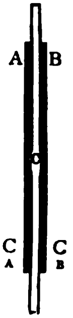

Fig. 45.

We will refer to these in the order in which they have been named, as the first, second, and third fields respectively. Let us begin by supposing the coating B connected with the room: then the boundaries of the third field being at the same potential, its charge is zero, and the coating A becomes a common boundary of the first and second. Putting V for the potential of the room, and V_0 for the initial potential of A, the charge of the first field is $C_A\,(V_0 - V)$, and that of the second field, the field of the condenser is

$$Q_0 = C\,(V_0 - V).^1$$

Now, after insulating B, connect A with the room, thus bringing its potential to the value V. The effect of this is that the charge of the first field vanishes, and that that of the second spreads into the third field, the coating B, whose potential we will denote by

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[1] It may be noted that the electric force in both these fields is directed from the surface A, if V_0 is greater than V, and towards A, if V_0 is less than V,—that is, in either case, the force is oppositely directed in space in the two fields.

V_1', becoming a common boundary of the second and third fields. Expressing by an equation the fact that the combined charges of these two fields is the same as the previous charge of the second field, we have

$$(C + C_B)(V - V_1') = C(V_o - V)$$

which gives for the present difference of potentials between the coatings of the condenser—

$$V - V_1' = \frac{C}{C + C_B}(V_o - V).$$

That is to say, it is the original difference of potentials, $V_o - V$, multiplied by the proper fraction $\frac{C}{C + C_B}$. Denoting this, for shortness, by m, we may write

$$V - V_1' = m(V_o - V).$$

Next, let A be again insulated, and let B be connected with the room, as in the initial stage. The charge of the third field again disappears, and that of the second spreads into the first, the surface A being a common boundary. If we put V_1 for the resulting potential of A, the equation expressing identity between the combined charges of the first and second fields and the previous charge of the second is

$$(C_A + C)(V_1 - V) = C(V - V_1')$$

whence

$$V_1 - V = \frac{C}{C_A + C}(V - V_1') = n(V - V_1'), \text{ say,}$$

or, putting in the value just found for $V - V_1'$,

$$V_1 - V = mn(V_o - V).$$

The charge of the condenser is now $Q_1 = C(V_1 - V)$, and therefore

$$\begin{aligned} Q_1 &= mn\, C(V_o - V) \\ &= mn\, Q_o. \end{aligned}$$

That is, the charge of the condenser after each coating has been successively connected with the room and then insulated is equal to the original charge multiplied by the constant fraction

$$mn = \frac{C^2}{(C_A + C)(C + C_B)},$$

and by following out again exactly the same course of reasoning and calculation, we should find that the charge after repeating the same process would be

$$\begin{aligned} Q_2 &= mn\,Q_1 &&= (mn)^2\,Q_0 \\ Q_3 &= mn\,Q_2 &&= (mn)^3\,Q_0 \\ &\vdots \quad \vdots && \quad \vdots \\ Q_n &= mn\,Q_{n-1} &&= (mn)^n\,Q_0. \end{aligned}$$

It appears, then, that the charges remaining in the condenser after the surfaces have been uninsulated 1, 2, 3, . . . times, form the terms of a decreasing geometric series.

The successive decreases of charge due to repetitions of the process also form a similar series: thus, the first time, the loss of charge is

$$Q_0 - Q_1 = Q_0(1 - mn);$$

the second time it is

$$Q_1 - Q_2 = Q_1(1 - mn) = Q_0 mn(1 - mn);$$

the third time

$$Q_2 - Q_3 = Q_2(1 - mn) = Q_0 m^2 n^2 (1 - mn),$$

and so on.

The process of discharge by successive contacts may take place when the surfaces of a condenser are successively connected with any insulated conductor, and it then takes place essentially in the way that has been traced out. It is illustrated by various familiar experiments which need not be more than mentioned, as the electric chimes (Fig. 46), and Franklin's spider (Fig. 47), in which a small insulated conductor B or B′, suspended like a pendulum, strikes successively against knobs connected with the two coatings of a Leyden jar, being attracted in turn by each after coming in contact with the other.

The experiment known as *Lichtemberg's figures* is connected with the same kind of action. We take hold of a charged Leyden jar by the outer coating and draw the knob connected with the inner coating along the surface of a cake of resin, tracing out with it figures of any kind. Then, putting the jar on an insulating stand, we take it by the knob and trace other figures on the resin with the outside of the jar. On afterwards dusting over the cake of resin a mixture of flowers of sulphur and red-lead, the particles of red-lead collect along the lines traced with one surface of the jar, and the sulphur along those traced with the other surface. By their mutual friction the particles of the two powders become electrified, the red-lead positively and the sulphur negatively, and they consequently attach themselves to parts of the surface of the resin which have been electrified in the opposite way to themselves. The lines formed present marked characteristics; the yellow lines of sulphur, corresponding to the positively charged parts of the resin, show very fine ramifications, while the red marks, corresponding to the negatively charged parts, are broader and often resemble drops or splashes.

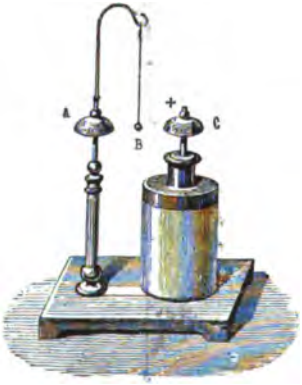

FIG. 46.

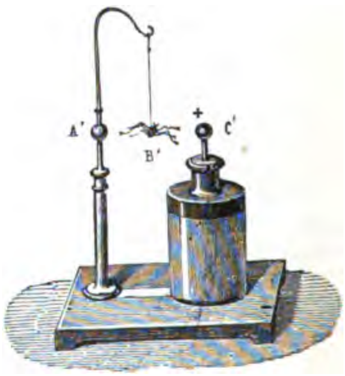

FIG. 47.

62. Energy of a Condenser.—The energy of a condenser, like that of any electric field (§ 55), is represented by

$$W = \tfrac{1}{2} Q (V - V') = \tfrac{1}{2} C (V - V')^2 = \tfrac{1}{2} \frac{Q^2}{C},$$

or (§ 52) by

$$W = \frac{SK}{8\pi e}(V - V')^2 = \frac{2\pi e}{SK}Q^2,$$

which shows that, for a given difference of potentials between its surfaces, the energy of a condenser is proportional directly to the area of the surfaces, and inversely to the thickness of the dielectric; while, for a given charge, the energy is directly proportional to thickness and inversely proportional to surface.

It follows from these formulæ, that if a charged Leyden jar has its surfaces connected with the corresponding surfaces of a second uncharged jar (which for simplicity we will suppose exactly similar to the first), so that its charge is shared between the two, the combined energy of the two jars is now only half the original energy of the first jar. By combining the two jars, we have virtually a condenser of twice the capacity, and therefore, with the same charge, only half the energy. The electric energy that is lost during the transfer of part of the charge from one jar to another is converted into heat during the process.

If several jars with their similar coatings connected are charged simultaneously to a given difference of potentials, the energy of the combination is obviously the sum of the energies of the separate jars. Thus if there are n jars each having capacity C, their total energy is

$$W = n \cdot \tfrac{1}{2} C (V - V')^2$$

or n times the energy of a single jar charged to the same difference of potentials.

If n similar jars, connected "in cascade," are charged so that the difference of potentials between the first and last coatings is $V - V'$, the difference between the coatings of each jar is $(V - V')/n$ and therefore its energy is $\frac{1}{2} C \left(\frac{V - V'}{n}\right)^2$, and the energy of the whole number is

$$W = n \cdot \tfrac{1}{2} C \left(\frac{V - V'}{n}\right)^2 = \frac{1}{n} \cdot \tfrac{1}{2} C (V - V')^2$$

or the nth part of the energy of a single jar charged to the same difference of potentials.

63. Effect of the Insulating Material.—It was first observed by Faraday that the capacity of a condenser depends not only on the size and shape of its surfaces, but also on the nature of the insulating material which fills the space between them. He had two spherical condensers made as nearly alike as possible (Fig. 48), the inner sphere, A, being supported concentrically with the outer one by a thick cylinder of shellac, *l*, fitting into a cylindrical neck, and the outer sphere being formed of two hemispherical cups fitting each other air-tight when put together. When one of the condensers had been charged, the fixed ball of a torsion-balance was put into contact with the knob *a* connected with the inner sphere A, and the charge of the ball was measured. The charge of the condenser was next shared between the two condensers, by connecting the two outer spheres by a wire, and then bringing the knob *a* of one into contact with the knob of the other. In this way the corresponding surfaces of the two condensers were brought to the same potentials. Then the fixed ball of the torsion-balance was again put into contact with the knob *a* and its charge measured as before. It was thus found that when both condensers were filled with air, the charge, measured by the torsion-balance after the sharing, was just half as great as that measured at first. On the other hand, if the space between the surfaces of one condenser was filled with some solid dielectric, such as glass, sulphur, or shellac, then the charge measured after sharing was more than half that measured before sharing, if the condenser first charged was that containing the solid dielectric; but it was less than half if the air-condenser was the one first charged.

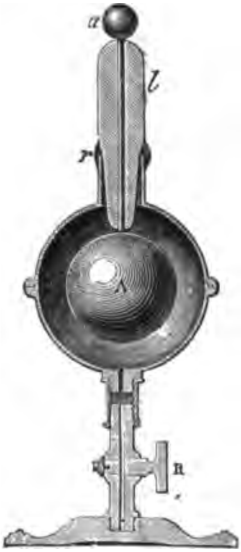

FIG. 48.

Throughout the experiments the outer surfaces of both condensers were connected with the room, so that the charge taken by the ball of the torsion-balance was in each case proportional to the excess of the potential of the inner sphere above that of the outer sphere, and the process was virtually a comparison of the difference of potentials between the surfaces of one condenser when first charged with the difference of potentials when it had given up part of its charge to the other. Putting V' for the constant potential of the room and outer surfaces of the condensers, V_1 for the initial potential of the inner surface of the charged condenser, and V_2 for its potential after sharing, we have

$$\frac{V_1 - V'}{V_2 - V'} = \frac{q_1}{q_2}$$

if q_1 and q_2 are the two charges measured by the torsion-balance. When the condensers are connected, the first loses a certain quantity of electricity, and the potential of its inner surface falls from V_1 to V_2; at the same time the second gains the same quantity, and the potential of its inner surface rises from V' to V_2. If now we call C_a and C_b the capacities of the two condensers respectively, and write an equation expressing that these are inversely as the changes of potential caused by the loss or gain of the same quantity of electricity, we get

$$\frac{C_b}{C_a} = \frac{V_1 - V_2}{V_2 - V'} = \frac{(V_1 - V') - (V_2 - V')}{V_2 - V'} = \frac{q_1}{q_2} - 1.$$

In this way the ratio of the capacities of the two condensers was obtained, and Faraday's results showed that the substitution of a solid dielectric for air increased the capacity. He gave the name *specific inductive capacity* to that property of an insulating material on which the capacity of a condenser formed of this material depends, or, in other words, to that property of the material occupying an electric field, of given shape and size, which determines the charge required to produce a given difference of potentials between the boundaries of the field.

64. Specific Inductive Capacity, or **Dielectric Coefficient.**—The specific inductive capacity, or, as it is otherwise called, the dielectric coefficient, of an insulating or dielectric material, is the quantity denoted in our formulæ by the symbol K. To see how its numerical value may be determined, we may, in the first instance, go back to the fundamental formula of electric force (§ 10), which, in the case of two equal quantities of electricity concentrated at points, we may write

$$Q^2 = fr^2K.$$

From this, if Q_1 is the quantity which repels an equal quantity at unit distance with unit force, we have

$$Q_1^2 = K$$

and, therefore, K may be defined numerically as equal to the square of the quantity Q_1, just defined.

But when the dielectric medium is air, the quantity which repels an equal quantity at unit distance with unit force is, by definition (§ 10), a unit of electricity. Hence, as already pointed out, our definition of the unit of electric quantity involves the numerical value $K = 1$ for air.

Another way of arriving at the value of K is by considering the relation between charge, difference of potentials, and dimensions of an electric field. Taking the simple case of a field bounded by plane parallel surfaces (§ 51), we have for the charge

$$Q = \frac{(V - V')\,KS}{4\pi e}$$

or, writing σ for Q/S, the surface-density, and transposing,

$$K = 4\pi\sigma \frac{e}{V - V'};$$

or, again, if σ_1 be the surface-density for which the difference of potentials is unity when the distance between the boundaries of the field is unity

$$K = 4\pi\sigma_1.$$

In words, this is equivalent to saying that the dielectric coefficient of a given substance is equal to 4π times the charge per square centimetre of a field, bounded by large flat parallel surfaces one centimetre apart and filled by that substance, when the difference of potentials between the boundaries is unity.

Seeing that the dielectric coefficient for air is numerically $= 1$, an experimental process for measuring its value for any other substance resolves itself into a comparison of the dielectric coefficient of that substance with the dielectric coefficient of air. Methods of carrying out such a comparison have been founded on the definition of K in the forms in which we have stated it above, but those most generally adopted depend, like Faraday's method, on the comparison of the capacities of condensers or of the same condenser with different dielectrics. By reference to the formula (§ 50) for the capacity of a tube of force, or to the results deduced from it for special cases of electric fields (§ 51), it will be seen that the expression consists of a factor depending solely upon the geometrical character of the tube, or field, multiplied by the value of K

for the particular dielectric medium occupying the field. Hence if we take two condensers whose electric fields are geometrically identical, but are occupied one by air and the second by any other dielectric, the ratio of the capacity of the second condenser to that of the first gives the value of the dielectric coefficient for the medium in question in terms of that for air. It is evident that instead of using two condensers for this purpose, one condenser will serve if we can compare its capacity, when its electric field is filled with a given dielectric medium, with its capacity when the field is filled with air.

Without entering into details of methods for the comparison of capacities, we may point out in a few words the general principles of the methods commonly adopted. Let two condensers to be compared be distinguished as A and B, then using corresponding suffixes to distinguish symbols referring to them respectively

$$\frac{C_A}{C_B} = \frac{Q_A (V_B - V'_B)}{Q_B (V_A - V'_A)}.$$

It is frequently possible, without knowing the difference of potentials, to ensure its being the same for each condenser, in which case a determination of the ratio of the charges gives the ratio of the capacities; or we may be able to make the charges equal, in which case the capacities are inversely as the differences of potentials.

The following are the approximate values of the specific inductive capacities of a few solid and liquid dielectrics, the inductive capacity of air being taken as unity :—

	Specific Inductive Capacity.
Paraffin wax	2.3
Rosin	2.6
Ebonite	3.2
Sulphur	3.8
Glass (plate or common flint)	6 to 7
Petroleum oil	2.1
Turpentine oil	2.2
Benzine	2.3

On the same scale, the specific inductive capacity of a vacuum is about 0.9994.

65. Residual Charge.—If the difference of potentials of a condenser with a solid dielectric is measured immediately after the condenser has been charged, and then again a few minutes or half-an-hour later, it is generally found, even when there is perfect insulation, that the second measurement gives a lower result than the first one. This diminution, when there is good insulation, does not go on continuously, but is comparatively rapid at first, and then becomes slower and slower, and at last imperceptible.

Again, if the surfaces of such a condenser, after it has been charged for some time, are connected for an instant by a thick wire, so as to discharge the condenser—that is, to bring its coatings to the same potential—it is found that a difference of potentials of the same sign as that previously existing makes its appearance, increasing comparatively quickly at first, and then more slowly, towards a constant value. This phenomenon is known as the *residual charge*, and is correlative with the decrease of difference of potentials spoken of above. The residual charge does not amount to more than a small proportion of the original charge, but a condenser that has been charged strongly for a long time will often allow of several successive discharges of progressively diminishing amount being obtained at intervals, one after another.

The formation of the residual charge implies that the ratio $Q/(V - V')$ for a condenser in which it occurs depends upon the time that has elapsed since the charge was imparted, or, more generally, upon the recent electrical history of the condenser, as well as upon its material and geometrical characters. This circumstance introduces complication into the experimental determination of the capacity of a condenser, which, as has been explained, is the name given to the ratio in question. The disturbance due to this cause may be avoided to a great extent by measuring the capacity of a condenser as soon as possible after it is charged,—that is to say, before the residual charge has had time to form to an appreciable extent.

66. Condenser with Removable Coatings.—According to the view we have adopted, the charging of a condenser, or of any electric field, consists in setting up a peculiar condition of stress within the dielectric. To do this the conducting surfaces which bound the dielectric must be connected with the conductor of an electric machine, or equivalent arrangement, between which an electric field is produced by friction or otherwise. By virtue of the connecting wires, the stresses existing in the original field are able to relieve themselves by causing an expansion and partial transfer of the field to the dielectric of the condenser. The function of the metallic coatings of a condenser appears to be simply to facilitate the transfer of electric stress to and from the dielectric material between them. So long as the stress exists, the dielectric is charged, even if the metal coatings are removed. This is well illustrated by means of a Leyden jar with removable coatings of thin tin-plate (Fig. 49). If this has been charged in the ordinary way, and placed on a cake of resin, or other insulating support, the glass jar can be lifted out from the outer metal vessel, and then the inner conductor can be lifted out from the jar by a glass rod put under the hooked stem. On bringing the two metal coatings together, they give only a slight discharge, if any; but if, after the metal coats have been shown to be electrically neutral, the jar is put together again, it will give a discharge almost as powerful as if it had never been taken to pieces.

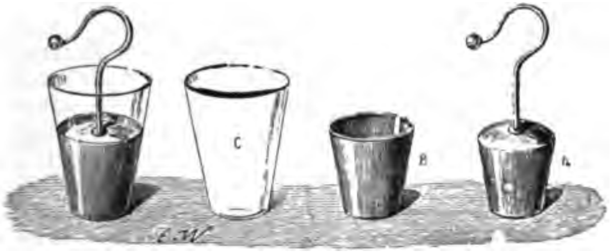

FIG. 49.

The same experiment can also readily be made with an Æpinus's condenser (§ 58).

CHAPTER VII.

EFFECTS OF THE DISCHARGE.

67. Conductive Discharge and Disruptive Discharge.—The energy which is accumulated in an electric field is expended during the discharge in various forms, which will now be briefly discussed.

When the boundaries of the field are put in conducting communication with each other, a spark is always produced. The experiment may be made in such a manner that this spark absorbs either the greater part of the available energy, or, on the contrary, only a very small portion. In the latter case the work of the discharge is chiefly expended in the conductors, and the discharge is said to be *conductive;* in the former case it is said to be *disruptive.*

68. Resistance of Conductors.—Whatever may be the nature of the conductors by the aid of which the discharge is effected, they always offer a certain *resistance* to the motion of electricity, and a greater or less part of the work available is expended in overcoming this resistance, and in so doing it produces an equivalent quantity of heat.

The quantity of heat disengaged in any portion of the conducting wire is readily measured, at any rate comparatively, by means of Riess's thermometer (Fig. 50). The portion, S, of the conductor to be investigated is placed between binding screws, A and B, in a glass globe to which is connected a slightly inclined capillary tube terminating in a much wider vertical tube, E. The whole rests on a hinged board, which can be fixed at any desired angle by the screw K. The capillary tube contains a coloured liquid, and when the discharge is passed, the increase of pressure produced by the heating of the gas is manifested by a displacement of the top of the capillary column. The heating takes place so rapidly that there is not time for appreciable loss of heat to occur; and as the volume of gas does not sensibly alter, the displacement is proportional to the change of pressure, which in turn is proportional to the rise of temperature, and therefore to the quantity of heat disengaged in the wire S.

In order to compare two conductors, they are both placed simultaneously in the circuit, each in turn, in the bulb of the thermometer, and the other outside, and the same quantity of electricity is discharged through the circuit. Experiment shows that the quantity of heat liberated, and consequently also the resistance, is, for a given metal, directly proportional to the length of the wire, and inversely proportional to its area of cross-section. It varies, moreover, with the material of the wire. In this way the resistance of the metals may be determined in comparison with any one taken as a standard. If ρ is the *specific resistance* of a metal—that is to say, the resistance expressed in any units which it offers for unit length and unit section—the resistance, R, of a conductor of this metal, of length l, and section s, in terms of the same unit, will be

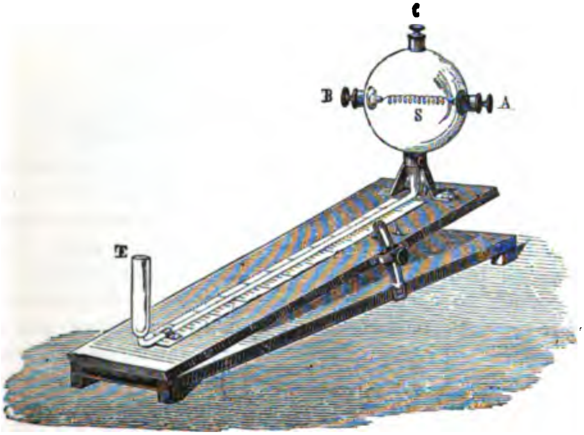

FIG. 50.

$$R = \rho \frac{l}{s}$$

We shall hereafter become acquainted with methods more convenient than that of Riess's thermometer for determining the resistance of a conductor, and of obtaining it not merely in relative, but in absolute value. The practical unit of resistance is the *ohm.* This will afterwards be defined theoretically; for the present it will be sufficient to know that it is the resistance presented at 0° by a column of mercury a square millimetre in cross-section and 106.3 centimetres in length.

69. Fusion and Volatilisation of Metals.—In the conductive discharge all the energy is converted into heat in the conductor itself, provided no external work is done. We have already seen (§ 39) that if we express quantity of electricity in coulombs, difference of potentials in volts, and capacity in farads, electric energy is expressed in joules. On the other hand, a *gramme-degree*,—that is, the quantity of heat necessary to raise the temperature of one gramme of water at 0° through *one* degree,—is equal to 4.17 joules. This number is called the *mechanical equivalent of heat*, and is denoted by the letter J. If H is the number of gramme-degrees disengaged, we have

$$JH = W = \tfrac{1}{2}\,Q\,(V - V') = \tfrac{1}{2}\,C\,(V - V')^2 = \tfrac{1}{2}\,\frac{Q^2}{C}.$$

The quantity of heat corresponding to a given discharge is a fixed quantity; it is independent of the conditions in which the discharge takes place, and in particular of the nature of the conductor. If this latter is made up of two parts, K' and K'', experiment shows that the quantity of heat divides between them in proportion to their respective resistances. Accordingly, by taking very stout conductors of very small resistance, and interposing between them a wire of great resistance, we may concentrate in the latter almost the whole of the heat produced by the discharge. Thus with an apparatus called the *universal discharger* (Fig. 51), we may raise the temperature of a wire of any material stretched between the two knobs D and D′ sufficiently to melt it, or even to volatilise it.

If H is the quantity of heat available, m the mass of the metal, d its density, and c its mean specific heat, the temperature, t, to which it will be raised from 0° by the quantity of heat, H, is

$$t = \frac{H}{mc} = \frac{H}{sldc}$$

if we disregard any losses by radiation.

Hence, for a given metal, the temperature depends only on the mass of metal; it is higher as the mass is less—that is to say, as the wire is finer and shorter. For two wires of the same mass but of different metals, it is inversely as the specific heat; lastly, for two wires of different metals, of the same section and the same length, it is inversely as the product of the density into the specific heat—that is to say, the thermal capacity of unit volume.

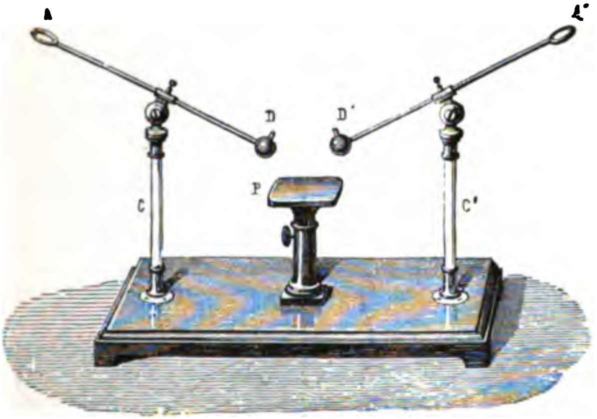

FIG. 51.

If the wire is to be fused by a given quantity of heat, H, the mass of the wire may be determined by the equation

$$m\,(cT + \lambda) = H$$

in which T is the melting point and λ the heat of fusion of the metal.

All metals may thus be either melted or volatilised; iron is suddenly reduced to droplets, which burn in the air, throwing out sparks, and producing oxide of iron. Silk threads interwoven with fine gold, silver, or copper wire, are very well suited for these experiments. A quite moderate discharge is sufficient to volatilise them with explosive violence. The metal disappears completely in fumes, which rapidly settle. They are easily collected on a card placed against the wire (Fig. 52), leaving a deposit on it which is characteristic of

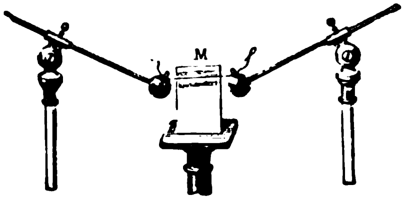

FIG. 52.

the metal, both in shape and in colour. The silk usually remains unchanged. The explosion which accompanies the volatilisation of the metal is an indication that the air is violently disturbed. If the experiment is made by immersing the ends of the rods between which the wire is stretched in a vessel of water (Fig. 53), the disturbance which takes place when the discharge is passed, being transmitted through an incompressible liquid, smashes the glass with a great noise. The effect resembles that of a torpedo.

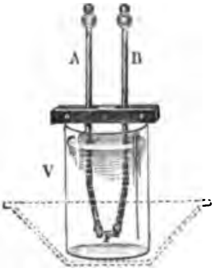

FIG. 53.

In what is known as *Franklin's experiment* (Fig. 54), a piece of gold-leaf, FF′, is applied against a piece of paper, B, in which a portrait of Franklin is cut out; on the other side is placed a white silk ribbon, C, and the whole is strongly clamped between two slips of wood, P. When a discharge is passed, the gold is volatilised, its vapours pass through the parts cut out, and reproduce the design in a violet-brown colour on the ribbon.

70. Passage of the discharge through Bad Conductors.—If a bad conductor is interposed in the circuit, the greater part of the energy is expended in the mechanical operations of tearing and of rupture. A sheet of paper, a card, a thin sheet of glass are easily perforated even by rather weak discharges.

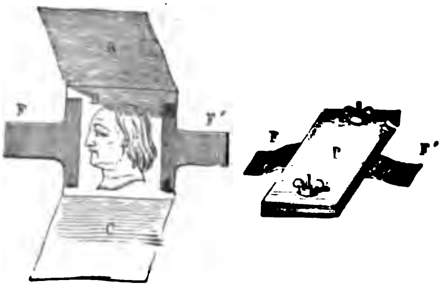

FIG. 54.

In the experiment with the card represented in Fig. 55, when the discharge is passed, the perforation produced exhibits a kind of welt round the edges, and if the two points are not at the same height, the hole is nearer the negative point.

Electricity passes less easily through glass. The experiment may be made by means of the apparatus represented in Fig. 56. The greatest difficulty in this experiment is to prevent the spark from travelling over the surface of the glass plate. This should be well warmed, and a drop of oil placed round the upper point. With powerful batteries glass plates of some centimetres in thickness may be perforated; in this case the two points should be completely imbedded in a bad conductor, such as resin or wax, or, still better, a mixture of the two.

71. Passage of the discharge over the surface of Bad Conductors.—The property which the discharge possesses of travelling over the surface of bad conductors gives rise to remarkable luminous phenomena. When a battery is so strongly charged that it is on the point of discharging spontaneously between the two coatings, a crackling sound is heard, and at the same time numerous lines of light spread over the uncoated surface of the jars.

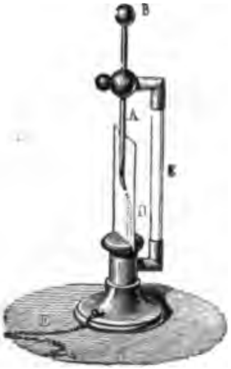

FIG. 55.

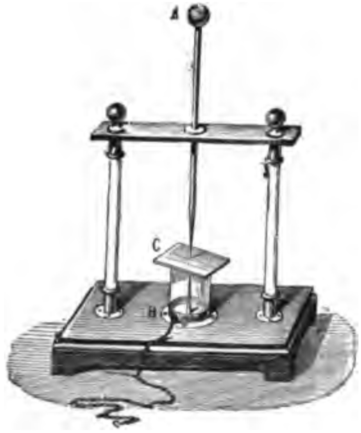

FIG. 56.

Very beautiful effects are obtained by covering a glass plate with tin-foil on one side only. One of the limbs of the discharger is placed in contact with the tin-foil, and the other at a small distance in front of the uncoated surface. Each discharge gives rise to very beautiful ramifications.

The discharges which travel thus over the surface of the glass leave a permanent trace, which can be made visible by breathing on the surface so as to deposit a little moisture.

The propagation of the spark along the surface of badly conducting bodies is facilitated by portions of conducting material dispersed over them. Very beautiful effects are thus obtained with what is known as the ***luminous jar*** (Fig. 57); this is a Leyden jar, the outer coating of which, instead of being continuous, consists of metal filings fastened on with varnish. The inner coating is prolonged by a tongue of tin-foil to within a short distance from the outer coating. Every spark which passes between the two armatures branches out over the latter.

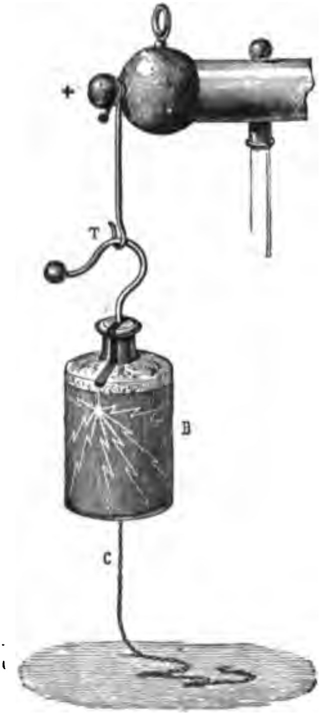

FIG. 57.

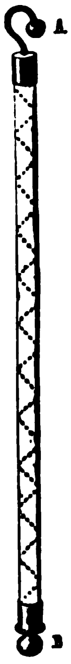

FIG. 58.

With these phenomena may be associated the luminous tubes and panes (Fig. 58), obtained by fastening on the surface of glass a series of small lozenges of tin-foil, between which sparks pass simultaneously, producing the effect of a continuous line of light.

72. Disruptive Discharge.—The disruptive discharge (§§ 20, 67) presents a great variety of forms, which may be referred to three types—the *spark*, the *brush*, and the *glow*.

All these forms have one common character, which is that, notwithstanding their short duration, they are not simple phenomena. When examined by means of a rotating mirror, a discharge is usually seen to be an intermittent phenomenon consisting of a great number of successive discharges in opposite directions, which have a well marked oscillatory character, as if the electricity went from one armature to another until it had lost all its energy.

The *spark* when it is short appears as a bright line of light, which is the more dense the greater the quantity of electricity. In proportion as the distance increases or as the capacity of the condenser diminishes, the line is less defined; it becomes narrower and less luminous, and acquires a zigzag form (Fig. 59), passing into that of a brush.

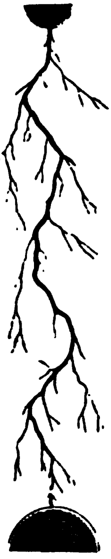

FIG. 59.

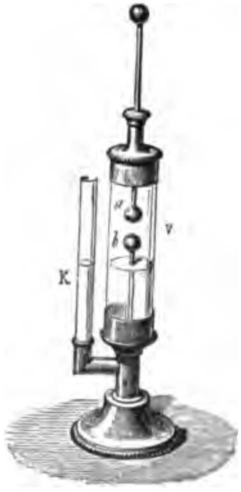

FIG. 60.

The production of the spark is always accompanied by a hard crackling sound, due to a violent mechanical disturbance of the medium, as may be illustrated by means of *Kinnersley's thermometer* (Fig. 60). When the discharge passes between the two knobs, the water is driven out of the larger tube into the smaller one.

If the spark is passed through water in the apparatus represented in Fig. 61, the disturbance is sufficient to break the tube.

The ***brush*** (Fig. 62) has a pale violet tint, and is accompanied by a peculiar continuous hissing sound, very different from the sharp crack of the spark discharge. The whole has an ovoidal form, and the ramifications which proceed from the positive pole are connected with it by a kind of brightly luminous stem. The negative pole appears as though it were clothed with a luminous layer; if it ends in a point, this is tipped with a small brilliant star.

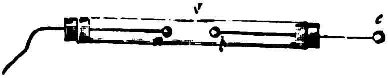

FIG. 61.

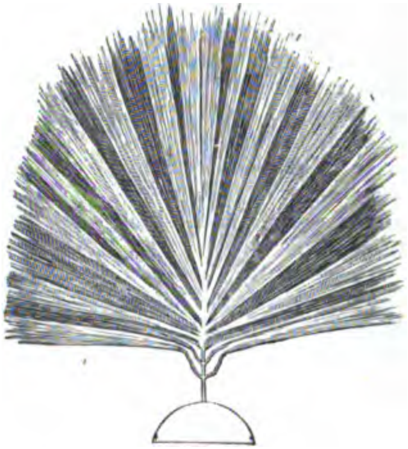

FIG. 62.

The *glow* is produced when the pressure of the gas is diminished. This can be investigated by means of the *electrical egg* (Fig. 63) or what are known as *Geissler's tubes* (Fig. 64). These are filled with various gases and are hermetically sealed after being exhausted. Platinum wires fused into the glass serve as conductors for the electricity.

The glow always seems to start from the positive pole; the negative pole is surrounded by a violet aureole followed by a darker space. Its colour varies with the nature of the gas; with air it is rose violet, white with carbonic acid, and bluish violet with hydrogen. The colour changes also with the pressure of the gas and the density of the flow of electricity which traverses it. With the tube represented in Fig. 64 filled with rarefied hydrogen, the glow which is blue in the wider parts is reddish crimson in the capillary portion which joins them.

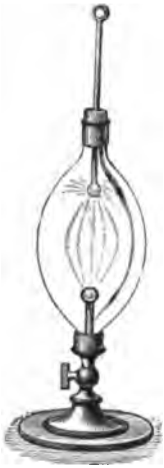

FIG. 63.

When the rarefaction attains a certain point, the light is formed of parallel striæ alternately bright and dark; this is the phenomenon of *stratification;* it is apparently due to the intermittent nature of the discharge. The striæ appear always to start from the positive pole. Their number, their shape, and the sharpness of their outline, depend on circumstances which are difficult to define. A conductor, the hand for instance, when brought near the tube has a perceptible action on the phenomenon.

The glow-light has the property of readily exciting fluorescence. The glass tube itself becomes more or less fluorescent according to its composition. If it contains a little uranium it acquires a very beautiful green phosphorescence. Very beautiful effects are also obtained by surrounding the tube through which the discharge passes with solutions of sulphate of quinine or other fluorescent substances.

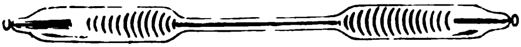

FIG. 64.

If the rarefaction is pushed still further, so that the pressure does not exceed a few millionths of an atmosphere, the phenomenon acquires a special character, which has been very completely investigated by Mr. Crookes. The glow-light disappears entirely, and the passage of electricity is only manifested by the fluorescence of the glass tube, which acquires an extraordinary lustre in the part opposite the negative pole.

The phenomena occur as if electrified particles were shot from the negative electrode at right angles to the surface.

Thus if one electrode has the shape of a small concave mirror of suitable curvature, the discharge which starts from it forms a cone having its apex on the opposite surface, where it forms a brightly luminous spot. In Fig. 65, the electrode, *a*, is a small convex mirror, while *b* is a cross of aluminium which is so fastened by a small joint that it can be raised or lowered.

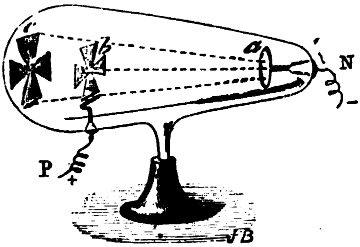

FIG. 65.

When the discharge passes, the glow-light proceeds in straight lines, and impinging on the glass makes the whole of it fluorescent, except that portion corresponding to the cross, which thus throws a shadow. If now the cross, *b*, is lowered, the part which was previously in shadow is now more brightly luminous than the rest, so that a bright cross stands out on a less bright ground.

The experiment shown in Fig. 66 seems to demonstrate the material character of these radiators. A small wheel with vanes of mica rests with its axle on two glass rails; two small electrodes of aluminium are fused into the ends of the tube just above the middle. When the discharge is passed, it acts on the upper half of the wheel and moves it from one end of the tube to the other towards the positive pole.

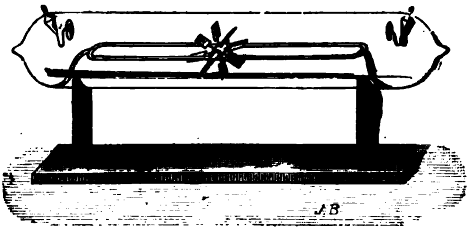

FIG. 66.

Spectroscopic examination of these luminous phenomena produced by the discharge throws some light on their nature. The light of the spark shows lines which are characteristic of the metals of the electrodes between which the spark passes, and at the same time of the gas in which it is formed. The light of the former predominates especially in the case of strong sparks. If the electrodes are of two different metals, the lines due to both metals are found. A portion of the energy is therefore expended in detaching and volatilising particles of metal from the two electrodes. It is observed, for instance, that after a somewhat powerful discharge between a gold knob and a silver one, silver is found on the gold, and gold on the silver knob.

In the light of the glow discharge no trace of metallic lines is met with; the spectrum is the same whatever the nature of the metal, and is due solely to the incandescent gas.

73. Striking Distance.—The length of the spark in air under the ordinary pressure, or what is known as the *striking distance*, depends on the difference of potentials of the two electrodes. It varies a little with the shape of the electrodes; thus, for the same striking distance, the difference of potentials required between two spheres is greater than between two plates. It must be observed that in the case of two parallel plates, which are large in proportion to their distance apart, if the difference of potentials increased in proportion to the striking distance, the discharge would always take place with the same electrical force, and consequently for the same density of the electrical layer (§ 43). As a matter of fact, the difference of potentials increases less rapidly than the distance, the rate of increase becoming less rapid the greater the distance. This is seen from the following table, which refers to the discharge between two knobs each one centimetre in diameter:—

Distance of the Knobs. Cm.	Difference of Potentials.	
	In Electrostatic Units.	In Volts.
0.1	16.1	4,830
0.5	56.3	16,890
1.0	84.7	25,440
1.5	97.8	29,340
2.0	104.5	31,350
3.0	124.0	37,200
5.0	153.0	45,900
10.0	187.0	56,100
15.0	206.0	61,800

The striking distance in air is sensibly the same for the spark properly so called as for the brush. This is readily demonstrated by a Holtz machine, with or without the Leyden jars, which increase the capacity of the conductors (§ 91). Without the jars, the brush is formed continuously and the difference of potentials between the two electrodes is sensibly constant. Using the jars, the sparks pass intermittently, the difference of potentials increases until the spark is produced, when it falls almost to zero.

When the pressure is diminished, the difference of potentials corresponding to a given striking distance diminishes rapidly to a certain limit, beyond which it again increases with extreme rapidity. There appears to be one particular pressure at which the resistance to the discharge is a minimum. This varies in different gases, and with one and the same gas it varies with the dimensions of the tube. With air it is about 3 mm. in a space of an ovoidal form, but it is considerably smaller in tubes. Lastly, as the pressure is gradually diminished a point is reached at which the spark does not pass, whatever may be the difference of potentials. It seems to result from this, that matter is necessary for the transmission of electricity, and that it is the molecules of the dielectric itself which serve as the vehicle.

74. Chemical Effects of the Spark.—The spark in air ignites combustible bodies like alcohol and ether; it ignites mixtures of oxygen with combustible bodies like hydrogen and its compounds, carbonic oxide, &c. The eudiometer is based on this principle. In this, as in *Volta's pistol* (Fig. 67), a spark is caused to pass through a detonating mixture of gases. The rod, DE, is insulated from the side of the vessel, and the moment a spark passes between the electrified body and the knob D, a spark also strikes between the knob E and the side of the vessel which is connected with the earth.

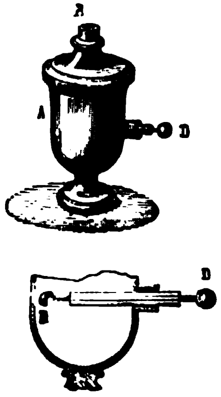

FIG. 67.

A succession of sparks decomposes ammoniacal gas into its elements, nitrogen and hydrogen, until a certain proportion of these gases has been produced. If hydrogen and nitrogen are present in larger proportion, the passage of sparks causes the gases to combine and reform ammonia. By means of an apparatus such as that shown in Fig. 68, it was proved by Cavendish that the passage of electric sparks through a mixture of nitrogen and oxygen produced nitric acid. The ends of the V-tube filled with the mixture of the two gases are placed in two vessels of mercury, one of which is connected with the prime conductor of an electrical machine, while the other is put to the rubber. The gases should be slightly moist, and when the machine is worked the mercury rises in the tube in proportion as the gases combine to form the acid.

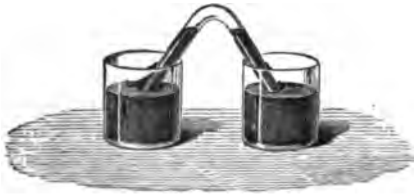

FIG. 68.

The brush discharge, or *electric effluvium*, produces in some cases characteristic effects different from those of a sudden spark discharge. An important example is the transformation of oxygen into ozone, which takes place when a current of oxygen is passed between two glass plates, or two glass tubes one inside the other, each coated on the farther side with tin-foil connected with the terminals of an induction-coil or of a Holtz machine. The proportion of ozone formed is increased when the temperature is kept low. If concentric tubes are employed, the tin-foil coatings may with advantage be replaced by strong sulphuric acid. Fig. 69 shows a convenient form of apparatus.

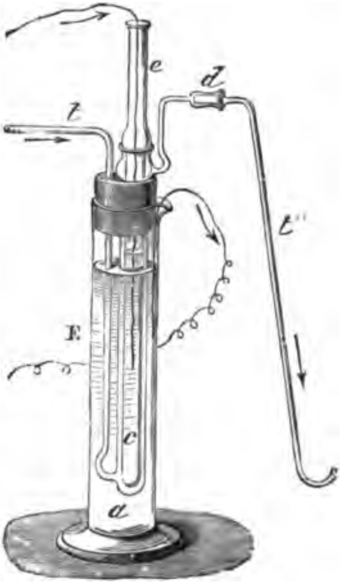

FIG. 69.

75. Physiological Effects of the Discharge.—When the discharge is passed through the human body, a shock is felt which, according to the energy of the charge, may be either a simple prickling or a disabling shock. The experiment is made by holding a charged Leyden jar in one hand by its outside coating while the other hand is brought to touch the knob connected with the inner coating. The shock may also be taken by a large number of persons at once, by joining hands in a chain, while the first grasps the outside, and the one at the end brings his hand to the knob.

Experiment shows that the physiological effect depends on the energy of the discharge, that is to say, jointly on the fall of potential and on the quantity of electricity. Thus we may take with impunity sparks from an ordinary machine 20 to 30 cm. in length, while one of only a few millimetres from a battery of great capacity could not be borne. In like manner, for the same quantity of electricity the discharge by cascade gives a more violent shock than that of a single jar. Another circumstance which comes into play is the duration of the discharge and the manner in which it is effected; thus a battery which would give a violent shock under ordinary circumstances only gives a very feeble one when the discharge takes place through a moist cord forming part of the circuit. The effect of this is to increase the duration of the discharge, and at the same time, without altering the total energy of the discharge, to cause a considerable part of it to be converted into heat.

CHAPTER VIII.

APPARATUS FOR ELECTROSTATIC MEASUREMENT.

76. Coulomb's Balance.—This fundamental apparatus, by which a quantity of electricity may be measured in absolute value, has been previously described (§ 9). If the two balls are equal, and q is the charge on each, then for an angle α, which does not exceed 20°, the force between them

$$f = \frac{q^2}{d^2} = \frac{q^2}{l^2\alpha^2}.$$

On the other hand, the condition of equilibrium between the moment of this force and the moment of the torsion-couple is expressed by the equation

$$fl = C(A + \alpha)$$

from which

$$q^2 = Cl(A + \alpha)\alpha^2.$$

77. Gold-Leaf Electroscope.—This instrument has already been described (§ 13) as well as the manner in which it is used. A few words may here be added to complete the theory.

In order that the indications of the instrument may be definite, the cage which contains the leaves should be at a known potential—that of the earth, for instance. It is advisable to make the jar a conductor by lining the interior to within a convenient distance from the rod with strips of tin-foil, or with brass gauze of somewhat large mesh. This as well as the knobs are in connection with the ground. An excellent earth-contact is obtained by connecting the apparatus with gas or water pipes.

If V is the potential of the leaves and V_0 that of the cage, the charge of the leaves is proportional to their capacity and to the difference of potentials $V - V_0$ (§ 49). The divergence, which depends on the charge, serves to measure either the charge or the potential. If the capacity were independent of the divergence, the potential and the charge would vary proportionally to each other, and the same graduation would serve for both cases.

Faraday's cylinder (§ 15) furnishes a very simple means of graduating an electrometer for comparing charges.

78. Condensing Electroscope.—We have sometimes to deal with electrification which is very great as to quantity, but which is so distributed as not to produce a difference of potentials sufficient to cause a visible divergence of the leaves of an electroscope employed as hitherto described. In such cases it is advantageous to use a *Volta's condenser* (Fig. 70) in connection with the electroscope. This consists of two metal plates varnished on the surfaces facing each other, the two layers of varnish forming an insulating plate.

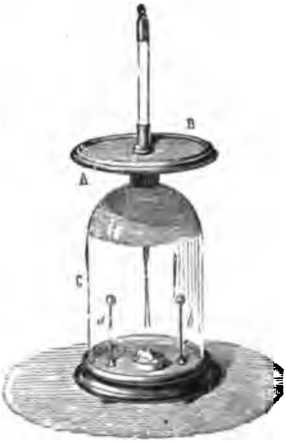

FIG. 70.

The two boundaries of the electrified field to be tested are connected, one with one of the plates of the condenser, and the other with the other. If, as is the commonest case, one of the boundaries consists of an insulated conductor and the other of the surface of the room, the former is connected, say, with the lower plate, while the upper plate is connected with the room either through the body of the experimenter, who touches it with his finger, or, still better, by a wire. The connections with both plates are removed after a moment or two, and then the upper plate is raised by the insulating handle. The capacity of the Volta's condenser which is very great when the plates are together, is thus greatly diminished, and the potential of the lower plate and leaves, which was previously nearly the same as that of the room, now differs from it enough to make the gold leaves diverge.

The phenomenon of electrical absorption (§ 65) explains why it is necessary that the insulating layer should consist of two parts: it is necessary that the upper plate should remove, with the adjacent half of the insulating layer, the charge which the latter may have absorbed.

79. Quadrant Electrometer.—This instrument (Fig. 71), which was invented by Lord Kelvin, serves for the same purpose as the gold-leaf electroscope, but is far more sensitive and is better adapted for accurate quantitative measurements.

It consists of two pairs of quadrants, AA′, BB′ (Fig. 72), which together form a flat cylindrical metal box cut into four equal sectors by two diametral sections at right angles to each other. Each quadrant is supported by an insulating glass stem, and the alternate pairs of quadrants, AA′ and BB′, are in conducting communication with each other. Each pair is connected also with a small insulated rod which is called the *electrode*, and which serves to connect up with bodies on the outside.

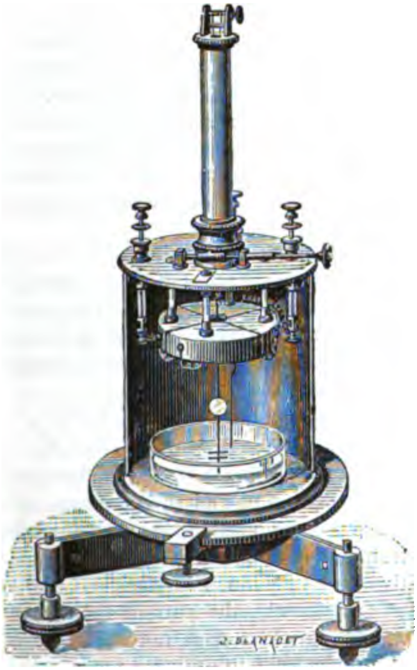

FIG. 71.

In the middle of the quadrants is suspended what is called the *needle*, which has the shape represented in the figure, and consists essentially of two quadrants of thin sheet aluminium joined at the centre, parts being cut out, as shown in the figure, for still greater lightness.

The needle hangs by two cocoon fibres, which constitute what is called a *bifilar suspension.*

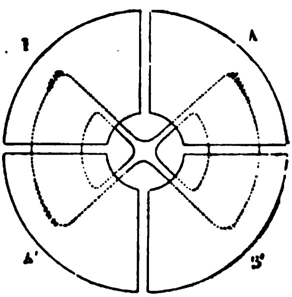

FIG. 72.

The position of equilibrium of the needle is that in which the two fibres are in the same plane. For a small deflection the force which tends to bring the needle back to its position of equilibrium is proportional to the angle of deflection. The suspension must be adjusted so that the position of equilibrium of the needle is symmetrical in reference to the space which separates the quadrants.

The needle diverges from its position of equilibrium when the potentials of the two alternate pairs of quadrants differ from each other. If V, V_1, and V_2 are the potentials of the needle and of the two pairs of quadrants respectively, and A is a factor which depends on the construction of the instrument, the deflection θ is given by the equation

$$\theta = A\,(V_1 - V_2)\left[V - \frac{V + V_2}{2}\right],$$

that is, it is jointly proportional to the difference of potentials of the two pairs of quadrants, and to the excess of the potential of the needle above the mean potential of the quadrants.

As the electrometer is commonly used, the potential V of the needle is positive and very much higher than that of either pair of quadrants. Consequently, the part of the above formula enclosed within square brackets does not differ much from V, and we may write, approximately, in ordinary cases

$$\theta = A\,(V_1 - V_2)\,V.$$

In order to keep the potential of the needle as nearly constant as possible, it is connected with the inner surface of a Leyden jar, generally formed by a glass vessel forming part of the outer case of the electrometer. In the more complete forms of instrument, there is a small electrical machine (called a *replenisher*) whereby the potential of the jar and needle can be brought to any required value; and also a subsidiary electrometer (the *gauge*) which shows when the right value is attained. These two organs are referred to in more detail below (§§ 83 and 90).

Any two conductors, the difference of whose potentials is to be measured, are connected one to each of the electrodes belonging to the respective pairs of quadrants and the resulting deflection θ is observed (see § 80). The required difference of potentials would then be given by

$$V_1 - V_2 = \theta/(A\ V)$$

if V and the factor A were known for the electrometer used. But as these hardly admit of being accurately determined directly, the usual plan is to observe the deflection, say θ_{ab}, corresponding to some known difference of potentials $V_a - V_b$, and then to eliminate A and V from two equations similar to the last. We thus get the required difference of potentials in the form

$$V_1 - V_2 = (V_a - V_b)\ \theta/\theta_{ab}.$$

If it is required to determine the amount by which the potentia of an insulated conductor differs from that of the earth, the electrode of one pair of quadrants is connected with the room and the other with the conductor to be tested.

A different mode of employing the quadrant electrometer is often adopted by Continental experimenters. The two pairs of quadrants are maintained at equal opposite potentials (so that their mean potential is zero) by connecting them respectively with the opposite terminals of a large number of small galvanic cells connected in series, the middle of the series being put to earth. Thus arranged, the electrometer is adapted for indicating the difference of potentials between an insulated conductor and the earth. The conductor is connected with the needle, and its potential, reckoned from that of the earth (or the room) taken as zero, is given by

$$V = \frac{\theta}{A(V_1 - V_2)}.$$

If a difference of potentials between two insulated conductors is to be measured, each is connected separately with the needle and the corresponding values of θ are observed. Their difference gives the required difference of potentials, thus

$$V - V' = \frac{\theta - \theta'}{A\ (V_1 - V_2)}.$$

As before, the factor A can be eliminated by observing the effect due to some known difference of potentials.

Another way of using the electrometer is to connect the needle with one pair of quadrants, and with one of the two bodies whose difference of potentials is required, while the second of these is connected with the second pair of quadrants. In this case, $V_1 = V$, and the deflection becomes

$$\theta = \tfrac{1}{2} A \, (V - V_2)^2$$

and is thus always in the same direction, whether the difference $V - V_2$ is positive or negative. This circumstance makes this method of employing the quadrant-electrometer of great value when, as sometimes happens, a rapidly alternating difference of potentials has to be measured.

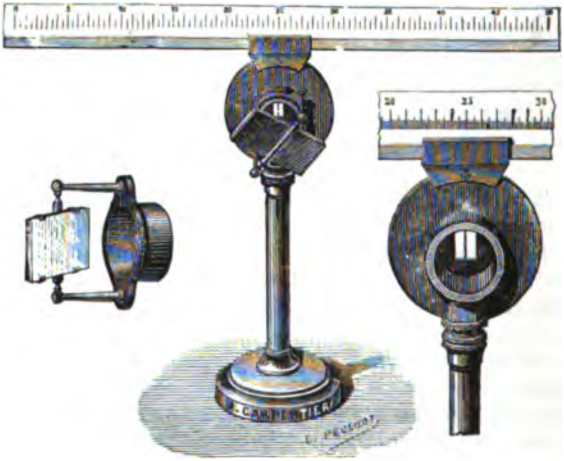

FIG. 73.

80. Reading by Scale and Mirror.—The deflections are measured by the method of reflection. A small concave mirror (Fig. 71) is supported by the rod of the needle, and is movable with it. A vertical slit of light and a scale divided into millimetres (Fig. 73) are placed in the vertical plane passing through the centre of curvature and perpendicular to the axis of the mirror, at equal distances above and below the centre of curvature. The image of the slit of light is produced in its real size on the scale. Instead of a narrow slit, a wider one may be used, in the middle of which an opaque wire is stretched vertically (Fig. 73). The image of the opening moves along the scale, and, as the region where the shadow of the wire falls is bright, the divisions are easily read off. Very good results are also obtained with a scale on ground glass, which is read from behind.

Let p (Fig. 74) be the division of the scale which corresponds to the position of equilibrium of the needle; if the needle is deflected through an angle α, the image is formed on the division x, such that

$$x - p = d \tan 2\alpha$$

d being the distance of the scale from the mirror. As the angles are always very small, we may take the arc for the tangent and replace $\tan 2\alpha$ by 2α, which gives

$$\alpha = \frac{x - p}{2d}.$$

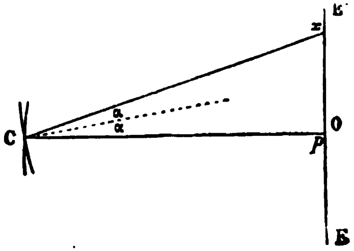

FIG. 74.

In order to deduce the potential of a needle from the deflection observed in any case, it is sufficient to know the deflection produced by a known potential.

81. Difference of Potentials between a point in the Air and the Earth.—If an insulated conductor is placed in an electric field, the surface of the conductor is everywhere at the same potential, and becomes continuous with some one equipotential surface of the field. Those parts of the surface of the conductor which lie to one side of this equipotential surface extend into regions where the undisturbed potential of the field would be higher than that of the conductor, and the other parts of the surface extend into regions where the potential would be lower. The electrification assumed by the conductor is such as to keep the potential of the first-mentioned parts down, and to bring that of the remaining parts up to the actual uniform potential possessed by the conductor as a whole. That is to say, the parts of the surface of the conductor which extend into the region of higher potential acquire a negative charge, and those parts which extend into the region of lower potential acquire a positive charge. If now a portion of the surface of the conductor were to become detached, say from the part which is negatively electrified, carrying its electrification with it, the potential of the whole remaining part of the conductor would rise; and if the part that has been detached were in some way to grow again, a conductor of the original form would be reproduced, but it would have a somewhat higher potential than before. Now, imagine this process of successive separation and reproduction of a portion of the surface of the conductor to go on, over and over again, always at the same part: it is obvious that the potential of the conductor as a whole will go on rising (if the portions that become detached are negatively electrified, or, in the opposite case, that it will go on falling) until the surface-density of the detached portions becomes zero, when there will be no further change.

To apply this principle to the experimental determination of the potential at a point in the air, a vessel of water is placed on an insulating stand (Fig. 75), and provided with a long, very narrow tube, from the end of which the water escapes in a fine stream. The whole vessel is thus gradually brought to the potential of the point at which the stream of water breaks into separate drops. To observe this, one pair of quadrants is connected with the water-vessel, and the other pair with the earth. The flame of a spirit-lamp connected by a fine wire with the electrometer acts in the same way as a water-jet, or a piece of touch-paper prepared by soaking paper with a solution of nitrate of lead may be used.

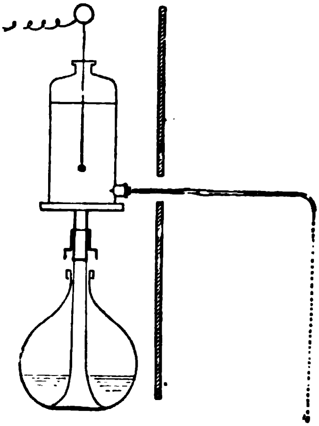

FIG. 75.

82. Absolute Electrometer.—This is based on the attraction which is exerted between a movable plate and an infinite plane parallel to each other and at different potentials.

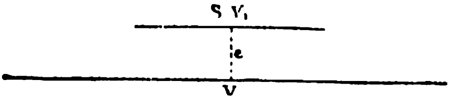

FIG. 76.

Let V_1 be the potential of the movable plate, V that of the plane, and e their distance apart. Neglecting perturbations arising from the edges, the lines of force are straight, perpendicular to the planes. In other words, the tubes of force extending between the opposed surfaces have the same cross-section from end to end, and the intensity of force is therefore uniform throughout and equal to $4\pi\sigma/K$, where σ is the equal surface-density at each end of the tubes, positive at one end and negative at the other (§§ 43, 45). But the electrification at each end of the tube contributes equally to the force within it, hence the force due to each end is $2\pi\sigma/K$, and this represents the pull exerted by one plate on each unit of electricity on the other (for the mutual forces exerted by the various portions of the charge on either plate cannot have a resultant tending to move that plate). The pull is, therefore, $2\pi\sigma^2/K$ per unit of surface, or $F = 2\pi\sigma^2 S/K$ for the whole surface.

Since, further,

$$\frac{\sigma}{K} = \frac{V_1 - V}{4\pi e},$$

we have

$$F = \frac{2\pi\sigma^2 S}{K} = \frac{SK}{8\pi e^2}(V_1 - V)^2.$$

In order to avoid the disturbing effect of the edges, the movable plate S is cut out of a larger plate, AB, which is called the *guard ring* (Fig. 77). The plate S can move freely in this, with which it is in metallic connection, and thus at the same potential. The plate AB is the bottom of a box, ABCD, which completes the system, and prevents any action which might be exerted from the outside on the upper face of the plate.

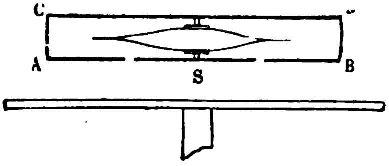

FIG. 77.

The plate, S, is supported by a spring which keeps it a little above the plane of the ring. The experiment consists in raising the lower plate A (Fig. 78), which represents the infinite plane and is at the potential V, until the plate S, as observed by help of the lenses l and l', appears exactly in the plane of the guard ring.

On the other hand, it is easy to determine the mass P, whose weight causes the plate S to sink through the same distance. The attraction is, then, equal to Pg dynes.

We thus have $F = Pg$, and therefore the difference of potentials becomes

$$V_1 - V = e\sqrt{\frac{8\pi Pg}{SK}}.$$

It is somewhat difficult to measure accurately the distance e of the two plates. This difficulty is got rid of by the following device : The lower plate is put to earth so that its potential $V = 0$,

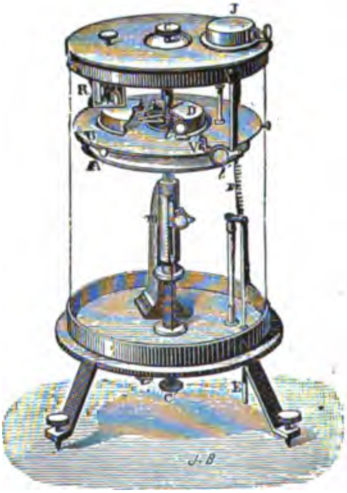

FIG. 78.

and the reading of the micrometer m, which satisfies the condition above stated, is noted : suppose the corresponding distance between the surfaces is e'. The plate is then put in connection with the body at the potential V, and the distance is again adjusted : if it is now e, we have

$$V_1 = e'\sqrt{\frac{8\pi Pg}{SK}}$$

$$V_1 - V = e\sqrt{\frac{8\pi Pg}{SK}},$$

from which by subtraction

$$V = (e' - e)\sqrt{\frac{8\pi Pg}{SK}}.$$

It is thus sufficient to measure the difference $e' - e$ of the two positions of the plate A.

83. In order to keep this plate at a constant potential, V_1, the whole system consisting of the plate, the guard ring, and the box is connected with the inner coating of a Leyden jar, which is formed by the glass case of the instrument. One of two accessory pieces of apparatus serves to maintain, and the other to verify, the constancy of the potential, V_1. The first, R, which is called the *replenisher* (§ 90), increases the charge of the jar when the knob is turned in one direction, and lessens it when it is turned in the contrary direction. The second, which is called the *gauge*, J, makes it possible to recognise when the potential has acquired a given value. This is an apparatus which is based on the same principle as the electrometer itself, and consists of a movable plate of aluminium, *p* (Fig. 79), with its guard ring, and a plate which attracts it, the whole being contained in the box, J (Fig. 78). Both the plate *p* and the guard ring are connected with the outer coating of the electrometer; the lower plate is at a fixed distance from the plate *p*, and is in connection with the inner coating of the electrometer, hence the plate *p* always comes back to the plane of the guard ring for the same value of the potential.

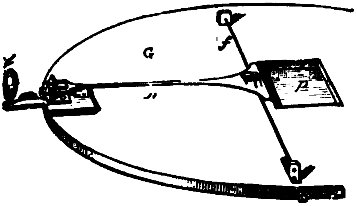

FIG. 79.

The plate *p* is attached to a stretched platinum wire, *f*, which it twists slightly in coming into the plane of the guard ring; a projecting tongue, *h*, carries at its end a fine wire which can be observed by means of the lens *l*, and, when the plate *p* is in the plane of the ring, ought to be exactly between two marks on a white card.

84. Lane's Unit Jar.—This apparatus is used to measure comparatively large quantities of electricity, such as the charge of a Leyden battery. It consists of a Leyden jar (Fig. 80), the outer coating of which is in conducting communication with a knob, B, which by means of a micrometric screw, F, may be set at any measured distance from the knob, A, connected with the internal coating. If the knob A is connected with the source of electricity, a spark passes between the two knobs whenever the difference of potential between them reaches a value depending on their distance apart. For the same distance between the knobs, every spark that passes represents the same quantity of electricity.

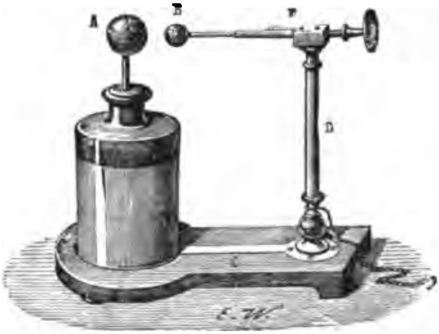

FIG. 80.

In order to measure the charge of a battery, the unit jar may be insulated and interposed between one terminal of the machine and the inner coating of the Leyden battery; or the battery may be insulated and the unit-jar interposed between the outer coating of the battery and the second terminal of the machine. In either case the number of sparks serves as a comparative measure of the quantity of electricity given to the battery.

CHAPTER IX.

ELECTRICAL MACHINES.

85. Electric Machines in General.—Any contrivance which serves for the continuous generation of electrical energy may be called an *electrical machine*. Such contrivances are frequently spoken of as sources of electricity, although no process is known which can be properly described as the production or generation of electricity. All that can be done by any electrical process is to *transfer* electricity in such a way that the quantity which enters any closed space across one part of the bounding surface is equal to the quantity which simultaneously leaves the same space across some other part of the surface. Or, if we prefer to speak of two different kinds of electricity, we must say that whenever a given quantity of positive electricity is transferred in one direction across a given surface, an equal quantity of negative is simultaneously transferred in the opposite direction across the same surface. It is thus impossible either to increase or decrease the absolute quantity of electricity within a given space.

The energy of an electric field (§ 55), namely $\frac{1}{2} Q (V - V')$, being proportional jointly to the charge and to the difference of potentials, is increased when either of these factors separately, or their product, is increased. In the various kinds of electrical machines, the mechanical energy expended is partly expended in overcoming the ordinary friction of the moving parts, but in addition to this there is always an expenditure of energy in maintaining motion in opposition to electric force, and the electric energy generated by the machine is equal to the mechanical energy thus expended.

Electrical machines may be divided into two classes, namely, those in which the friction of heterogeneous substances plays an essential part in the electrical action, and those whose action depends on electrical induction. Machines of the former class are called *friction machines*, those of the latter class are called *induction machines*.

86. General Mode of Action.—In either case an electric field is established, one boundary of which is formed by a moving part of the machine, which may be called the *carrier*, and the other boundary usually by the surface of the room. The motion of the machine brings the carrier into a position in which either the whole of it at once, or the various parts of it in succession, are more or less completely surrounded by an insulated conductor, the *collector* of the machine. Consequently, the field of force extending between the carrier and the room is cut into two parts, one extending from the carrier to the collector, and the other from the collector to the room. If the carrier is formed of conducting material and comes into connection with the collector as it passes, or if, being a non-conductor, its surface comes sufficiently near to a number of fine points projecting inwards from the inner surface of the collector, the first mentioned part of the field is abolished, the carrier passing away unelectrified and ready to be electrified afresh by friction or by induction. The result is thus to set up an electric field between the collector of the machine and the room. The charge, and therewith the difference of potentials of this field, increases with the continued action of the machine. By appropriate connections a greater or less portion of the charge and energy of the field may be transferred to a field bounded by any given conducting surfaces. In this way a Leyden jar or any other condenser may be charged.

87. Law of Increase of Charge.—In general, in a friction machine, each portion of the surface of the carrier is in the same electrical condition every time it comes under the influence of the collector, and consequently it increases the charge of the field established by the machine by the same amount every time; in other words, the charge of the field, and the difference of potentials which is proportional to the charge, increase in arithmetical progression. In other cases, however, particularly with induction machines, the charge frequently varies as the terms of a geometrical progression. Consider, for example, the following arrangement, which may be taken as a type of all inductive machines. Two metal jars, A and B, are placed on insulating stands, the potential of A exceeding that of the room by some small amount V_0, and the potential of B exceeding that of the room by V'_0. A metal ball is hung by an insulating thread inside each jar, and while there is connected for a moment with the room. Each jar thus becomes one boundary of an electric field, the second boundary of which is formed partly by the ball hanging inside the corresponding jar, and partly by the surface of the room. If c is the capacity of the part of the field extending between the inner surface of a jar and its ball, and C the capacity of the part of the field extending between the outer surface of a jar and the room, the total capacity of each field is $C + c$, and the electrification of the ball in the jar A is $-cV_0$, and that of the other ball is $-cV'_0$. Now transfer each ball to the other jar, letting it first go down to the bottom and then raising it a little by the insulating thread. The balls thus give up their electrification completely to the jars, which therefore acquire the potentials $V_1 = V_0 - \frac{c}{C+c}V'_0$ and $V'_1 = V'_0 - \frac{c}{C+c}V_0$ respectively, or, more shortly, $V_1 = V_0 - mV'_0$ and $V'_1 = V'_0 - mV_0$. This gives, for the difference of potentials

$$V_1 - V'_1 = (1 + m)(V_0 - V'_0).$$

Next let both balls be again connected for a moment with the room and then retransferred to the jars they were in before, allowed to touch the bottom and raised. The potentials of the jars thus become $V_2 = V_1 - mV'_1$ and $V'_2 = V'_1 - mV_1$ respectively, and the difference becomes

$$V_2 - V'_2 = (1 + m)(V_1 - V'_1) = (1 + m)^2(V_0 - V'_0).$$

If the same series of operations be repeated over and over again, the difference of potentials becomes progressively greater and greater, being, after n times,—

$$V_n - V'_n = (1 + m)^n(V_0 - V'_0).$$

88. Frictional Machine.—This consists of a glass plate, P, movable about a horizontal axis (Fig. 81); of two pairs of cushions, C, placed at the ends of a vertical diameter; and of two U-shaped insulated conductors, S and S′, provided with points on the side towards the plate. These conductors are called *combs*. Electrification results from friction of the glass against the cushions, and as each part of the electrified glass comes between the arms of the combs, the electrification is transferred to the latter, and the plate passes on almost unelectrified; accordingly a quantity of electricity, equal to that which the plate has brought, is communicated to the insulated *prime conductor*, DD′, connected with the combs.

Glass is the best material for the plate, but as there is considerable variation in the quality of glass, some care is needed in selecting it; that which is least hydroscopic is the best. All parts of the machine should be kept thoroughly dry.

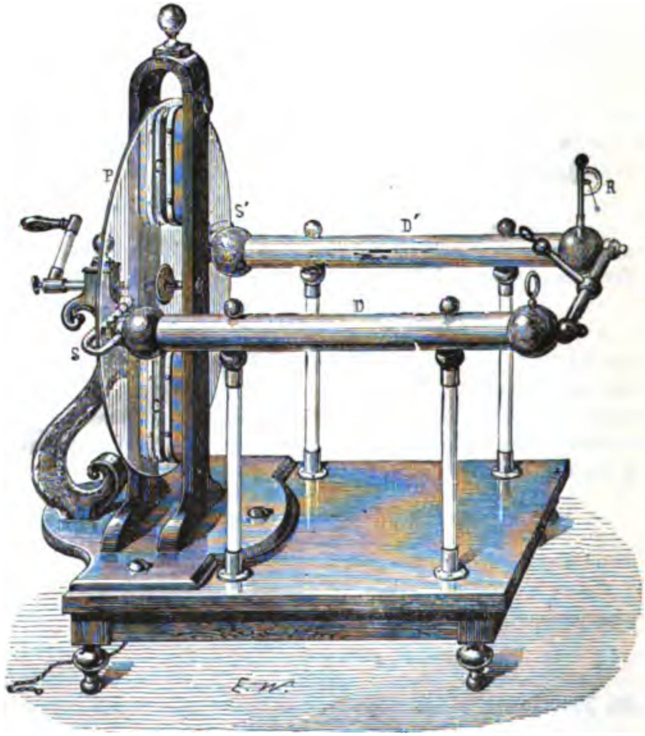

FIG. 81.

The cushions are ordinarily of leather stuffed with horse hair, and are kept pressed against the plate by springs. If the pressure is sufficient to bring the rubbers into good contact with the glass, further pressure does not increase the electrification. The surface of the rubber should be covered with a conductor such as that variety of sulphide of tin known as *aurum musivum*, or, still better, with *Kienmayer's amalgam*, which is composed of zinc, tin, and mercury; this is reduced to powder, and a little lard having been applied to the cushion, some of the powder is spread evenly over it.

The contact between the glass and the rubber sets up a fixed difference of potentials between them, the glass becoming positively electrified, and the rubber negative. This difference of potentials is at first very small, since the equal oppositely electrified surfaces are very close together. But as each portion of the glass moves away from the rubber, its potential acquires a very high value, and, in order to prevent loss, quadrant-shaped pieces of oiled silk, not shown in the figure, are often fixed to the rubbers so as to enclose on both sides the parts of the plate between the rubbers and the combs. These are kept in contact with the plate by the electrical attraction.

The rubbers are usually put in connection with the earth by strips of tin-foil which pass down the support; the conductor of the machine thus gives positive electricity. But machines are also constructed in which the cushions are supported by glass legs, and connected with a second system of conductors. Either positive or negative electricity may thus be obtained at will. Whether the rubbers are insulated or not, the yield of the machine is the same for the same atmospheric conditions, as is also the difference of potentials between the rubbers and the conductors. It is only the absolute value of the potential which changes.

In order to take sparks from either the positive or the negative conductor, the experimenter must be electrically connected with the other conductor. If this is not insulated the table and floor generally serve as a sufficient connection.

Similarly, to charge a Leyden jar or battery, the rubbers of the machine must be connected with one coating and the prime conductor with the other. Here, again, if the rubber is uninsulated, the table generally forms a sufficient connection between it and the outer coating. But if either the rubber or the outer surface of the jar is insulated, some conducting connection must be provided.

In order to estimate the potential of the conductor, a small electroscope known as *Henley's electrometer* (R, Fig. 81) is used. This is a small pendulum formed of a thread with a pith-ball at the end, movable about an axis in front of a scale. The divergence increases with the potential. Similarly the diminution of the divergence indicates the loss of electricity.

In order to increase the capacity of conductors and obtain stronger sparks, the prime conductor used formerly to be connected with large hollow metallic cylinders suspended from the ceiling by silk, which were called secondary conductors. It is simpler and less cumbersome to connect the conductor with the inner coating of a Leyden jar, the outer coating of which is connected with the room. If the potential of the machine should exceed that which the jar can support, a cascade arrangement may be used (§ 59).

89. The unit jar (§ 84) furnishes a very simple means of measuring the yield of the machine, and of investigating the conditions which modify it. We have already said that this yield is independent of the pressure of the rubbers when once there is sufficient contact between them and the glass. The yield is likewise independent of the capacity of the conductors, provided the causes of loss are the same. It is proportional to the surface of the glass which passes between the rubbers; it is, therefore, proportional to the length of the rubbers; thus, if one pair of rubbers is used instead of two, the yield is reduced to one half. The yield is further proportional within wide limits to the speed of rotation of the plate.

In some machines only a single pair of rubbers and a single comb is used, with the object of pushing back the limit at which discharge is produced between the conductor and the rubber. A greater difference of potentials is obtained, but a lower yield.

90. Induction Machines—Replenisher.—We shall commence the description of these machines by that of Lord Kelvin's replenisher, which Figs. 82 and 83 represents in about its ordinary size.

A and B (Figs. 82 and 83) are two portions of a cylinder with the same axis, O, both insulated, and each of them acting alternately both as inductor and collector. In the interior are two curved metal plates, P and Q, which are fixed by a cross-piece of ebonite to an ebonite rod, T, so that by means of a milled head, E, they can be made to rotate within the cylinders A and B. Four springs, *a*, *b*, *c*, *d*, placed at the same distance from the axis, are touched in succession by the projecting parts of the plates P and Q; *a* and *b* communicate with A and B respectively, while *c* and *d* are connected with each other. The action of the replenisher is essentially the same as that of the two metal jars and balls discussed in § 87.

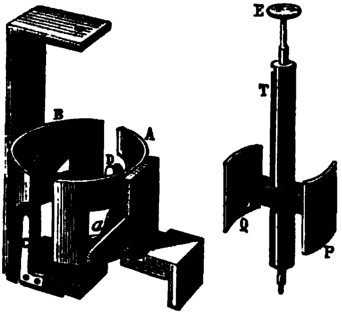

FIG. 82.

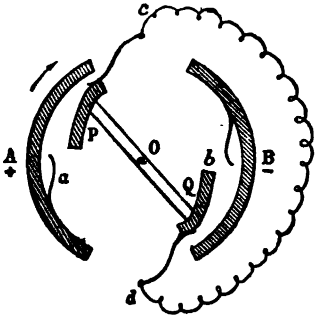

FIG. 83.

91. Holtz's Machine.—The action of Holtz's machine is analogous, except that the increase of the charge is continuous instead of intermittent.

It consists of two glass plates (Fig. 84) placed parallel at a small distance from each other; one is fixed by means of wooden knobs supported by glass rods, the other is movable about a horizontal axis. Besides the central aperture, through which passes the axis of the movable plate, the fixed plate has two apertures or *windows* at the ends of the same diameter. Along one edge of each of these windows is an armature of paper provided with a *tongue*, also of paper, terminating in a point in the centre of the opening. Opposite the armatures, on the other side of the free movable plate are two combs connected with two small insulated conductors, P and N. By an insulated handle it is possible to vary at will the distance of the two knobs, P and N, which will be called the poles of the machine. Two sliding knobs, P′ and N′, may be brought into contact with the corresponding poles, so that either one or the other may be put to earth.

To get the machine to work, it must first be *primed*. The two poles, P and N, are brought into contact, and the movable plate being put in rotation by the handle, so as to move towards the points, an ebonite plate or stick of sealing-wax charged by friction with negative electricity is brought near one of the armatures. When the machine is in action a peculiar hissing sound is heard; from the comb, near which the ebonite was held, a luminous sheet spreads in a direction opposite to the motion of the plate, this is the positive comb (§ 20); every point of the negative comb is tipped by a small star of light. The electrified ebonite may then be withdrawn, and the rod which connects the two poles drawn away; a brush discharge then passes between the two poles as long as the rotation continues. It is readily shown that the electricity of each of the poles is of the opposite kind to that which escapes from the corresponding comb. The action stops if the knobs P and N are drawn too far apart.

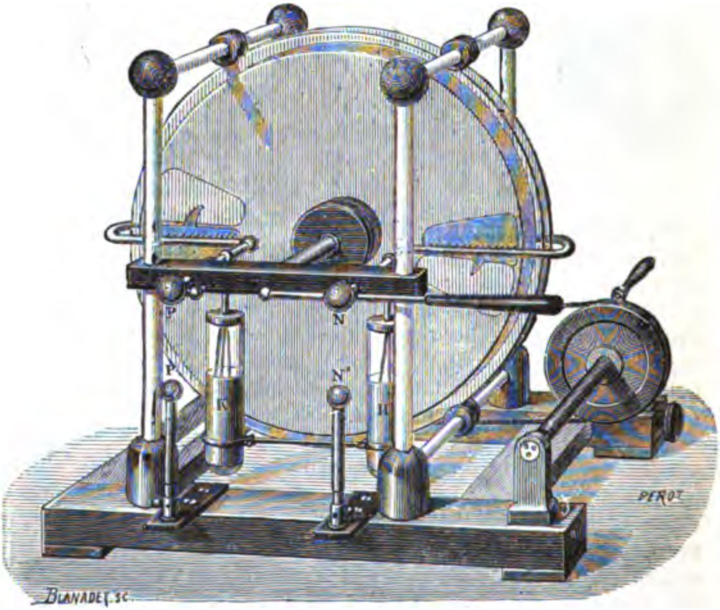

FIG. 84.

The brush discharge depends upon the small capacity of the conductors. In order to increase this capacity, the two conductors, P and N, may be connected with two Leyden jars, K and H, the outer coatings of which are connected with each other. The two jars form thus a cascade between the two poles, and each of them has half the difference of potentials of the two poles (§ 59). If the outer coatings are connected to earth, the two conductors are at equal and opposite potentials. When jars are used, the sparks are not continuous, as with the brush, but they follow each other at regular intervals, and are much more luminous, denser, and produce a louder crack. With a machine the plate of which is 60 cm. in diameter they may attain a length of 20 cm.

Most of these machines, as now constructed, are double; they have two fixed plates close to each other. The two movable plates, which are mounted on the same axis, rotate on the outside. The armatures are placed opposite each other, those which are similarly electrified being together; the same U-shaped comb surrounds the two movable plates. The action of the machine is in no way altered, except that the yield is double.

It is sometimes difficult to start the action of the machine, especially in damp weather; it then will not begin unless the ebonite plate is strongly electrified. The glass plate is usually made to rotate with a velocity of eight to ten turns a second.

In order to explain the action of the machine and to make the figures clearer, we will suppose the glass plates replaced by concentric cylinders (Fig. 85).

The circumference represents the movable cylinder; A′ and B′ the two paper armatures; the fixed cylinder is not shown, as it has scarcely other function than that of supporting the armatures; A*a* and B*b* are the two conductors represented as in actual contact.

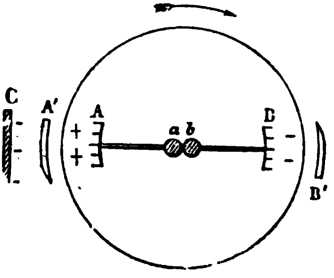

FIG. 85.

Suppose the plate to be at rest. The ebonite plate C charged with negative electricity is brought near the armature A′. It acts inductively on the conductor AB; positive electricity is attracted towards A, and negative towards B; but owing to the points the positive electricity escapes on to the glass at A and negative at B until equilibrium is established.

When the plate is turned, the positively electrified glass is carried away from A, and the negatively electrified glass from B, so that the conductor AB is again left subject to the inductive action of the ebonite, which, acting as before, causes a stream of positive electricity to escape from the comb A, and a stream of negative from B. Each part of the plate, as it passes A, thus receives a positive charge, and, as it passes B, a negative charge, until, after half a turn, the whole plate is electrified as indicated by Fig. 86. Henceforward the armatures come into play: the paper tongues that project from them through the windows in the fixed plate graze the surface of the approaching parts of the revolving plate, and thus the armature A′ acquires a negative charge, and B′ a positive charge. The electric force due to these charges acts in the same way as that due to the electrified ebonite, so that the inductive action on the conductor AB is increased, and a greater discharge of electricity—positive at A, negative at B—takes place on to the surface of the revolving plate. The consequence is that the armatures, in their turn, become more strongly charged, or acquire a greater difference of potentials, and that the action already described goes on still more actively, so that, after a few turns of the plate, the electrified ebonite may be removed and the machine continues to act. As described, the upper half of the plate carries positive electricity from left to right, and the lower half carries negative electricity from right to left, which is electrically the same thing. This transfer, due to the motion of the plate, is compensated by a flow of positive electricity from B to A through the conductor, and of negative electricity from A to B. When the difference of potentials between the armatures has attained a sufficiently high value, the electric flow through the conductor continues, even if the knobs P and N are separated, and it then becomes evident as a stream of sparks. If the knobs are separated too far, the action of the conductor ceases, and there being now no electrical interchange between the combs and the revolving glass-plate as it passes them, negative electricity is carried round to the positive armature B′, and positive electricity to the negative armature A′. The electrification of the armatures is thus lessened, and may be destroyed altogether, or even inverted. When the knobs are quickly put into contact again, if the electrification has only been diminished, or if it has been inverted in sign, the action

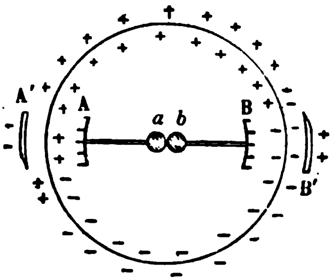

FIG. 86.

of the machine recommences, either in the same way as before or in the opposition, as the case may be.

If the jars are connected with the conductors, the flow, instead of passing continuously across the gap *ab*, charges the jars until the potential reaches the value corresponding to the striking distance; a spark passes and the same succession of phenomena is repeated.

When the jars are used, if the distance of the two poles is too great for the spark to pass, it sometimes happens that the action stops, then starts again, the signs of the poles being reversed, and the same series of alternations are reproduced. Each jar is periodically charged and discharged by the corresponding comb. This is specially liable to occur with batteries of large capacity.

Holtz's machine affords a means of making a curious experiment on *reversibility*. If the two combs of a machine in the ordinary state are connected with the poles of a second one, from which the driving band has been removed so as to allow the plate to turn freely, and if the first machine is then set in action, and a slight impulse is given to the plate of the second machine, it continues to rotate; the electricity thus transmits the motion of the first machine to the second.

92. Diametral Conductor.—Various attempts have been made to prevent the machine from ceasing to act or reversing its poles. A useful contrivance for this purpose is the *diametral conductor* (Fig. 87). Each armature occupies almost the whole of a quadrant, and an insulated conductor, A′B′, with combs at the ends, can be placed between them along any diameter. So long as the two poles are in contact, or when they are separated, so long as the machine continues in normal action, the diametral conductor has no effect, the conditions of its equilibrium between the two armatures being those which have been established by the principal conductor. But if the poles are separated so far that the principal conductor ceases to act, the diametral conductor replaces it, and the machine continues to work.

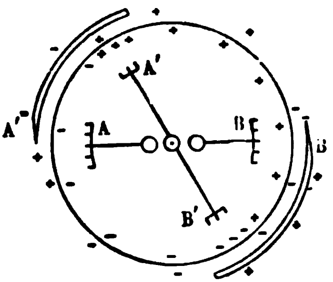

FIG. 87.

93. Voss's Machine.—Holtz's machine has been modified in many ways. Voss's machine (Fig. 88), which is now widely used, is a machine with a diametral conductor, and has some of the features of the Holtz machine and of the replenisher.

The movable plate is provided with metal studs, which act like the carriers of the replenisher. The inductors are represented by two strips of tinfoil fixed to the centre of the paper armatures. They are connected with two small brushes, which touch the studs just before their passing under the combs ; these brushes play the same part as the two springs *a* and *b* (Figs. 82, 83). Two other brushes placed in the middle of the comb of the diametral conductor act like the two springs, *c* and *d*.

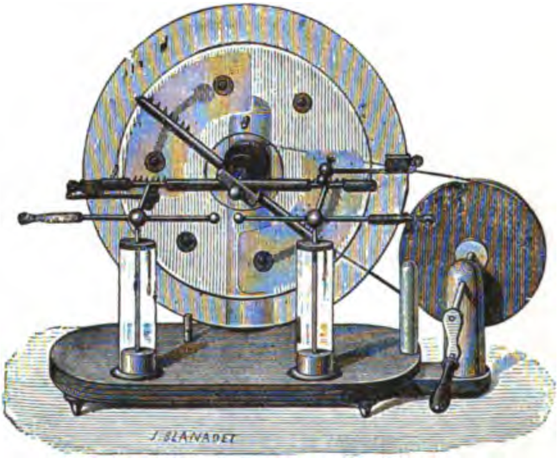

FIG. 88.

94. Wimshurst's Machine.—This consists of two identical plates of glass which are mounted on a horizontal spindle so as to rotate in contrary directions, and are provided on the outside with narrow strips of tinfoil arranged radially at equal distances apart (Fig. 89). Two horseshoe-shaped conductors provided with points enclose the two plates at opposite ends of the horizontal diameter, and are in connection with the two poles, and also with two Leyden jars. Diametral conductors, making an angle of from 60° to 90° with each other, are placed, one opposite each plate ; they have small brushes at their ends which graze against the bands of tinfoil.

The direction of rotation of the plates is such that a radius, which at any instant is horizontal, becomes parallel to the corresponding diametral conductor by turning through an acute angle. To explain the action, suppose that, by the approach of an electrified body or otherwise, the potential near one end of one of the diametral conductors,—say at the left hand upper part of the figure,—is made higher than elsewhere. A tinfoil sector passing through this region is connected by the conductor with the sector at the opposite end of the same diameter, and the two are, therefore, brought for the moment to the same potential, by the passage of positive electricity through the conductor from the first to the second. The motion of the plates brings these two sectors simultaneously opposite two sectors of the other plate, which are for the moment connected by the second diametral conductor. This second pair of sectors thus assume the same potential, although one is near a negatively charged sector of the first plate, and the other near a positively charged sector. This equalisation of potentials involves a passage of positive electricity through the

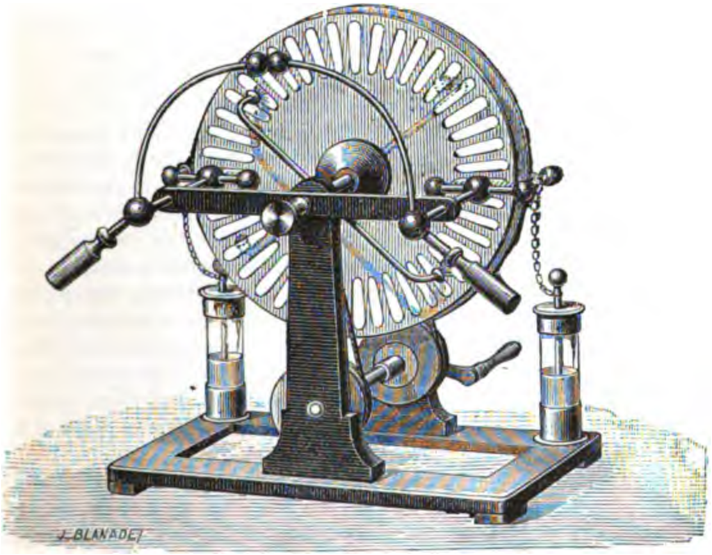

FIG. 89.

second conductor from the lower, left hand, end to the upper, right hand, end. These actions taking place with all the sectors in turn as they pass the diametral conductors, the result is a continuous succession of positively and negatively charged sectors on one plate, arriving simultaneously opposite the two ends of the diametral conductor applied to the other plate. As we have described the action, the sectors arriving opposite the left hand ends of both conductors would be positively electrified, and those to the right negatively. There would thus be a continuous flow of positive electricity through both conductors from left to right, charging the sectors positively which pass simultaneously through the left hand collecting comb. Similarly, those which pass simultaneously through the right hand comb would be negatively charged. The combs take up the electrification of the sectors that pass them, and there is, consequently, a flow of positive electricity from the left hand comb and discharging knob to the right hand comb, which compensates the transfer from right to left due to the motion of the plates and the action already described. The action of the machine is such as to increase the initial inequality of potential, and it, therefore, acts more powerfully as the motion goes on.

As a general rule, a machine like that of Holtz, where the electricity has to strike across an interval of air, can only begin to work with an appreciable charge. On the other hand, a machine with direct metallic contact or with a brush may start itself, however small the initial charge. It is very seldom that there is not sufficient difference of potentials between the different parts of a Wimshurst's machine to make it begin work.

95. Yield and Energy of a Machine.—By means of Lane's unit-jar (§§ 84, 89), it may easily be shown that the yield of inductive machines is proportional to the velocity of rotation, independent of the capacity of the conductor, and of the absolute value of the potential of the poles; it does not alter when one of these is connected with the ground; it diminishes as the difference of potentials between the poles increases. The yield of inductive machines is ordinarily much greater than that of frictional machines. With a Lane's jar with a striking distance of 1 mm., the yield for one turn of a plate machine, 98 cm. in diameter, being taken at 1, that of a Holtz machine of 55 cm. was 0.86. But the former made one turn in a second, and the latter ten turns, so that the yields per second were as 1 : 8.6.

The following numbers will give an idea of the absolute value of the yield: seven turns of a double plate Holtz machine charged a battery, the electrostatic capacity of which was 22,500 cm., so as to give a spark 0.1 cm. in length.

As the capacity of the battery was $\frac{22,500}{3^2 \times 10^5} = 0.025$ microfarads and since a striking distance of 0.1 cm. corresponds to a potential difference of 4830 volts (§ 73), we get for the charge of the battery

$$Q = VC = 4830 \times 0.025 \times 10^{-6} = 120.75. \times 10^{-6} \text{ coulombs,}$$

and for the corresponding work (§ 39)

$$T = \tfrac{1}{2} MV = \tfrac{1}{2} \times 4830 \times 120.75 \times 10^{-6} = 0.742 \text{ joules.}$$

These numbers correspond to seven turns of a machine, which makes ten a second; so that to get the numbers for one second we must multiply by 10 and divide by 7. We thus find that the machine would give per second 0.00017 coulombs of electricity, and 1.06 joules of work.

The *power* of a machine is defined by the work which it does in unit time. The *watt*, which corresponds to one joule per second, has been taken as unit of power. We should accordingly say that the power of the machine was 1.06 watts.

CHAPTER X.

GALVANIC OR VOLTAIC BATTERY.

96. Galvanic Element.—If a plate of zinc and a plate of copper dip side by side, without touching, in a vessel of water acidulated with sulphuric or hydrochloric acid, two pieces of copper wire attached one to each plate are found to differ in potential. If the wires are connected with the alternate pairs of quadrants of a quadrant-electrometer, a deflection is produced indicating a difference of potentials of about one volt, the potential of the wire from the copper plate being the higher.

Similar results would be obtained if almost any two metals were substituted for copper and zinc in the above experiment, and instead of acidulated water, ordinary tap-water, or water containing a small quantity of some salt in solution, may be used. The only difference in the result would be in the magnitude of the observed difference of potentials. The material of the connecting wires is of no consequence; copper is mentioned as being the material most likely to be at hand, and, on the whole, the most convenient.

Results equivalent to those just stated were observed in 1789 by Ludovico Galvani, professor of anatomy at Bologna. An arrangement such as we have described, or any modification of it producing like effects, is consequently often spoken of as a *galvanic element* or *galvanic cell;* and the study of the numerous phenomena for the discovery of which Galvani's observation formed the starting-point constitutes the science of *galvanism.*

97. Voltaic Pile — Galvanic Battery. — The difference of potentials between the wires of a galvanic cell depends only on the materials employed, and, to some extent, on their temperature; it is not affected by the size, shape, or relative positions of the plates, nor by their absolute electrical potential. If placed on an insulating stand, a galvanic cell may be strongly electrified either positively or negatively, but however high or however low the potential of the cell as a whole may be, the potential of the wire from the copper plate remains about one volt higher than that of the wire connected with the zinc.

A consequence of this is that, if several cells are arranged one after another, and the zinc plate of the first is connected by a wire with the copper plate of the second, the zinc of that with the copper of the third, and so on throughout, the zinc of the last cell but one being joined to the copper of the last, the difference of potentials between the wire from the first copper plate and that from the last zinc plate is equal to the difference found in the case of a single cell multiplied by the number of cells. To see how this follows, let us assume for simplicity that the difference of potentials in the case of a single cell is exactly one volt, and that the wire connected with the last zinc plate is in contact with the inside of the room where the experiment is made. We may assume then (§ 36) that its potential is zero. Accordingly the potential of the copper plate of the same cell and of the wire attached to it will be one volt. But this wire is fastened to the zinc of the last cell but one, and therefore the copper of this cell will have a potential = 2 volts. In the same way that of the next preceding cell will have a potential = 3 volts, and so on to the beginning of the series, the potential rising by one volt for every cell. Proceeding in the opposite direction along the series of cells, we should find a fall of potential equal to one volt at a time as we passed from one cell to the next; so that, if the first copper plate were kept at potential zero, the second would have a potential = – 1, the next – 2, and so on.

The fact that the difference of potentials resulting from a single pair of metals dipping into an aqueous liquid can be thus multiplied, by arranging in order a number of such pairs connected together, was discovered in 1800 by Alessandro Volta, professor of physics at Pavia. In order to produce the effect in a marked degree, he arranged a number of alternate discs of copper and zinc, each pair being separated from the next by a round of cloth soaked with acidulated water, as shown in section in Fig. 90, and in perspective in Fig. 91. This arrangement is known as *Volta's pile.*

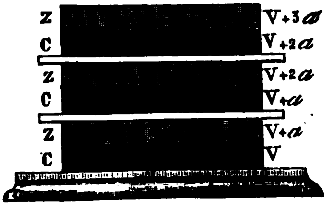

FIG. 90.

A corresponding combination of pairs of metal plates in separate vessels of liquid is called a *voltaic battery* or *galvanic battery*, and in general the terms voltaic and galvanic, and words formed from them, are used almost interchangeably in relation to the class of phenomena we have now to study.

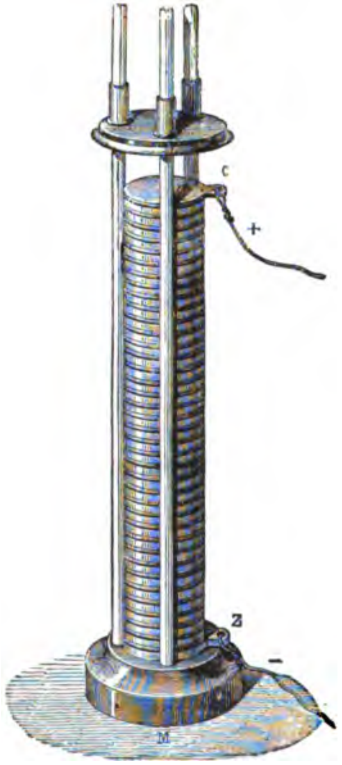

FIG. 91.

98. Volta's Theory of Electrification by Contact. — In order to explain facts of the kind referred to in the last two paragraphs, Volta supposed that the surface of contact of two metals, or more generally of any two different substances, is the seat of a peculiar force, which he termed *electromotive force*, the effect of which is to transfer positive electricity across the surface in one direction and negative in the other, until the tendency of the two electricities to reunite balances the force which tends to separate them. He supposed the electromotive force in any given case to depend solely on the nature of the materials in contact, and not upon the size or shape of the bodies, or the extent of the surface of contact, or on their electric state.

Among the numerous experiments which he devised to establish this principle the following may be cited :—

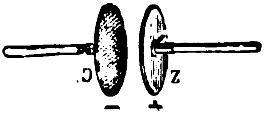

FIG. 92.

1. Two plates, one of zinc and the other of copper, supported by insulating handles (Fig. 92), and at first in the neutral state, are placed in contact and then separated. On applying these, each separately, to a

delicate electroscope, it is found that they are each electrified, the zinc positively and the copper negatively. The amount of the charge is proportional to the extent of the surfaces in contact. The two plates are charged as if they were the plates of a condenser between which a difference of potentials has been established. The two electricities form on the surfaces in contact two equivalent layers, which remain separated in conformity with the electromotive force of contact of Volta, notwithstanding the absence of an insulating plate.

2. A zinc strip joined to one of copper is held in the hand, and the copper end is applied against the lower plate of a condensing electroscope, the top plate of which is touched by the other hand (Fig. 93). If the connections are broken and the top plate is raised, the gold leaves diverge, with negative electricity; this proves that the potential of the copper was negative, for as the zinc was held in the hand, it was at the potential of the earth—that is, at zero. If the experiment is repeated in exactly the same manner, except that the copper is held in the hand and the zinc applied to the electroscope, the leaves do not diverge. The fall of potential from zinc to copper is evidently the same in

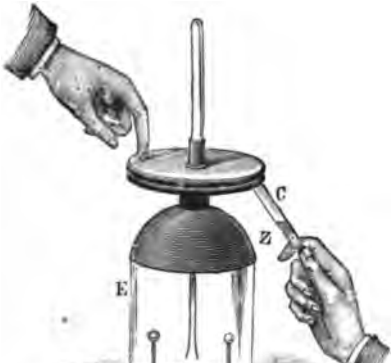

FIG. 93.

both cases, and the potential of the plate is zero like that of the copper held in the hand.

The experiments may be modified by interposing between the plate of the condenser and the strip of copper a disc of cloth or paper wetted with acidulated water. Nothing is changed in the former case; the divergence of the gold leaves is still negative. But in the second case, in which the copper is held in the hand, the leaves diverge with positive electricity, instead of remaining unaffected.

99. Objections to Volta's Theory.—While no question is raised as to the results of the fundamental experiments adduced by Volta and his supporters in confirmation of his views, there is by no means universal agreement among physicists as to the true interpretation of these results. The most serious objection to Volta's theory is that it does not afford a satisfactory account of the source of the energy the production of which it implies. Suppose S to be the capacity of the condenser used in either of the forms of experiment described in § 98, and V and V' to be the potentials acquired by the two plates respectively, then the production of the difference of potentials $V - V'$ would indicate a generation of electrical energy $= \frac{1}{2} S(V - V')^2$. If the plates of the condenser have a radius of 5 cm. and are $\frac{1}{80}$ cm. apart, the capacity is (§§ 52, 59) $S = \frac{\text{area}}{4\pi \times \text{distance}} = 500$ in C.G.S. units. If the difference of potentials is one volt, or $\frac{1}{300}$th of an electrostatic unit, the energy is $\frac{1}{2} \times 500 \times \frac{1}{300} \times \frac{1}{300} = 2.78 \times 10^{-3}$ erg. The only source from which this energy could be derived, consistently with Volta's hypothesis, is the heat contained in the zinc and copper. It is true that the withdrawal of the quantity of heat required to furnish the quantity of energy in question,—rather less than three thousandths of an erg,—would cause an utterly imperceptible fall of temperature in a compound bar of zinc and copper large enough to be used in the experiment, so that the suggested explanation cannot be rejected on this ground. Indeed, it is known from other experiments (see Chap. xiv.), that when a much larger quantity of electricity than there is any question of in the present case, is caused to pass from copper to zinc across the common surface, an ascertainable lowering of temperature really ensues. But when a numerical comparison is made, it is found that the quantity of heat which thus disappears is inadequate to account for the difference of potentials developed in

Volta's experiment. It is further found, when other pairs of metals are tried, that not only is the fall of temperature due to the passage of electricity across the separating surface generally insufficient to account for Volta's results, but that there is no recognisable relation between the magnitude of the change of temperature in the one set of experiments and the difference of potentials observed in the other set. In fact, there appears to be a real difference of potentials between different metals in contact, arising in some way solely from their own properties, which enable them to transform thermal energy into electric energy; but this difference is, in general, much too small to account for the effects now under consideration, and it does not always agree with these even in direction.

100. Alternative Theory.—If we no longer look for the source of the required energy in the surface of contact of the two metals, we may inquire whether it is not possible for it to be furnished by chemical action taking place between one or both of the metals and the air or liquid with which they are in contact. It is well known (see Chap. xv.) that chemical combination or decomposition is associated in a definite way with the transfer of electricity between the bodies taking part in the action, and that the occurrence of a fixed amount of chemical action involves the transfer of a fixed quantity of electricity. In order to charge a condenser, such as that supposed in § 99, of capacity 500, to a difference of potentials of 1 volt, or $\frac{1}{300}$th of an electrostatic unit, a charge of $\frac{500}{300} = 1\frac{2}{3}$ electrostatic unit would be required. Now the oxidation of one gramme of zinc would correspond with a charge of 9×10^{12}, or nine millions of millions of such units. From this it follows that if the electrification observed in such experiments as those of Volta were really furnished by chemical action, oxidation of zinc, the required amount of such action would be so small as to be utterly imperceptible by any direct mode of investigation.

The connection above alluded to between chemical action and transfer of electricity naturally suggests the idea that a definite electric charge is uniformly associated with each atom of a chemical substance. If, in addition to this, we admit that the tendency to combination which is recognised by chemists under the name of chemical affinity is capable of counterbalancing electric forces so as to produce a condition of mutual equilibrium, we may give some such account as follows of the action that results from the contact of two metals.

When a piece of zinc is exposed to oxygen or air, we may suppose that atoms of oxygen, each carrying with it a definite charge of negative electricity, unite with the metal and at the same time give up their charge to it, so as to lower its potential below that of the air, until the resulting electric force counterbalances the chemical force tending to cause further combination. On grounds that we cannot conveniently discuss at this stage, we infer that equilibrium is reached when the potential of the zinc is about 1.8 volt below that of the air. We may further suppose that when a piece of copper is exposed to oxygen, oxidation goes on in a similar way until again a condition of equilibrium is reached, this time when the potential of the metal is about 0.8 volt below that of the gas. We assume here that the force tending to cause combination of copper with oxygen is less than that tending to make zinc combine with oxygen, which is in harmony with the generally recognised chemical characters of the two metals.

If now a piece of zinc and a piece of copper, both in contact with air, and therefore, according to what we have just said, differing in potential by something like one volt, are brought into contact with each other, we must suppose equalisation of potentials to take place between them in accordance with the general properties of conductors. (We leave out of account at present the small difference of potentials referred to at the end of § 99.) The potential of the zinc is thus raised, so that it no longer causes equilibrium between the chemical and electrical forces, and we may therefore suppose a further oxidation to occur until the difference of 1.8 volt is again produced. Or we may suppose the same result to be produced by a *deoxidation* of the copper; or, again, we may imagine that both these actions occur simultaneously. Either way, the result would be *uniformity of potential throughout the two connected pieces of metal*, with a *difference of potentials between portions of the gas* in contact with the zinc and copper respectively. The potential of the gas just outside the oxidised portion of the zinc would exceed that of the metals by 1.8 volt, and that of the gas just outside the copper would exceed it by 0.8 volt. That is to say, there would be an electric field established in the gas, the difference of potentials between the boundaries being one volt, and the positive side of the field would adjoin the zinc. We must suppose that it is this difference of potentials which is observed in many experiments that are usually cited as evidence of a difference of potentials between the metals.

A point of considerable importance remains to be discussed in connection with the account just given of the probable origin of the phenomena referred to in the preceding paragraphs. We have assumed that the oxidation of a metal is accompanied by a transfer of negative electricity to the metal. This seems to imply that oxygen gas as such is negatively electrified, a conclusion inconsistent with the observed electrical neutrality of oxygen or air under ordinary conditions. In reference to this we may call to mind the fact that, on purely chemical grounds, chemists have concluded that a *molecule* of oxygen gas consists of *two atoms* which are in some way held together, and that the connecting force has been likened to, if not actually identified with, electrical attraction, a molecule of oxygen being represented by the symbols $\overset{+}{O}\ \overset{-}{O}$. If we follow out such ideas, we must look on each molecule as constituting in itself an electric field of almost infinitesimal extent, but the gas as a whole would be electrically neutral. If now combination of oxygen and zinc takes place, we must suppose that the *negative* member of a molecular pair unites with the metal, leaving the positive member in the state of a so-called dissociated atom. It seems likely that the metallic surface, with which a number of negative atoms have entered into chemical combination, would have the corresponding positive oxygen atoms adhering to it as an external layer held on by electric force. If so, we should have two parallel oppositely electrified surfaces separated by a molecular distance, and comparable with the surface of a condenser. The difference of potentials between the positive and negative layers would be the electric force multiplied by the distance, or, in the symbols we have commonly used—

$$V - V' = 4\pi\sigma e/K.$$

If we put $K = 1$, $V - V' = 1.8$ volt $= \frac{1.8}{300} = 0.006$ electrostatic unit, and $e = 1$ cm.$/10^8$ (one hundred-millionth of a centimetre), a value which there is reason to believe is somewhere near the truth, we get for σ the charge per square centimetre

$$\sigma = \frac{.006 \times 10^8}{12} = 5 \times 10^4.$$

Now we have already said that the oxidation of one gramme of zinc corresponds with the transfer of nearly 9×10^{12} electrostatic units of electricity. Hence, to produce a difference of potentials of 1.8 volt between a zinc plate and the adjacent air, we should require, according to the view we have been discussing

$$\frac{5 \times 10^4}{9 \times 10^8} = 5.5/10^8$$

grammes of zinc to be oxidised per square centimetre of surface. Or the oxidation of 5½ milligrammes would suffice for 10 square metres of surface.

The charge per unit surface of a copper plate in air or oxygen would be proportionately less than that of zinc, as its potential differs less from that of the gas.

101. Résumé.—The two theories we have been discussing, which we may distinguish as the contact theory and the chemical theory respectively, both profess to account for the same experimentally verifiable facts, that, under certain definite conditions, differences of potential are produced involving the generation of corresponding amounts of electric energy.

According to the contact theory, the seat of the action that gives rise to the difference of potentials is at the separating surface of the two metals, and this action would still go on if it were possible to make the experiment in an absolute vacuum. According to this theory, the source of energy is the heat contained in the metals, whose temperature must accordingly be lowered by an immeasurably small amount when they are put into contact.

According to the chemical theory, the seat of the action is the surface of contact between the metals and the surrounding air (oxygen), and the metals themselves are at the same potential (leaving out of account a very small difference to be discussed later). The energy is derived from the energy of chemical combination between one of the metals and the surrounding gas.

Hitherto no single experiment has been made which can be considered as separately conclusive either way, and the amounts of energy involved are so small that we can hardly expect such an experiment to be made hereafter. At any rate, for the present we can only infer what are admissible suppositions by appealing to general principles and observing the relations between thermal, chemical, and electrical energies in cases where they are brought into play to a measurable extent. Appealing to this test, we have in favour of the chemical theory the apparently conclusive facts that the energy of the galvanic battery, the action of which Volta's hypothesis was intended to explain, is admittedly derived from chemical action (§ 155); and that, in the case of thermo-electric action (§§ 140–144), where heat is undoubtedly the source of electric energy, Volta's theory of contact action has no application.

102. Electromotive Force of a Cell or Battery.—According to the contact theory, when a plate of copper and a plate of zinc dip into acidulated water without touching each other, there is no appreciable difference of potentials between them, because there is no metallic contact. According to the chemical theory. there is a difference of potentials of about one volt, but there is no difference of potentials between the air in contact with the zinc and that in contact with the copper, just because the liquid acts on the metals in virtually the same way as air, so that there is the same step of potential at the liquid-metal surfaces as at the air-metal surfaces, as is illustrated in Fig. 94, where vertical heights represent potentials. If a copper wire is attached to the zinc plate, then, according to either view, there is a difference of about a volt between this wire and the copper plate or a copper wire attached thereto. According to the contact theory, this result is attributed to the introduction of a copper-zinc contact; according to the chemical theory, the copper wire merely takes the potential of the zinc to which it is joined; but there is now an experimentally recognisable difference of potentials between the air in contact with the wire from the copper-plate and the air in contact with the wire from the zinc plate, because now the acid-metal contacts do *not* balance the air-metal contacts (see Fig. 95).

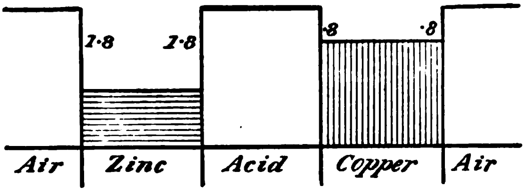

FIG. 94.

In §§ 96 and 97, in order to avoid raising the questions we have just been referring to, we were careful to speak of the difference of potentials between wires of the same metal (copper) attached to the two plates of a single cell, or to the two terminal plates of a battery or pile. But in any case, these questions are of theoretical and not practical importance: in practice the plates of a cell or a battery are always connected either with wires of the same metal (usually copper), or with binding-screws of the same metal (usually brass), and there is then no controversy as to the difference of potentials existing between these wires or binding-screws, as the case may be.

This difference is necessarily equal to the algebraic sum of the differences occurring at all the surfaces of contact of different materials intervening between the two extremities of the cell or battery; for since all the materials are conductors, and there is electric equilibrium, there is no difference of potentials except at the contacts. The wires or binding-screws connected with the first and last plates of a battery, or with the two plates of a single cell, are called the *poles* or *terminals* of the battery or cell; that one which is connected with the copper plate and has the higher potential is the *positive* pole or terminal, and that connected with the zinc is the *negative* pole.

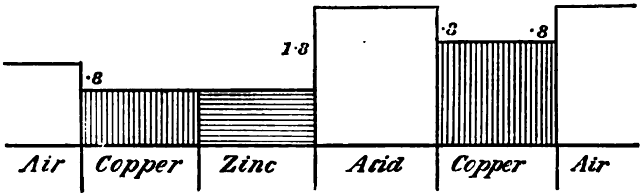

FIG. 95.

The algebraic sum of the differences of potential occurring at all the surfaces of contact of different materials intervening between the terminals of a cell is called the *electromotive force* of the cell. As we have just said, this is the same as the difference of potentials between the terminals when there is electrical equilibrium. If, however, the terminals are joined by a wire, a condition of statical equilibrium no longer obtains, and the difference of potentials between the terminals is then *less* than the electromotive force of the cell. The relation between these quantities is more fully discussed in Chaps. xi. and xii.

When several cells are connected one after another to form a battery, the algebraic sum of the electromotive forces of the several cells is the electromotive force of the battery. By connecting the terminals of each separate cell, and then those of the battery, with the quadrant electrometer, it is easily verified that the difference of potentials between the terminals of the battery is the sum of the differences between the terminals of the cells composing it.

103. A Galvanic Battery Comparable with an Electric Machine.—The fundamental property of a galvanic battery, by virtue of which it maintains a constant difference of potentials between its terminals, or between any conductors connected with them, makes it exactly analogous to an electric machine. By its action an electric field is established, the boundaries of which are the surfaces of the conductors attached to its terminals. If the theory set forth in § 100 is correct, at least in its main outlines, the charge of the field is derived from the atomic charges of the molecules which take part in the chemical action that goes on in the cell. Expressing the same thing in other words, we may say that the lines of force that extend between the boundaries of the field are transferred to them from the atoms. Fig. 96 may suggest the electric field set up when the terminals of a battery, B, are connected by wires with the surfaces of a condenser, C: the fine lines in the figure represent roughly the distribution of lines of force. If now the plates of the condenser be connected for an instant with each other, the lines of force between them disappear, but new lines immediately take their place. Lines previously extending between the connecting wires close in upon the condenser and

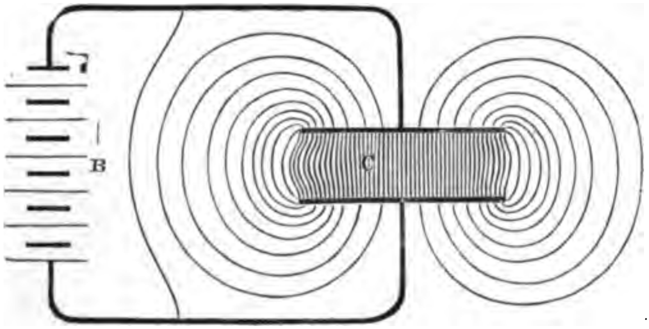

FIG. 96.

are followed by fresh lines generated by the battery, the ends of the lines travelling along the wires. We have already seen (§ 21) that a surface from which lines of force start is what is otherwise called a positively electrified surface, and that a surface on which lines terminate is negatively electrified. It follows that when lines of force flow through the dielectric medium from the battery to the condenser, their ends at the same time moving along the connecting wires, that the process may be otherwise described as a flow of positive electricity from the positive terminal of the battery along one wire to the corresponding plate of the condenser, and of negative electricity from the negative terminal along the other wire to the second plate of the condenser. But, as we cannot distinguish the transfer of positive electricity in one direction from the transfer of an equal quantity of negative electricity in the opposite direction, the process referred to may be equally well described as a flow of positive electricity from one pole of the battery to the condenser, and of an equal quantity of positive electricity from the condenser to the negative pole of the battery.

If the plates of the condenser are connected permanently, instead of only for an instant, or, more simply, if a continuous wire (sometimes called an *interpolar* wire) is used to connect the terminals or poles of the battery with each other, a continuous procession of lines of force must be thought of as spreading out from the battery, their ends travelling in opposite directions along the wire until they meet, and the lines, as it were, shrink into the wire. This process constitutes what is known as a continuous *electric current*, which may be described as the passage of equal quantities of positive and negative electricity in opposite directions round the circuit formed by the battery and wire, or as the circulation of a double quantity of positive electricity in one direction. The ordinary phraseology employed in reference to electric currents is founded on the conception of a single kind of electricity traversing the circuit. If we employ the conception of a double circulation of the two kinds of electricity in opposite directions, any statement of the quantity of electricity passing through the battery or along a wire must be taken as referring to the arithmetical sum of the quantity of positive going one way and of negative going the other way. This conception of two electricities in some ways connects itself most easily with the notion of an electric field employed in the previous part of this work; but, whatever the mode of expression employed, it is important to remark that, while what is called an electric current is maintained in a circuit, the battery is continuously giving off energy to the surrounding dielectric medium, and that the conducting wire is continuously absorbing energy from the medium. While this is going on, both the conducting wire and the dielectric medium exhibit special properties that we have to study in what follows.

104. Decrease of Electromotive Force by the Passage of a Current.—During the passage of a current through a galvanic cell, chemical action goes on among the materials. With cells of the construction we have hitherto spoken of, formed of two metal plates dipping into dilute acid, the result is a change in the chemical nature of the substances in contact, producing a diminution of electromotive force, and a consequent weakening of the action of the cell. Usually, on interrupting the conducting circuit and allowing the cell to rest for a while without producing a current, there is a more or less complete recovery of the original electromotive force, unless indeed the chemical action has gone so far as to nearly use up some essential constituent. This temporary decrease of electromotive force is often referred to as *polarisation* of the cell or battery. We shall discuss the effect more fully in connection with the chemical effects of the current (§ 153).

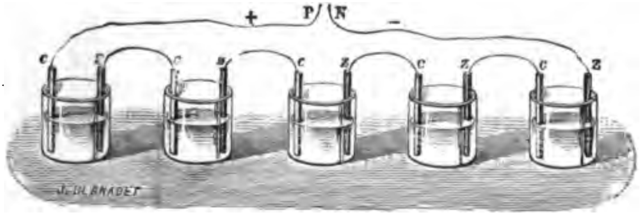

FIG. 97.

105. Modifications of Volta's Pile.—This decrease of power is shown in a marked degree by a Volta's pile if it is used to give a continuous current. It was at first attributed to the squeezing out of the liquid from the pieces of cloth at the lower part of the column by the weight of the metal discs above them, and to remedy it Volta adopted the arrangement shown in Fig. 97, and known as the *crown of cups*, consisting of a number of separate vessels containing dilute acid, with a strip of copper and a strip of zinc inserted in each and connected as before described. To make a battery of many cells more compact and portable, and to facilitate filling in and removing the acid, a further early device was Cruickshank's *trough battery* (Fig. 98), consisting of a number of double plates of zinc and copper cemented into grooves cut in the opposite sides of a wooden trough so as to divide the trough into a number of water-tight cells, each having one side formed of copper and the other of zinc.

The electromotive force of a Volta's cell, as measured on open circuit on the quadrant electrometer, is about one volt, but in whatever form it is constructed the electromotive force falls rapidly when the circuit is closed.

106. Use of Amalgamated Zinc.—If ordinary commercial zinc is used in the construction of a voltaic cell, it dissolves in the acid even when the circuit is not closed. But pure zinc, or common zinc that has been amalgamated, does not dissolve by itself in dilute sulphuric or hydrochloric acid. When used in a voltaic cell, it is attacked only while the current is passing, and the quantity that then dissolves is, as we shall see (§ 155), proportional to the current produced. To amalgamate a zinc plate, its surface should be cleaned by dipping it in dilute acid, and then, while it is still wet with the acid, pouring on to it a few drops of mercury, which readily spread over it.

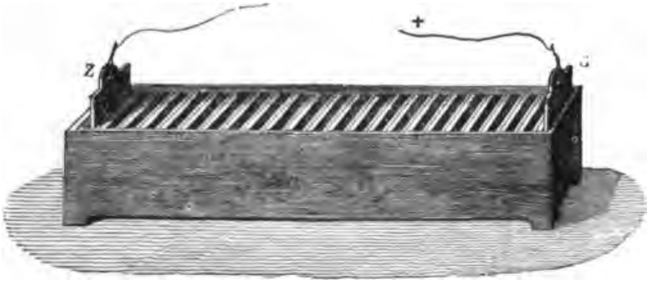

FIG. 98.

107. Constant Batteries.—The so-called polarisation of a Volta cell appears to be mainly due to the formation of a layer of hydrogen on the surface of the copper plate, to which it adheres with considerable obstinacy. To prevent this many expedients have been employed, mostly consisting in the employment of such materials that the chemical action taking place among them does not lead to the separation of hydrogen. In order that there may be no polarisation, in other words, that the electromotive force may be the same whatever the strength of the current, the materials of the cell should be such that the passage of the current and the accompanying chemical action cause no change in the nature of the substances in contact. In proportion as polarisation is avoided, a battery or cell is called *constant*. No perfectly constant form of battery has yet been devised, but there are two or three forms which make a fair approach to constancy when properly constructed and employed.

Daniell's battery (1836) was the earliest form of constant battery, and is still one of the most efficient. The metals employed are amalgamated zinc and copper, the zinc dipping in a dilute solution of zinc sulphate or in dilute sulphuric acid, and the copper in a saturated solution of copper sulphate. The two liquids are kept from mixing by a porous partition of unglazed earthenware, or sometimes by putting the heavier saturated copper solution in the bottom of a vessel and the lighter liquid above. A cell arranged in the former way is represented in Fig. 99, where C and Z are the copper and zinc plates respectively, and D is the porous partition. During the passage of the current, metallic zinc is converted into zinc sulphate, which dissolves in the liquid in contact with the zinc plate, and metallic copper separates from the solution of copper sulphate and is deposited on the copper plate. The metallic surfaces thus retain permanently their original

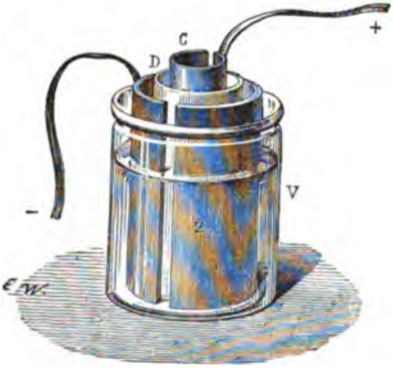

FIG. 99.

character, but the concentration of the solutions changes, especially in close contact with the metal plates, and this causes a small decrease of electromotive force while the current is passing. The normal electromotive force of a Daniell's cell is about 1.07 volt.

In *Grove's* battery (1840) the evolution of hydrogen is suppressed by the employment of strong nitric acid, which gives up part of its oxygen and converts the hydrogen into water. One of the metals is again amalgamated zinc dipping into dilute sulphuric acid (1 vol. of acid to 8 or 10 of water), which is separated from the nitric acid by a partition of porous earthenware. The use of nitric acid necessitates the replacement of copper by a metal, platinum, which the acid does not attack.

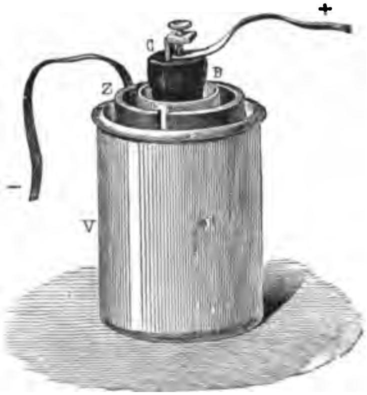

FIG. 100.

Bunsen's battery (Fig. 100) is a modification of Grove's, in which rods of dense gas-graphite are substituted for the expensive platinum plates. All else remains as in Grove's battery.

The electromotive force of a Grove's or Bunsen's cell is about 1.8 volt.

Among the many other forms of cells that are in more or less common use, we may mention the *bichromate* and the *Leclanché* cells. In both of these the plates consist of amalgamated zinc and graphite respectively: in the former, the plates dip in a solution of potassium bichromate acidulated with sulphuric acid; in the latter, the liquid is a saturated solution of ammonium chloride (sal ammoniac), and the carbon plate is closely packed round with black oxide of manganese in small fragments. The electromotive force of a bichromate cell is about 2 volts, but it has the disadvantage that the liquid acts on the zinc even when no current is passing; it must not, therefore, be left to stand with the plates in the liquid. A Leclanché cell has an electromotive force of not quite 1.5 volt on open circuit. It falls considerably when a current passes, but recovers when left to itself, and the plates may be left in contact with the liquid for long periods without injury.

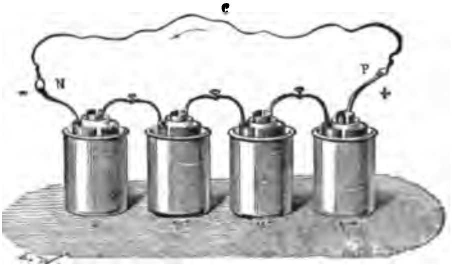

FIG. 101.

In all the cells mentioned the *zinc* terminal is *negative.*

Fig. 101 represents the mode of connecting a number of cells to form a battery, and shows a conducting wire joined to opposite terminals of the first and last cells.

CHAPTER XI.

ELECTRIC CURRENTS—RESISTANCE—OHM'S LAW.

108. The Electric Current.—We have already said (§ 103) that when the terminals of a battery are connected with the plates of a condenser, positive electricity passes to the plate in connection with the positive terminal, and negative electricity to that connected with the negative terminal. This accumulation of electricity on the two plates continues until each of them is brought to the same potential as the pole of the battery with which it is connected. When this state of things has been reached, there is a condition of equilibrium in which one condenser-plate, the corresponding wire, and the battery terminal to which it is attached are all at one potential, and the second condenser-plate, wire, and terminal are all at another potential, differing from that of the first plate by an amount equal to the electromotive force of the battery, —that is, the same change of potential occurs between the two sides of the dielectric of the condenser as between the two terminals of the battery, and the tendency of the condenser to discharge exactly balances the electromotive force of the battery.

If, however, the terminals are connected by a continuous wire, a condition of statical equilibrium is never established. The wire tends to bring the two terminals to the same potential, while the battery tends to maintain between them a difference of potentials equal to its own electromotive force. These two tendencies cannot be satisfied simultaneously: the result is that an intermediate state of things is reached, in which there is a continuous passage of positive electricity in one direction through the wire, and of negative in the opposite direction, tending to equalise the potential at the two ends; while, at the same time, the battery continues to supply positive electricity to, and to remove negative from, that end of the wire which has the higher potential, and to supply negative and remove positive at the other end, thus keeping up a difference of potentials between the ends. This difference of potentials falls short of equality with the electromotive force of the battery to an extent to be investigated later (§ 109).

As long as these conditions are maintained, the wire possesses certain special properties which are briefly indicated by saying that it is traversed by an *electric current.* These properties are not in all respects the same in both directions—that is, some of them differ according as, when we pass along the wire in a given direction, we are proceeding from the positive terminal of the battery to the negative, or the reverse. The same thing is commonly expressed by saying that the properties of the wire depend on the *direction of the current*, which is conventionally understood to mean the direction in which *positive* electricity is conceived of as passing round the circuit, namely, through the battery from the negative terminal to the positive, and through the connecting wire from the positive terminal to the negative.

Experiment shows that the characteristic properties associated with what is called the passage of a *current* are possessed by every point in the *circuit* formed by the battery and the conductor by which the terminals are connected, and, further, that the intensity with which these properties are exhibited, as well as their direction, is the same all round the circuit. It is found also by experiment that the potential at any one point of the circuit remains constant, which shows that there is no accumulation of electricity at any part. Hence it is concluded that the current, of which the circuit is the seat, is of the nature of a continuous circulation such that every section of the circuit is traversed simultaneously by the same quantity of electricity. The quantity of electricity which passes any section of the circuit in a given time determines what is called the *strength of the current.* The unit of current-strength, or, as it is also called more shortly, the unit of current, commonly employed for practical purposes, is that which corresponds to a coulomb per second. Such a current is called a current of one *ampere.* The strength of a current expressed in amperes therefore gives the number of coulombs which pass every cross-section of the circuit in one second.

If the strength of a current is constant, it is represented by the formula

$$C = q/t$$

where q is the quantity of electricity which traverses any section of the circuit in t seconds, and since the quantity which traverses every section of the circuit is the same, we may speak of this as the quantity which traverses the circuit. If the current is variable, its strength *at a given instant* is still expressed by the same formula if we understand by q the small quantity that passes during an indefinitely short interval of time, t, including the instant in question.

109. Variation of Potential along the Circuit.—A characteristic property of the current is that on each of the conductors which make up the circuit, the potential, instead of being uniform throughout, as in the condition of statical equilibrium, varies from one point to another, always decreasing in the direction of the current.

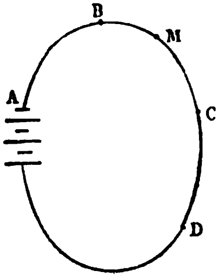

FIG. 102.

Let us suppose that the terminals of the battery are connected by a series of homogeneous cylindrical wires, AB, BC, CD . . . of different metals (Fig. 102). Take a quadrant electrometer, the needle of which is kept at a fixed potential, and put one pair of quadrants in connection with the point A, which may be, for instance, the positive terminal of the battery, and the other with a variable point, M, of the interpolar wire.

From A to M there will be a fall of potential which will vary with the position of the point M. So long as this point is on the wire AB, the fall is proportional to the distance of the point M from A, and its value in volts is equal to the product of the strength of the current C, expressed in amperes, into a factor, that we will denote by ρ_1, depending on the length AM measured along the wire, on the material of the wire, its thickness, and to some extent on its temperature and other physical conditions. Thus, letting A and M stand respectively for the potentials at the corresponding points, we may write

$$A - M = C\rho_1.$$

If we put r_1 for the value which ρ_1 assumes when M comes to B, the end of the first wire, we have for the difference of potentials between the points A and B

$$A - B = Cr_1.$$

Again, if the point B is connected with one pair of quadrants, and the point M connected with the other pair, is taken on the second wire, BC, we have, as in the previous case, for the difference of potentials between B and M

$$B - M = C\rho_2$$

ρ_2 being a factor depending on the material and dimensions of the second wire, or, when M coincides with C

$$B - C = Cr_2$$

and so on for each wire of which the circuit is made up.

If N be the potential at the negative terminal of the battery, the difference of potentials between the terminals, or $A - N$, is the sum of the differences $A - B$, $B - C$, . . . ; that is

$$A - N = C(r_1 + r_2 + r_3 + \ldots).$$

If we examine, similarly, by means of the electrometer, the difference of potentials between the terminals of the several cells of the battery, we find that in each case this is less than the corresponding difference when the circuit is not closed, and when therefore no current is passing, by an amount proportional to the strength of the current and to a factor, say ρ', depending on the nature of the materials forming the cell, and on the size and shape of its different parts. Thus if e be the electromotive force of a cell, or the difference of potentials between the terminals when no current is passing, the difference f when it is traversed by a current of strength C, is

$$f = e - C\rho'.$$

If the battery consists of n identical cells connected in series, its total electromotive force, E, is equal to n times the electromotive force, e, of one cell; and when the circuit is completed, so that the battery is producing a current of strength, C, the difference of potentials between its terminals is n times the difference between the terminals of a single cell, or

$$A - N = nf = ne - nC\rho' = E - Cr'$$

if we put r' instead of $n\rho'$.

Equating this value of $A - N$ with that previously found, we have

$$E = Cr' + C(r_1 + r_2 + r_3 + \ldots)$$

or

$$E = C(r' + r) = CR$$

when r is put for the sum of the separate quantities, r_1, r_2, r_3, . . ., and R for the total sum $r' + r$.

110. Definition of Resistance.—Writing the expressions for the difference of potentials between two points of a circuit as follows:—

$$r_1 = \frac{A - B}{C}, \; r_2 = \frac{B - C}{C}, \; r_3 = \frac{C - D}{C}, \text{ \&c.}$$

We see that the factors r_1, r_2, . . . applicable to the metallic parts of the circuit may be defined as the ratio for each conductor of the difference of potentials between its ends to the strength of the current which traverses it. By experiment it is found that this ratio is constant. If, for example, various differences of potentials, $A - B$, $A' - B'$, $A'' - B''$, . . . be maintained between the ends of a conducting wire, the wire is in each case traversed by a current of definite strength, such that if C, C', C'', . . . represent the currents in the several cases, we have

$$\frac{A - B}{C} = \frac{A' - B'}{C'} = \frac{A'' - B''}{C''} = \ldots = \text{a constant} = r_1.$$

This ratio, which for every conductor has a definite value characteristic of that conductor, is called its *resistance.*

In order that any conductor which is not itself the seat of electromotive force may be traversed by a current, it is essential that, as the result of an external electromotive force applied to it, the potential at the part where the current enters should exceed that at the part where the current leaves—that is to say, there must be a *drop of potential* as we pass with the current through the conductor. The resistance of the conductor may then be shortly defined as the *drop of potential corresponding to a current of unit strength* traversing the conductor.

A galvanic cell is a conductor which is the seat of electromotive force. Where there is no current, there is a difference of potentials, say e, between the terminals equal to the electromotive force of the cell. When the terminals are joined by a wire, a current traverses the cell from the terminal which has the lower potential to that which has the higher, and the difference of potentials assumes a value, say f, less than the electromotive force of the cell, so that there is a decrease in the difference of potentials, in consequence of the passage of the current, equal to $e - f$. This decrease, divided by the strength of the current, is again a constant quantity, characteristic of each particular cell, and is called its *resistance.* Using accents to distinguish the values in different cases, this may be expressed in symbols as follows :—

$$\frac{e-f}{C} = \frac{e-f'}{C'} = \frac{e-f''}{C''} = \ldots = \text{a constant.}$$

Accordingly, the resistance of a galvanic cell may be defined as the *decrease of difference of potentials between its terminals due to the passage of unit current.*

111. Resistance of Combinations of Conductors.—If several conductors are connected end to end, or *in series*, so as to be traversed by the same current, the resistance of the combination is, by definition, the drop of potential between the points where the current enters and leaves the series divided by the strength of the current; and, as appears at once on writing down the values, it is equal to the sum of the resistances of the separate conductors. Thus, using the same notation as previously, we have for the resistances of the first, second, third, . . . conductors taken by themselves

$$r_1 = \frac{A-B}{C}, \quad r_2 = \frac{B-C}{C}, \quad r_3 = \frac{C-D}{C}, \text{ \&c.}$$

and for the resistance of the combination

$$r = \frac{A-N}{C}$$

or

$$r = r_1 + r_2 + r_3 + \ldots$$

since

$$(A-B) + (B-C) + (C-D) + \ldots = A - N.$$

Similarly, for a battery formed of a number of cells connected in series; if e_1, e_2, . . . are the electromotive forces of the separate cells when no current is passing, and f_1, f_2, . . . the differences of potential between their terminals when the circuit is closed, and if E and F have corresponding meanings for the whole battery, the resistances of the separate cells are

$$r_1 = \frac{e - f_1}{C}, \quad r_2 = \frac{e_2 - f_2}{C}, \quad r_3 = \text{\&c.}$$

and the resistance of the battery is

$$r = \frac{E-F}{C}.$$

Now it is found by experiment (§§ 97, 102) that $E = e_1 + e_2 + \ldots$, and $F = f_1 + f_2 + \ldots$, hence

$$r = \frac{e_1 - f_1 + e_2 - f_2 + \ldots}{C} = r_1 + r_2 + \ldots;$$

or, again, the resistance of the combination is the sum of the separate resistances.

If a number of conducting wires whose separate resistances are r_1, r_2, r_3, . . . are employed to connect the same points, as A and B (Fig. 103), the total current of strength, say C, divides among the wires, so that the sum of the currents in the separate branches is equal to the undivided current, or

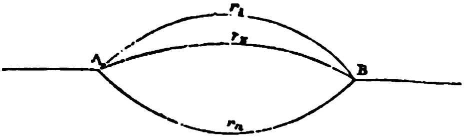

FIG. 103.

$$C = c_1 + c_2 + c_3 + \ldots$$

If we put $A - B$ for the difference of potentials between the points A and B, the resistances of the separate wires are given by

$$r_1 = \frac{A - B}{c_1}, \quad r_2 = \frac{A - B}{c_2}, \quad r_3 = \frac{A - B}{c_3}, \text{ \&c.}$$

and the resistance of the combination is

$$r = \frac{A - B}{C} = \frac{A - B}{c_1 + c_2 + c_3 + \ldots}$$

whence,

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \ldots$$

Conductors connected in this manner are variously spoken of as being connected *abreast*, or in *multiple arc*, or in *parallel circuit.* A case that occurs in practice very frequently is where two conductors are so combined. In this case the resistance of the combination is $r = \frac{r_1 r_2}{r_1 + r_2}$, the product of the separate resistances divided by their sum.

If two galvanic cells whose separate resistances are r_1 and r_2

respectively, and whose electromotive forces are e_1 and e_2, are connected in multiple arc, their positive terminals being joined at A (Fig. 104) and their negative terminals at B, the difference of potentials between the points A and B may be found from the following consideration. The two cells form a closed circuit, round which there will in general be a current which will traverse one of the cells, namely, that one which has the greater electromotive force, from the negative to the positive terminal (this may be called the *natural* direction), and the other from the positive to the negative terminal (or in the *inverse* direction). Let c denote the strength of this current; then, according to § 109, the difference of potentials between A and B will be less than the electromotive force, say e_1, of the stronger cell, and greater than that, e_2, of the weaker cell, by the product of the current strength into the resistance of the corresponding cell. If E be this difference of potentials, we have accordingly

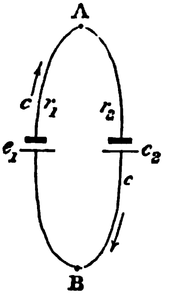

FIG. 104.

$$E = e_1 - cr_1 \text{ and } E = e_2 + cr_2.$$

Or, eliminating c between these two values,

$$E = \frac{e_1 r_2 + e_2 r_1}{r_1 + r_2}.$$

Now suppose the points A and B connected by a wire of resistance r (Fig. 105). This wire will be traversed by a current flowing from A to B of strength $C = c_1 + c_2$, if c_1 and c_2 denote the currents now flowing from B to A through the two cells respectively. As the result of this current, the difference of potentials between A and B will now be less than before: let it be denoted by F. Then, by the definition given at the end of § 110, we have for the resistance of the single cell which would be equivalent to the given combination of two cells

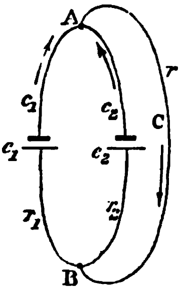

FIG. 105.

$$R = \frac{E - F}{C}.$$

To determine F, we may apply the same definition to the two cells taken separately; we thus get

$$r_1 = \frac{e_1 - F}{c_1} \text{ and } r_2 = \frac{e_2 - F}{c_2},$$

while the consideration of the connecting wire gives

$$r = \frac{F}{C}.$$

Remembering that $C = c_1 + c_2$, these equations give

$$F = \frac{e_1 rr_2 + e_2 rr_1}{rr_1 + rr_2 + r_1 r_2}$$

$$\text{and } C = \frac{F}{r} = \frac{e_1 r_2 + e_2 r_1}{rr_1 + rr_2 + r_1 r_2}.$$

Putting these values into the expression for R, we have

$$R = \frac{E}{C} - \frac{F}{C} = \frac{rr_1 + rr_2 + r_1 r_2}{r_1 + r_2} - r = \frac{r_1 r_2}{r_1 + r_2}.$$

Hence it appears that when two conductors are connected in multiple arc, the resistance of the combination, or, in other words, the resistance of the single conductor by which the two given conductors can be replaced without altering the strength of the current in any other part of the circuit, is in all cases equal to the *product divided by the sum* of the separate resistances, whether these are simple metallic wires, or whether they are galvanic cells, or other arrangements which form the seats of electromotive forces. In the first case, the single equivalent conductor must not contain an electromotive force; in the second case, it must be the seat of an electromotive force $E = (e_1 r_2 + e_2 r_1) / (r_1 + r_2)$.

112. Ohm's Law.—The nature of electric resistance, and the fact that the resistance of any given conductor has a definite numerical value characteristic of that conductor, were first clearly recognised by G. S. Ohm. The properties of the electric circuit were discussed by him from this point of view in a work published in 1826, and the relations that he established between the electromotive force or forces acting in a circuit, its resistance, and the strength of the current by which it is traversed, form the starting-point of all calculations involving these magnitudes. This relation is usually referred to as *Ohm's Law*, and may be expressed in symbols by the following equation—

$$R = \frac{E}{C}, \quad C = \frac{E}{R}, \quad E = CR$$

which may be applied either to a single conductor or to a complete circuit. R is the resistance of the conductor, or the total resistance of the circuit, as the case may be ; E the electromotive force acting in the conductor, or the resultant electromotive force of the circuit ; and C the strength of the current.

The unit, in terms of which resistances are commonly expressed, or the *practical unit of resistance*, is called an *ohm*, and is such that an electromotive force of one volt applied to a homogeneous metallic conductor having this resistance will maintain in it a current of one ampere. The ohm will be more fully discussed in the chapter on Electrical Units (Chap. xxx.), but we may state here that it is very nearly equal to the resistance at 0° C. of 14.452 grammes of mercury in the form of a column of uniform cross-section and 106.3 cm. long.

Ohm not only showed that the resistance of a given conductor in a given condition is a constant quantity depending on its material and dimensions, but he also showed that the resistance of a conductor of uniform cross-section s, and length l, is expressed by

$$R = \rho \frac{l}{s}$$

where ρ is a constant characteristic of the material, called its *specific resistance*. The full statement of Ohm's law must be taken as including this equation.

113. Specific Resistance.—If we put R_1 for the resistance of a conductor of length 1 cm., and cross-section 1 cm.^{2}, l and s in the above expression both become unity, and we have $R_1 = \rho$, which gives us the definition of the specific resistance of a material as the *resistance of a conductor of that material* 1 *cm. long and having a cross-section of* 1 *cm*^{2}. The shape of the cross-section makes no difference, so that, for the purposes of this definition, we may suppose that we take 1 cm. length along a square rod measuring 1 cm. across each way ; such a piece would be a centimetre-cube, and its resistance would be the specific resistance of the material. But it is important to notice that, when specific resistance is defined as the resistance of a centimetre-cube, it is to be understood that the flow of electricity takes place between opposite faces and is uniformly distributed throughout the cross-section.

The specific resistances of the metals increase with temperature, and therefore also the actual resistances of conducting wires. If the resistance of a wire at 0° C. is denoted by R_0, its resistance at $t°$ may be represented to a first approximation by

$$R = R_0 (1 + at),$$

where a is a coefficient depending on the material. For several pure metals, the value of a has been found to be from 0.0036 to 0.0038, or nearly equal to the coefficient of expansion of gases. For alloys, the value of a is much smaller, and bodies are known for which its value is actually negative beyond a certain moderate temperature.

Recent experiments by Professors Fleming and Dewar show that at very low temperatures, such as can be obtained by the evaporation of liquid oxygen, the specific resistances of the pure metals are very much less than they are at ordinary temperatures. The following numbers are taken from their results :—

Approximate Temperature.	Specific Resistance.			
	Platinum.	Silver.	Copper.	Iron.
0° C.	10974	1489	1564	9115
− 200°	3339	390	289	1220
− 220°	2439	...	144	660

The specific resistances of non-metallic liquids are enormously greater than those of metals, and they decrease with rise of temperature.

A table giving numerical values for a number of substances, both metallic and non-metallic, is given at the end of this volume.

CHAPTER XII.

APPLICATIONS OF OHM'S LAW.

114. Current in a Simple Circuit.—Ohm's law enables us to calculate immediately the strength of the current that a given battery will produce through any conductor of known resistance which may be connected with its terminals. For instance, suppose the battery to consist of n cells connected in series, each having electromotive force e and resistance r', and that its terminals are connected by a wire of resistance r. Then the total electromotive force of the circuit is $E = ne$, and the total resistance is $R = nr' + r$; consequently the current is

$$C = \frac{E}{R} = \frac{ne}{nr' + r}.$$

If the cells composing the battery, instead of being all identical, have various electromotive forces and resistances, e_1, e_2 . . . and r_1', r_2', . . ., we shall have $E = e_1 + e_2 + \ldots$, and $R = r + r_1' + r_2' + \ldots$ Again, if any of the cells are connected with their poles in the opposite direction to the rest, the values of e for them must be taken as negative—that is to say, E must be taken as the *algebraic* sum of the separate electromotive forces. The resistance of a cell comes into account in the same manner, whichever way it is connected in the circuit; hence the value of R is not affected by the reversal of any of the cells.

115. Geometrical Representation. — The influence on the strength of the current of the electromotive force of the battery, and of the relative resistance of the battery and the rest of the circuit, can be conveniently expressed geometrically by the construction of Fig. 106. Here horizontal distances represent resistances, and vertical distances electromotive forces or differences of potential. MN represents the resistance of the battery, NO the external resistance, MK the electromotive force, and NL the difference of potentials between the battery terminals.

The strength of the current is given by the slope of the line KO, that is, by the tangent of the angle at O, or if we draw KP parallel to MO, to represent one ohm (represented in the figure by 1.5 cm.), and PQ at right angles to KP, the current is represented by the length of the single line PQ. For, by construction

$$C=\frac{E}{R}=\frac{\text{MK}}{\text{MO}}=\tan\text{ MOK}=\tan\text{ PKQ}=\frac{\text{PQ}}{\text{PK}}=\text{PQ}.$$

The figure also shows clearly the relation between F, the difference of potentials at the terminals of the battery, and the electromotive force $E=ne$. For NL, which represents F, is equal to MK − LL′, and LL′ = MN × $\frac{\text{MK}}{\text{MO}}$ which is equivalent to

$$F=E-bC=E\frac{r}{b+r},$$

where the single symbol b is put for nr', the resistance of the battery, in order to recall the fact that, as long as the battery remains unaltered, its resistance is a constant quantity.

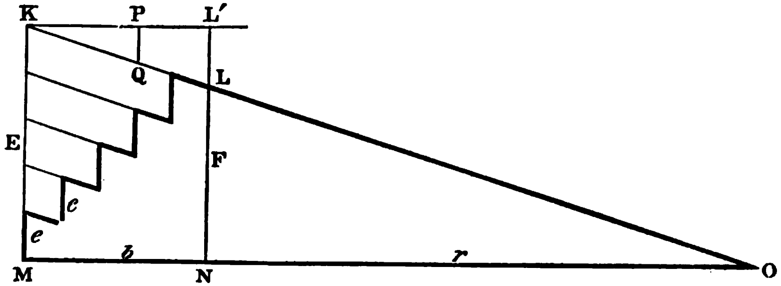

FIG. 106.

It is easy from the same figure to see, at least generally, how the strength of the current depends on the external resistance. If this is great, O will be very far from N, the line KO will be nearly parallel to MO, and PQ, which is proportional to the current strength, will be small; obviously, however, PQ does not vanish for any finite value of NO or r. On the other hand, if r diminishes, O approaches N, and finally coincides with it. In this case the current attains its maximum value, $C = E/b = e/r'$, which is the same as that of the maximum current obtainable from a single cell. When the terminals of a cell or battery are connected through a conductor of no appreciable resistance, the battery is often spoken of as being *short-circuited.* Fig. 107 represents the effect of short-circuiting a battery of twelve similar cells in series, the resistance of each cell being represented by an arc of a circle instead of by a distance along a straight line.

In Fig. 106, a vertical ordinate drawn from any point in NO would represent the difference of potentials between that point and O, which corresponds with the negative terminal.

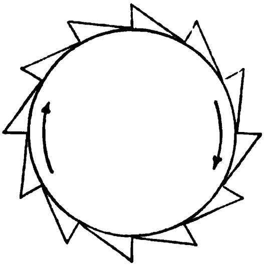

FIG. 107.

116. Kirchhoff's Laws.—The two following deductions from Ohm's law, often referred to as *Kirchhoff's rules*, are useful in calculations relating to complex circuits :—

1. *If several conductors meet at the same point, the algebraic sum of the strengths of the currents in the several conductors, reckoned positive when the direction of the current is towards the point, is equal to nothing*, or

$$\Sigma c = 0.$$

2. *If two or more conductors form a closed figure, the algebraic sum of the products of the resistances of the several conductors into the strengths of the currents through them, reckoned positive the same way round the figure, is equal to the algebraic sum of the electromotive forces acting round the figure in the same direction*, or

$$\Sigma cr = \Sigma e.$$

The first of these theorems may be regarded as self-evident, it being simply an expression of the fact that under the given conditions there is no accumulation (positive or negative) of electricity at the point considered (Fig. 108).

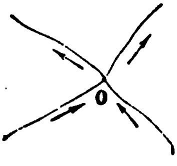

FIG. 108.

The second depends upon the relation already pointed out (§ 109), that the difference of potentials, f, between the ends of a conductor which is the seat of electromotive force e, is less than e when the conductor is traversed by a current in the direction of that which the electromotive force tends to produce, by the product of the strength of current into the resistance, or

$$f = e - cr.$$

Applying this result in succession to each of the conductors forming a closed circuit, we get from any selected starting-point back to the same point again; accordingly, the sum of the differences of potential round the circuit must be zero, or

$$\Sigma f = 0, \text{ whence } \Sigma cr = \Sigma e.$$

An important special case falling under this theorem is when a number of metal wires form a closed circuit (Fig. 109): here e is equal to nothing for each conductor, and therefore also for the sum, or $\Sigma cr = 0$.

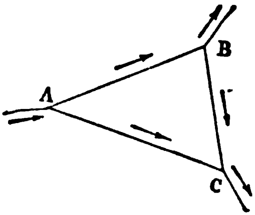

FIG. 109.

117. Conductors in Multiple Arc.—By aid of the foregoing results it is easy to solve many problems which present themselves in connection with complex combinations of conductors. We will consider first a number of metallic conductors joining the same points A and B; three conductors (Fig. 110) will be enough to illustrate the general case. If r_1, r_2, r_3, are the resistances of the separate conductors, we have already seen (§ 111) that the resistance R of the combination is

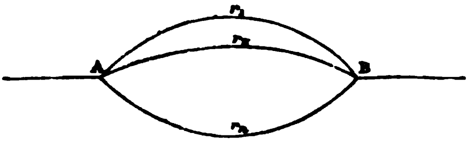

FIG. 110.

$$R = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}.$$

The strength of the current in each branch may be expressed in terms of the total current C as follows: putting $A - B$ for the difference of potentials between the points A and B, we have

$$c_1 = \frac{A - B}{r_1}, \quad c_2 = \frac{A - B}{r_2}, \quad c_3 = \frac{A - B}{r_3},$$

whence

$$C = \frac{A - B}{r_1} + \frac{A - B}{r_2} + \frac{A - B}{r_3} = c_1 \left(\frac{r_1}{r_1} + \frac{r_1}{r_2} + \frac{r_1}{r_3}\right)$$

or

$$c_1 = C\frac{r_2r_3}{r_1r_2 + r_2r_3 + r_3r_1}.$$

Similarly

$$c_2 = C\frac{r_3r_1}{r_1r_2 + r_2r_3 + r_3r_1}; \quad c_3 = C\frac{r_1r_2}{r_1r_2 + r_2r_3 + r_3r_1}.$$

The case that occurs oftenest in practice is the simple one of two conductors. This is included in the above formula if we suppose r_3 to become infinite, when we get

$$c_1 = C\frac{r_2}{r_1 + r_2}; \quad c_2 = C\frac{r_1}{r_1 + r_2}.$$

As one among many instances of the application of these formulæ, we may refer to the measurement of currents. It often happens that it is not desirable to allow the whole of the current to pass through the measuring instrument. In such a case, supposing the resistance of the instrument to be r_1, its terminals may be connected by a wire of resistance, r_2, frequently called a *shunt*. Then if the total current is C, and the current indicated by the measuring instrument is c_1, the strength of the total current is given by

$$C = c_1\frac{r_1 + r_2}{r_2} = c_1\left(1 + \frac{r_1}{r_2}\right).$$

For example, if $r_1/r_2 = 9$, $C = 10c_1$; if $r_1/r_2 = 99$, $C = 100c_1$.

118. Voltaic Cells or Batteries in Multiple Arc.—Let e_1, e_2, e_3, be the respective electromotive forces of three single voltaic cells or batteries connected as in Fig. 111; r_1, r_2, r_3, the corresponding resistances measured between the points A and B; c_1, c_2, c_3, the

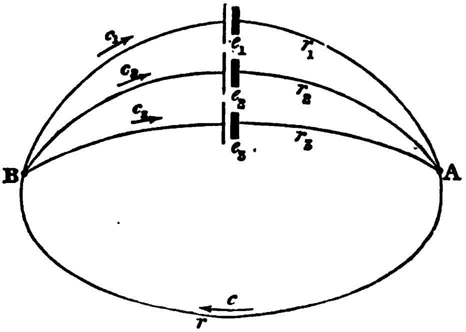

FIG. 111.

currents through the several cells; lastly, r the resistance of a wire connecting A and B, and c the current in it. By applying Kirchhoff's second rule to the circuit formed by taking the cells, one at a time, together with the connecting wire r, we get the equations

$$cr + c_1 r_1 = e_1, \text{ or } c\frac{r}{r_1} + c_1 = \frac{e_1}{r_1}$$

$$cr + c_2 r_2 = e_2, \text{ or } c\frac{r}{r_2} + c_2 = \frac{e_2}{r_2}$$

$$cr + c_3 r_3 = e_3, \text{ or } c\frac{r}{r_3} + c_3 = \frac{e_3}{r_3}.$$

Then, adding, reducing to a common denominator, and simplifying, we get for the current in the external conductor

$$c = \frac{e_1 r_2 r_3 + e_2 r_1 r_3 + e_3 r_1 r_2}{r_1 r_2 r_3 + r r_2 r_3 + r r_1 r_3 + r r_1 r_2}.$$

For the resistance R of a single cell equivalent to the combination, we have (§ 111)

$$R = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}.$$

To obtain the electromotive force of the equivalent single cell, we have

$$E = c(R + r)$$

which, putting in the values of c and R and reducing, gives

$$E = \frac{e_1 r_2 r_3 + e_2 r_1 r_3 + e_3 r_1 r_2}{r_2 r_3 + r_1 r_3 + r_1 r_2}.$$

We may note the following special cases:—

1. All three cells have the same electromotive force, say e, then

$$E = e$$

which shows that the resultant electromotive force of any number of equal cells connected in multiple arc is the same as that of a single cell of the same kind.

2. All the cells have the same resistance; in this case the formula gives

$$E = \frac{e_1 + e_2 + e_3}{3}$$

or, the resultant electromotive force of any number of cells of equal resistance connected in multiple arc is the arithmetic mean of their separate electromotive forces.

To get the currents in the separate branches, we may start from the equation

$$c_1 = (e_1 - cr)/r_1$$

and the corresponding expressions for c_2 and c_3. Putting in the value of c already found, we get

$$c_1 = \frac{e_1(r_2 r_3 + r r_3 + r r_2) - e_2 r r_3 - e_3 r r_2}{r_1 r_2 r_3 + r r_2 r_3 + r r_1 r_3 + r r_1 r_2}.$$

The values of c_2 and c_3 may be obtained similarly, or they may be written down from the last expression by cyclically changing the suffixes. It is evident that the denominator is not altered thereby.

119. Best Arrangement of Cells.—Suppose that n similar cells, each of resistance r and electromotive force e, are available for sending a current through a conductor of resistance r': the cells may be grouped in various ways, as all in series (Fig. 112), or all in multiple arc (Fig. 113); or they may be arranged in an intermediate manner, being divided into two or more series and these again being joined in multiple arc (Fig. 114). In general, any change in the arrangement of the cells will cause an alteration in the strength of the current in the external conductor, and the question arises, how must they be combined in order to make this current as strong as possible? Let p be the number of cells in each series, and q the number of series, so that $pq = n$. The resistance of each series will be pr, and the resistance of the q series in multiple arc will be $\frac{p}{q}r$. Again, the electromotive force of each series will be pe, and as the series are all equal, this will also be the electromotive force of the whole combination. Consequently the strength of the current will be

$$C = \frac{pe}{pr/q + r'} = \frac{ne}{pr + qr'}.$$

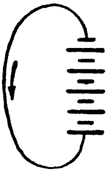

FIG. 112.

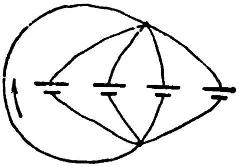

FIG. 113.

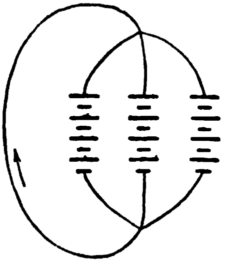

FIG. 114.

The numerator of the expression for C, in its second form, is constant, and C will therefore be greatest when the denominator is least. But the product of the two terms of the denominator is $pqrr' = nrr'$, a constant quantity, and therefore the denominator has its smallest value when its two terms are equal, or $pr = qr'$, that is, when the resistance of the battery is equal to the external resistance, or, what is the same thing (since the last equation is the same as $p/q = r'/r$), when the number of cells in each series is to the number of series as the external resistance is to the resistance of a single cell. Seeing that p and q must be whole numbers, and $pq = n$, it is not generally possible to satisfy accurately the condition for a maximum current; all that can then be done is to choose the arrangement that comes nearest. If r', the external resistance, is very much greater than r, the resistance of one cell, we must make p/q as great as possible, or $p = n$ and $q = 1$; on the other hand, if r'/r is small, we must make $p = 1$ and $q = n$.

120. Conduction in Two Dimensions.—If, in place of being a thin wire, the conductor is a uniform flat sheet to which electricity is supplied or from which it is withdrawn at a given point, the current, instead of taking place in one direction only, will spread out from the point on all sides, or converge towards it, as the case may be. If we suppose the uniform sheet to extend without limit in all directions, it is evident that the current will spread uniformly, and that the flow will take place along straight lines drawn through the point. If the quantity of electricity given to or removed from such a sheet in a second is Q coulombs, and we draw straight lines through the point, making with each other a common angle a, such that

$$a = 2\pi/Q,$$

the total flow between each pair of consecutive lines will be one ampere. A quantity equal to Q will flow in every second across any circumference having its centre at the *electrode* or point where electricity is supplied or withdrawn, and the flow across equal parts of such a circumference will be equal. The flow per second across *unit length* of a circumference drawn through any point of the sheet may therefore be taken as the measure of the *intensity* or *density* of the current at that point. Thus at any point at a distance r from the electrode, the intensity of the flow is $Q/2\pi r = 1/ar$, and the total flow across an arc of length s is $\frac{Qs}{2\pi r} = Q\frac{\theta}{2\pi}$, if we write θ for the angle $\frac{s}{r}$ subtended by the arc s.

It is evident from symmetry that the potential at every point on a circle drawn about the electrode as centre must be the same, or that any such circle must be an *equipotential* line. And since the direction of flow in a conductor is the direction of decreasing potential, the potential must *decrease* with increasing distance from an electrode that supplies electricity to the sheet, that is, from a *source*, and must *increase* with decreasing distance from a *sink*, or an electrode that removes electricity from the sheet. Consider (Fig. 115) two concentric equipotential circles of very nearly equal radii, r and $r + dr$, and draw two radii, making a very small angle $d\theta$ with each other. The two circles and the two radii will intercept a small area on the conducting sheet, which, in the limit, may be treated as a rectangle of length dr, in the direction of the current, and of breadth $rd\theta$, at right angles to the current. If δ is the thickness of the sheet and ρ its specific resistance, the resistance of this elementary portion may be taken as

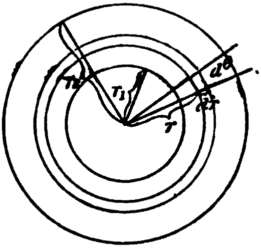

FIG. 115.

$$\frac{\rho dr}{\delta r d\theta}.$$

The whole circular strip bounded by the two circles contains $2\pi/d\theta$ such areas, which may be considered as conductors connected in multiple arc, and therefore the resistance of the whole strip to the radial flow taking place across it is

$$\frac{\rho dr}{2\pi r\delta}.$$

To get the resistance of an annular belt contained between circles of radii r_1 and r_2 differing by a finite amount, we require to integrate this expression between the corresponding limits. Thus the resistance of the belt is

$$R = \frac{\rho}{2\pi\delta}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{\rho}{2\pi\delta}\log_e\frac{r_2}{r_1} = 0.3665\frac{\rho}{\delta}\log_{10}\frac{r_2}{r_1}.$$

But (§ 112) the difference of potentials between two equipotential circles is equal to the resistance of the intervening portion of the conductor multiplied by the strength of the current: we have therefore for the amount by which the potential at radius r_2 *exceeds* that at radius r_1

$$V_2 - V_1 = -0.3665 \frac{Q\rho}{\delta} \log_{10} \frac{r_2}{r_1}, \text{ if the centre is a } source,$$

$$\text{and } V_2 - V_1 = 0.3665 \frac{Q\rho}{\delta} \log_{10} \frac{r_2}{r_1}, \text{ if the centre is a } sink.$$

It follows that a series of circles drawn about the same electrode have a constant difference of potentials between each one and the next if their radii are in geometrical progression, and if V_1 be the potential at unit distance from an electrode, the potential at any distance r is given by $V = V_1 - 0.3665 \frac{Q\rho}{\delta} \log_{10} r$, the sign of Q being taken positive for a source and negative for a sink.

121. Two Equal Opposite Electrodes—Lines of Flow.—Now consider a source A (Fig. 116) supplying Q coulombs per second to a conducting sheet, and a sink B removing electricity at

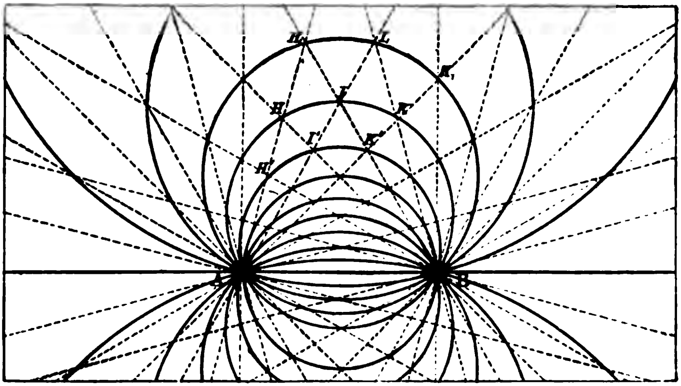

FIG. 116.

the same rate. In this case there is no change in the total quantity of electricity possessed by the sheet, and therefore, with a limited sheet, if the shortest distance from the electrodes to any point of the boundary is great in comparison with the distance between the electrodes, the state of things does not differ appreciably from what would exist in the corresponding portion of an unlimited sheet, and can be deduced by simply superposing on each other the conditions which we have already seen would apply in the case of a source or a sink respectively existing by itself in an unbounded sheet. Let AH, AI, AK, be drawn through A, making a constant angle $\alpha = \frac{2\pi}{Q}$ with each other; and let BH, BI, BK . . . be drawn similarly through B. Considering the effect of the source A by itself, the flow between AH and AI into the quadrilateral HH$_1$ II′ across HI′, would be one ampere. Similarly, considering the sink B by itself, the flow due to it between BH and BI would be one ampere out of the same quadrilateral across I′I. The resultant flow due to the co-existence of the source and sink must then be such as to satisfy simultaneously the conditions of a flow of one ampere inwards (with respect to the quadrilateral considered) across HI′ and outwards across I′I. Obviously, also, corresponding conditions must apply to the flow across the boundaries of any of the quadrilaterals into which the whole surface of the conducting sheet is cut up by the mutual intersection of the two sets of rays drawn outwards from A and inwards towards B. Again, consider the flow at one of the points of intersection, say H: the flow due to A alone is along HH$_1$, and that due to B alone is along HI′. The resultant flow must have an intermediate direction. Similarly at I the resultant flow must have a direction intermediate between II$_1$ and IK′. Now, trace a curve, such as HIK, through alternate angles of successive quadrilaterals, and consider one of the resulting triangles, such as HI′I: we have already seen that the flow into this triangle across the side HI′ is equal to the outward flow across I′I. Therefore, on the whole, there can be no flow inwards or outwards across the third side, HI, of the triangle. Hence we see that lines drawn like H′I′K′, HIK, H$_1$I$_1$K$_1$, in the figure are such that, on the whole, no electricity flows across them; and by taking the common angle between the constructive lines of the figure smaller, so that the flow between consecutive lines due to source or sink alone is less than one ampere, we may determine as many points as we please between the points H, I, K, . . . and thus trace continuous curves such that no electricity flows across them anywhere. But such lines must coincide everywhere with the direction of the current—in other words, they must be *lines of flow* of the combined system of source and sink. We see, then, that the lines of flow for the case we are considering are continuous curves traced through the alternate intersections of equiangular pencils of rays drawn through the source and sink respectively, and that, if the common angle between the rays corresponds to a flow of one ampere, the flow between consecutive lines of the resultant system, *e.g.*, between H'I'K' and HIK, or HIK and $H_1I_1K_1$, will also be one ampere. From the geometry of the figure it is easy to see that the angles AHB, AIB, AKB, . . . are equal, and therefore that the points H, I, K, . . . are points on a circle passing through A and B,—and similarly for the points on any other line of flow, as H'I'K', &c.

122. Equipotential Lines.—The system of equipotential lines due to a source and a sink in a conducting sheet can be obtained by a process of superposition quite analogous to that employed for the lines of flow. In Fig. 117, let A and B again represent a source and a sink, and let the two sets of dotted circles, of which these points are the respective centres, represent systems of equipotential lines which would correspond to each electrode if it existed by itself. Further, let the smallest circle round A correspond to potential V, and the smallest circle round B to potential $-V$, and let there be a common difference of one volt between any one circle and the next in each set. Then at the point H in the figure the potential due to the source is $V - 6$, the potential due to the sink is $-V + 6$, and the resultant potential, being the algebraic sum of these, is $= 0$. Similarly, the potential at I and K $= 0$. At H_1 the potential due to the source is $V - 5$, and that due to the sink is $-V + 6$, consequently the resultant

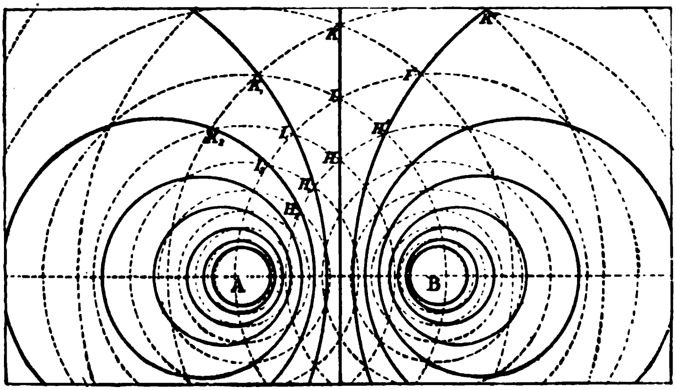

FIG. 117.

potential is + 1. At I_1 the potentials due to the electrodes separately are $V-6$ and $-V+7$ respectively, and therefore the resultant is again + 1. Similarly at K_1. It follows, then, that the line of zero potential must pass through the points H, I, K,—it is, in fact, the straight line that bisects AB at right angles,—and that the line of potential + 1 must pass through H_1, I_1, K_1. From similar considerations it is evident that the line of potential + 2 must pass through the points H_2, I_2, K_2. In like manner the line of potential − 1 must pass through H′, I′, K′; the line of potential −2 through H″, I″, K″; and so on.

If we put r for the distance of any point on the conducting sheet from the source, and r' for the distance of the same point from the sink, it is easy to see that the potential must be the same at all points for which the ratio of the distances r and r' is constant, and that it must be positive for all parts of the sheet for which r/r' is less than unity, and negative for all parts for which this ratio is greater than unity.

The same result is easily obtained algebraically. Let V be the potential which the source, if existing alone, would produce at a given point, and V' the potential which the sink alone would produce at the same point: then, if we put U for the resultant potential due to both source and sink, we have (§ 120)

$$V = V_1 - \frac{Q\rho}{2\pi\delta} \log_e r$$

$$V' = -V_1 + \frac{Q\rho}{2\pi\delta} \log_e r'$$

and therefore

$$U = V + V' = \frac{Q\rho}{2\pi\delta} (\log_e r' - \log_e r) = \frac{Q\rho}{2\pi\delta} \log_e \frac{r'}{r}$$

whence it appears that the potential is proportional to $\log \frac{r'}{r}$, and therefore, as before, it is constant when r'/r is constant.

The curves which fulfil the condition, $r'/r =$ constant, are circles with their centres in the line AB.

123. Resistance.—Suppose a portion of a conducting sheet, in which there are a source and a sink of equal strength, bounded by two lines of flow and by two equipotential lines of potentials, U_1 and U_2 respectively: we can deduce the resistance of this portion of the sheet to the current through it from the formula

$$R = (U_1 - U_2)/C,$$

derived from Ohm's Law.

For U_1 we may write $\frac{Q\rho}{2\pi\delta}\log_e\frac{r_1'}{r_1}$, and $\frac{Q\rho}{2\pi\delta}\log_e\frac{r_2'}{r_2}$ for U_2, so that the difference

$$U_1 - U_2 = \frac{Q\rho}{2\pi\delta}\left(\log\frac{r'}{r_1} - \log\frac{r_2'}{r_2}\right) = \frac{Q\rho}{2\pi\delta}\log\frac{r_1' r_2}{r_1 r_2'}.$$

The strength of the current C is to the total flow Q as the angle θ, at which the flow-lines bounding the portion of the sheet considered meet each other at the electrodes, is to 2π. That is $C = Q\frac{\theta}{2\pi}$: hence

$$R = \frac{\rho}{\theta\delta} \cdot \log\frac{r_1' r_2}{r_1 r_2'}.$$

If we have to deal with the whole space between two complete equipotential lines, we have $\theta = 2\pi$ and therefore $C = Q$, and consequently

$$R = \frac{\rho}{2\pi\delta} \cdot \log_e\frac{r_1' r_2}{r_1 r_2'}.$$

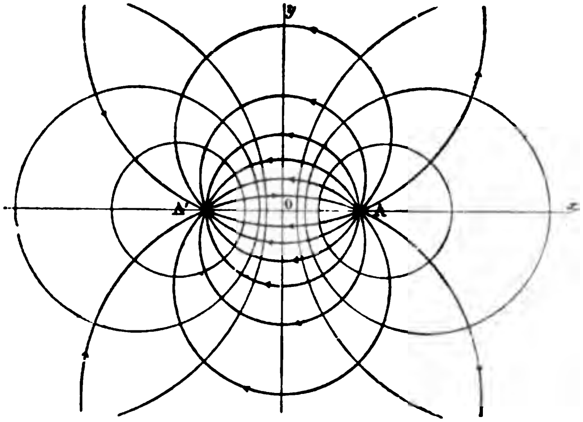

FIG. 118.

Fig. 118 shows the lines of flow and the equipotential lines for the case we have been discussing combined in one diagram. It will be seen that the two sets of lines intersect at right angles.

CHAPTER XIII.

WORK DONE BY THE ELECTRIC CURRENT

124. Energy of the Circuit.—We have seen (§§ 34, 35) that, in order to transfer electricity from a point of lower to a point of higher potential, work has to be done *against* electric force, the amount of work being numerically equal to the product of the quantity of electricity transferred into the difference of potentials; and also that, when electricity passes from a higher to a lower potential, a corresponding amount of work is done *by* electric force. Accordingly electric energy is generated when electricity passes from a lower to a higher potential, and is expended when it passes from a higher potential to a lower. In the former case, energy of some other kind must be expended; and in the latter case, energy of some other kind is produced.

Let us consider, from this point of view, the processes that go on in the circuit of a galvanic battery whose terminals are connected by a conducting wire. To make the discussion definite, suppose the electromotive force of the battery to be E, and let b stand for the resistance of the battery itself, and r for that of the connecting wire. Then the circuit is traversed by a current of strength $C = E/(b + r)$, and in t seconds a quantity of electricity, $Q = Ct$, passes round it. By the action of the battery this quantity of electricity is raised in potential to the extent E: consequently a quantity of work, $W \doteq EQ$, is done against electric force, and therefore electric energy equal to W is generated.

It is the function of the battery to supply this energy, and it does so, as will be further explained in Chapter XV., at the expense of the chemical energy of the materials consumed in it. During the passage of the current, chemical action goes on in the battery, such that the products of the action possess less chemical energy than the materials from which they are formed, and the difference is available for the production of electrical energy.

In all cases energy must be continuously supplied to the circuit in order to maintain a current; but, according to the nature of the case, the source from which it is derived may vary. Thus, in a thermo-electric circuit, energy is supplied in the form of heat; in the case of the circuit of a dynamo-electric machine, it is mechanical energy that is supplied. From our present point of view, it may be said that the function of a galvanic battery of a thermo-electric pile, or of a dynamo-electric machine, is identical: they are all contrivances for effecting the transformation of other kinds of energy into the energy of an electric current, and the differences in their construction and in the details of their action depend on the kind of energy which they respectively transform.

125. Electric Work.—Turning now to the other aspect of the matter, we have to consider the electric circuit as giving out energy, or the electric current as doing work. The form in which energy is given out, or the nature of the work done, depends on the nature and conditions of the various parts of the circuit, and much will have to be said on these points in subsequent chapters, but certain general laws as to the amount of energy given out may be considered at this stage.

It follows from the simple consideration that the energy of the circuit does not on the whole either increase or decrease during the passage of a current, that the amount of energy given out by it in a given time must be the same as the amount received by it in the same time, though the form may be wholly or partially different. Another consideration which leads to the same conclusion is that the same quantity of electricity passes every section of the circuit in the same time; hence, while the potential of a quantity of electricity, Q, is raised by the amount E, by the action of the battery or other equivalent organ (thermopile, dynamo, &c.), and while, therefore, energy to the amount EQ is received by the circuit, the same quantity of electricity has its potential lowered in an equal degree, energy being thus given out by the circuit to the amount EQ.

126. Available Energy.—We have already seen (§§ 109, 111) that the difference of potentials, F, between the terminals of a battery is less than the electromotive E by the product of the resistance of the battery, b, into the strength of the current, or $F = E - bC$. Hence the fall of potentials which the current undergoes outside the battery is F, and the energy generated in the part of the circuit external to the battery and given out by it is FQ. This represents the whole of the energy of the battery which is available for doing external work, and it is less than the total energy of the circuit by $(E - F)\ Q = bC^2t$.

The difference between the total energy and the available energy of the circuit is analogous to that between the total work done by a steam-engine and that available for external purposes. A certain amount of the work done on the piston by the steam is consumed within the engine itself in processes essential to its working and in overcoming unavoidable friction, and it is only the remainder that can be made use of for driving external machinery or other purposes. In like manner, the external or available energy of the electric circuit is the excess of the total energy above that consumed in the battery.

The available energy given out per second is $FQ/t = FC$. This quantity is the *rate of doing work* by the circuit, and is comparable with the horse-power of a steam-engine. If the difference of potentials, F, is measured in volts, and the strength of the current, C, in amperes, the product, FC, is expressed in watts or volt-amperes, one watt being the rate of doing work represented by 10^7 ergs per second (§§ 39, 95).

By analogy with the horse-power of an engine, the rate at which work can be done by a battery may be called the *power* of the battery. It is a quantity which depends in part on the construction of the battery, but in part also on the conditions of its employment, just as the horse-power of an engine is not entirely determined by the characteristics of the engine itself, but also to some extent upon the conditions of working, such as the speed, steam-pressure, &c.

127. Joule's Law.—Returning to the case of a battery with its terminals connected by a wire, as supposed in § 124, it is found that the temperature begins to rise as soon as the circuit is completed, and if the electromotive force amounts to a few volts, and the total resistance to an ohm or less, the elevation of temperature may become very appreciable, the wire becoming red, or even white-hot, if it does not previously fuse or burn. As soon as the temperature of the wire has risen at all above that of surrounding objects, it begins to lose heat by conduction and radiation, and the temperature continues to rise until the rate at which heat is thus given out becomes equal to the rate at which it is generated in the wire by the current. In this case, heat is the only form of energy generated, and during the passage of a quantity of electricity, Q, the amount of heat, H, is such that

$$JH = EQ,$$

where J is the mechanical equivalent of heat. Assuming the strength of the current constant and applying Ohm's law, we may write this

$$JH = ECt = E^2t/R = C^2Rt,$$

where R is put for $b + r$ the total resistance of the circuit. Of the whole amount of heat produced, a part, say H_1, is generated within the battery, and the rest, H_2, in the connecting wire, the relative proportions being such that

$$H_1/H = b/(b + r) \quad \text{and} \quad H_2/H = r/(b + r).$$

The law of the generation of heat in the circuit, which we have here deduced from general considerations relating to electric energy, and have expressed above in three different but equivalent formulæ, was discovered experimentally by Joule about 1843, at a time when the true nature of electromotive force was very imperfectly recognised. It is consequently known as *Joule's Law.*

The subjoined diagram (Fig. 119) may be taken as a geometrical representation of Joule's law: MK represents the electromotive force of the battery, MN the battery resistance, NO the external resistance, and the ratio MK : MO the strength of the current. Then if we draw KP at right angles to KO, the length, MP, represents the energy expended in the circuit per second,—that is to say, the energy converted into heat per second in the whole circuit. To get the heat generated in the external conductor and in the battery respectively, we must draw NL perpendicular to MO, LQ perpendicular to KO, and LSR parallel to OP: then NQ represents the external energy ($= JH_2$), and SR the internal energy ($= JH_1$).

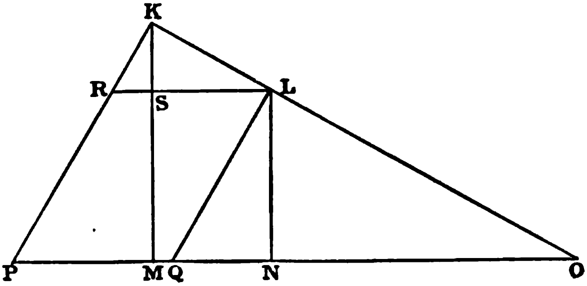

FIG. 119.

If the units for the electrical quantities in the above formulæ are the volt, the ampere, and the ohm respectively, the energy developed is expressed in joules (§§ 39, 112). In order to get it expressed in units of heat, gramme-degrees, we have to remember that 1 joule = 10^7 ergs, and that 1 gramme-degree = 4.18×10^7 ergs. Hence, the heat in gramme-degrees is given by dividing the number of joules by 4.18, or by multiplying it by 0.239. Thus, for the heat produced per second, we have

$$H = 0.239\, EC = 0.239\, E^2/R = 0.239\, C^2R.$$

These formulæ apply either to the heat developed in the complete circuit, or to that developed in any part of it, provided that appropriate values are assigned to the symbols.

The experimental verification of Joule's law is obtained by allowing a measured current to pass for a known time through a wire of known resistance, r, immersed in a calorimeter, and measuring the resulting quantity of heat, h, which, when the experiment is properly performed, is found to satisfy the equation

$$h = 0.239\, C^2rt.$$

Taking the law itself as established, the same kind of experiment as that just indicated may be employed for determining r, J, or C, when in each case the other quantities are known.

128. Heating of Wires.—If a chain be formed of bits of metal wire—platinum, for instance—alternately thick and thin, and a current be passed through, the thin wires will become red hot, while the stout wires are scarcely heated. In like manner with a chain formed of alternate pieces of platinum and silver wire of equal diameter, the former become red hot while the latter are still dark.

Again, if we take a long thin platinum or iron wire through which a current is passing, and we gradually diminish the length of the wire interposed in the circuit, it will be seen that this becomes more and more heated until it fuses. The strength of the current increases as the length of the wire diminishes. In like manner, if we take a long piece of iron wire and regulate the strength of the current so that the wire is at a dull red heat, and then cool a part of the wire by dipping it in water, the remainder becomes brightly incandescent. By cooling the wire its resistance is lessened (§ 113), and therefore the strength of the current is increased; the effect is the same as if part of the wire had been taken out of the circuit.

If there were no loss of heat by radiation, the temperature of a conductor traversed by a continuous current would go on indefinitely increasing. The temperature which it reaches is that for which the gain at each instant is equal to the loss. We may admit Newton's law of cooling as at any rate approximately applicable to this case,—that is to say, the loss by radiation at each instant is proportional to the excess of temperature of the wire over the surrounding medium,—then the quantity of heat lost in a second by a wire of length l, of radius r, of emissive power ϵ, for the excess of temperature θ, will be $\epsilon . 2\pi r l . \theta$; and if ρ is the specific resistance of the metal, and C the strength of the current, we have the equation

$$J\epsilon 2\pi r l\theta = \frac{l\rho}{\pi r^2} C^2$$

from which we get

$$\theta = \frac{1}{2\pi^2 J} \frac{\rho}{\epsilon r^3} C^2.$$

This formula shows that for a given metal with the same conditions of surface, the heating remains the same if the diameter of the wire varies as the two-thirds power of the strength of the current. If further, as shown by experiment, ϵ and ρ vary with the temperature in such a manner that their ratio remains sensibly constant, and θ and θ_0 are the temperatures which for a given wire correspond to the currents C and C_0, we have

$$C = C_0 \sqrt{\frac{\theta}{\theta_0}}.$$

The conditions vary, however, according as the wire is bare or is covered with an insulator, and according as it is stretched out or coiled on a reel. With copper it is in practice assumed that the current must not exceed six amperes for each square millimetre of cross-section if the wire is bare, and two or three if it is covered.

129. Generalisation of Joule's Law.—We have already stated (§ 125) that under proper conditions the energy of the electric circuit can be employed, not only in generating heat, but in doing work of various other kinds. Without inquiring what kinds of work are done by the current between any two given points of the circuit, we may say that if C is the strength of the current in amperes, and e the difference of potentials between the points in volts, the total work done by the current in one second in this part of the circuit is

$$W = eC \text{ joules.}$$

Suppose, for example, a circuit made up as in Fig. 120, where the current of the battery, B, is sent through some piece of apparatus enclosed in a sealed box, A, entering by the binding-screw, p, and leaving at q. We can measure the difference of potentials between p and q, and the strength of the current, C, and thus determine eC. Without its being needful to open the box, or to know anything about the apparatus contained in it, or the effects that the current produces within it, this gives the algebraic sum of all the work done per second within the box. If r is the resistance of the part of the circuit within the box, there will be a generation of heat, h, per second, such that

$$Jh = C^2r.$$

But there may be work of other kinds as well. Let w_1, w_2, . . . represent for one second the amounts of work of various kinds, other than generation of heat; then

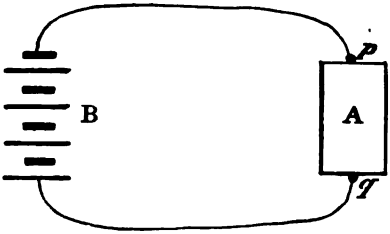

FIG. 120.

$$\begin{aligned} eC &= w_1 + w_2 + \ldots + C^2r, \\ &= w + C^2r, \end{aligned}$$

if, for shortness, we put w for $w_1 + w_2 + \ldots$, the sum of the work of all kinds except production of heat.

If $w = 0$, the whole of the energy expended between p and q in a second is transformed into heat, and the quantity of this energy may be represented indifferently by C^2r or by eC. This is Joule's law in the special sense. But we may generalise the law so as to make it express the rate of expenditure of energy independently of the form or forms which that energy assumes. In words, the more general form of the law may be stated thus: the rate of expenditure of energy in any part of an electric circuit is equal to the strength of the current multiplied by the amount by which the potential at the point where the current enters exceeds that at the point where it leaves. In symbols we may write it—

$$W = eC = w + C^2r.$$

130. Inverse Electromotive Force.—Suppose the resistance of the complete circuit to be $R = r' + r$ ohms, r' being the resistance of the battery and of everything not included in A, and let the electromotive force of the battery be E, then we have the equation

$$EC = w + C^2(r' + r) = w + C^2R$$

or

$$E = w/C + CR.$$

Now, by Ohm's law, CR must be equal to the algebraic sum of all the electromotive forces of the circuit: let this be denoted by E'. Then

$$E' = E - w/C.$$

That is, when the energy of a current is not wholly expended in generating heat, in order to obtain the effective electromotive force of the circuit, we must *deduct* from the electromotive force of the battery the quotient obtained by dividing the sum of the work of other kinds done per second by the strength of the current. Hence, we conclude that the doing of work by a current, other than the generation of heat, involves the existence of a *negative* electromotive force

$$\epsilon = w/C.$$

131. Strength of the Current—Maximum Rate of doing Work.—Seeing that the battery, in the case now under consideration, is not the only seat of electromotive force, we cannot get the strength of the current by considering it alone: we require to take also into account the rate at which work is done. The equation used in the last paragraph—

$$EC = w + C^2R$$

gives for the strength of the current

$$C = \frac{E}{2R} \pm \sqrt{\frac{E^2}{4R^2} - \frac{w}{R}};$$

or

$$C = \tfrac{1}{2}C_0 \pm \sqrt{\frac{\tfrac{1}{4}C_0E - w}{R}}$$

if we put C_0 for the strength of current E/R that the given battery would maintain in the circuit if all the energy were transformed into heat. This equation indicates two strengths of current for any given value of w: for example, if $w = 0$, we may have $C = 0$ or $C = C_0$. It is represented geometrically by the annexed diagram (Fig. 121), in which distances measured to the right denote values of w, and distances measured upwards denote values of C. The electromotive force of the battery is taken as 4 volts, and the resistance of the circuit as 0.04 ohm, so that $C_0 = 100$ amperes. The diagram indicates the double value of the current for a given value of w; for example, $C = 20$ and $C = 80$, both correspond with $w = 64$. Again, the diagram shows very clearly that, however great the current may become, there is a maximum rate of work that cannot be exceeded. This corresponds with $C = \frac{1}{2}C_0$, and,

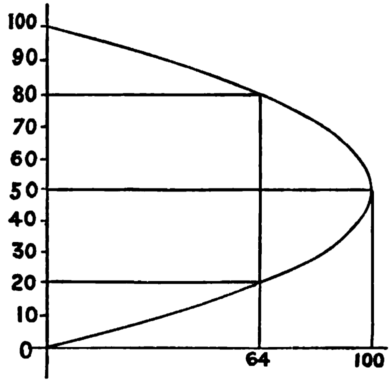

FIG. 121.

with the values used in the diagram, is equal to 100. The same thing appears from the algebraical formula, in which the quantity under the sign of the square root would assume an impossible value if w were greater than $\frac{1}{4}C_0E$. With this limiting value of w, the quantity under the sign of the square root vanishes, and we get $C = \frac{1}{2}C_0$, as from the diagram.

132. Efficiency.—When a battery of electromotive force, E, maintains a current of strength, C, it gives out energy at the rate EC, of which the amount C^2R per second is converted into heat, while the remainder, $EC - C^2R$, is available for doing work of other kinds. When a battery is employed, say, for driving an electromagnetic motor, or for producing chemical decomposition, the heat generated represents a loss of energy so far as the intended purpose is concerned, and it is only the remainder that is actually used. The ratio of the energy converted into useful work in a given time to the total energy given out by the battery in the same time, or

$$\frac{EC - C^2R}{EC} = \frac{C_0 - C}{C_0} = u$$

is called the *efficiency* of the circuit. Since C_0 is a constant quantity for the given circuit, it is evident that the value of u increases as C diminishes, and that it attains its maximum value $= 1$ when $C = 0$, and its minimum value $= 0$ when $C = C_0$. Efficiency $= 1$, or perfect efficiency, represents the conversion of all the energy given out by the battery into useful work; the other extreme case, $u = 0$, is when the whole of the energy of the battery is converted into heat. The actual rate of doing useful work is equal to the rate at which energy is given out by the battery multiplied by the efficiency, or

$$w = u \,.\, EC = (C_0 - C)\,CR.$$

This equation shows that in the extreme case of perfect efficiency ($C = 0$) the actual amount of work done would vanish, for no energy would be given out by the battery. It also again shows that the maximum rate of doing work is when the inverse electromotive force spoken of in § 130 is equal to $\frac{1}{2}E$ and therefore sufficient to reduce the strength of the battery current to $\frac{1}{2}C_0$; for the sum of the two factors $C_0 - C$ and C in the expression for w is constant, and therefore the product is greatest when the factors are equal, or $C = \frac{1}{2}C_0$. In this case $u = 0.5$, and

$$w = 0.25\,EC_0 = 0.25\,E^2/R.$$

That is, the maximum rate at which useful work can be done is equal to one quarter the rate at which energy would be given out by the battery if it were all converted into heat.

From the diagram of § 131 it is easily seen that, when the rate of work is 64 per cent. of the maximum, the efficiency may be 0.8 or 0.2. The greater value corresponds with the lower half of the curve, which represents the conditions that would always be employed in practice.

133. Energy of a Cell.—Suppose a single cell of electromotive force e and resistance ρ, and let the external resistance be negligible, then the maximum rate of work obtainable from the cell is

$$\frac{e^2}{4\rho},$$

which is a constant quantity characteristic of the cell. The greatest value of e is about two volts in cells actually used. It approaches this value in a Grove's or Bunsen's cell, and slightly exceeds it in secondary cells with lead plates. The value of ρ may vary greatly, according to the size and arrangement of the plates, but it is most easily made small in cells of the kinds for which e is great; hence the practical advantage of such cells.

Next suppose n similar cells arranged in q series of p cells each. The electromotive force of the combination will be $E = pe$, and the resistance $R = \frac{p}{q}\rho$; and, if we again suppose the external resistance to be negligible, the maximum attainable rate of work is

$$\frac{E^2}{4R} = \frac{p^2qe^2}{4p\rho} = n\frac{e^2}{4\rho};$$

that is to say, that the maximum rate of work is proportional to the total number of cells employed, but does not depend upon the manner in which they are combined.

CHAPTER XIV.

THERMO-ELECTRICITY.

134. Peltier's Phenomenon.—The generation of heat depending on resistance being (§ 127) proportional to C^2R, is the same whether the current is positive or negative. It is in this sense like the generation of heat by ordinary mechanical friction, which is independent of the direction of the motion, and it may consequently be spoken of as the frictional generation of heat. But it is evident that if there is anywhere a fixed difference of potentials between adjacent parts of a conducting circuit, existing independently of the passage of a current, as, for example, an electromotive force acting at the surface of contact of two different metals, this must give rise to a further production of heat if the current passes from higher to lower potential, and to a disappearance of heat if the current flows in the opposite direction. The sign of any such effect would then be *reversible* with the direction of the current, and the quantity of heat produced in the one case or destroyed in the other, in a given time, would be proportional to the strength of the current.

Conversely, a reversible thermal effect occurring at any part of a circuit must be attributed to the existence at this part of a difference of potentials existing independently of the current. If at a part of the circuit there is a *fall* of potential Π as we pass along the circuit in the direction of the current, the reversible quantity of heat H generated by a current of strength C flowing for t seconds will be given by

$$JH = C\Pi t.$$

Now, an effect of this kind taking place at the surface of contact of two metals was discovered by Peltier in 1834, and is consequently known as the *Peltier effect.* If a current passes from antimony to bismuth, heat is produced at the surface of contact; and if a current passes from bismuth to antimony, heat is destroyed: similarly in the case of any other two metals, heat is generated if a current traverses the surface of contact in one direction, and is consumed if it passes in the opposite direction, the quantity in a given time being in each case proportional to the strength of the current and to a coefficient (Π in the last equation) depending on the nature of the metals and on their temperature. This coefficient, known as the *Peltier coefficient*, or the coefficient of the Peltier effect, represents for a junction of any two metals the amount of energy converted into or produced from heat per second when a current of unit strength traverses the junction.

Experiments on the Peltier effect present some difficulties, from the fact that they are always complicated by the frictional generation of heat. It is not possible to observe the thermal effects occurring at the mere surface of contact of two metals: in any actual experiment we are obliged to include a finite portion of the circuit, extending from a point, say M, on one side of the junction to a point, say N, on the other side, and to observe the total amount of heat generated in a given time within this part of the circuit. If r is the resistance between the points M and N, the energy converted into frictional heat per unit of time is C^2r, and the energy converted into reversible heat is $C\Pi$. Hence, if H is the quantity of heat produced in t seconds when the current flows in one direction, we have

$$JH = (C\Pi + C^2r)t;$$

and, denoting the heat generated when the current flows in the opposite direction by H', we have

$$JH' = (-C\Pi + C^2r)t.$$

In order to determine the value of the Peltier coefficient for a given case, we may allow a current of constant strength to pass first in one direction, and then, for the same length of time, in the opposite direction: taking the difference of the two quantities of heat produced, we get in this way

$$\Pi = \frac{J(H - H')}{2Ct}.$$

To get good results, the frictional or non-reversible term C^2r should be made as small as practicable by making MN, the portion of the circuit over which the experiment extends, short, and therefore r small, and by using a small current.

If an iron wire, AB, be connected by copper wires with the terminals of a battery, on completing the circuit a current will flow from copper to iron at one end, say A, and from iron to copper at the other end. In this case heat will disappear at A, and will be generated at B; and if the current is sufficiently weak, there may be an actual fall of temperature at A, while the temperature at B rises. This effect may be shown in a striking form by surrounding the joint A with ice-cold water, and placing the joint B in ice. As the current passes, ice is formed at A, and an equal quantity is melted at B.

In a case like that just described, where two opposite junctions have the same temperature, the Peltier coefficient has the same value for both, but has opposite signs; hence the Peltier effects at the two junctions exactly compensate each other, and the quantity of heat generated in the circuit is that which a current of the same strength would produce in a homogeneous conductor of the same resistance.

A similar compensation occurs in a circuit containing any number of metallic junctions all at the same temperature; the sum of the reversible thermal effects vanishes, and Joule's law expresses the total generation of heat.

135. Thomson Effect.—Professor Sir William Thomson (Lord Kelvin) discovered in 1856 that a phenomenon analogous to the Peltier effect occurs in a conductor of one and the same material when a current passes from a hotter to a colder part, or *vice versâ*, whence we must conclude that parts at different temperatures are not at the same potential, even when there is electrical equilibrium. Thus, for instance, if the ends A and B of a copper bar (Fig. 122) are kept at 0°, and an intermediate point, C, is raised to some higher temperature, *T*, the potential increases continuously from A to C, and decreases by the same amount from C to B. Consequently, when electricity flows through the wire from A to B, there is an increase of electrical energy as it goes from A to C, and a corresponding quantity of heat is used up, while from C to B there is, to an equal extent, a decrease of electrical energy and a generation of heat. The final result thus shows itself as a transfer of heat from the part A C to the part C B—that is to say, in the direction of the current.

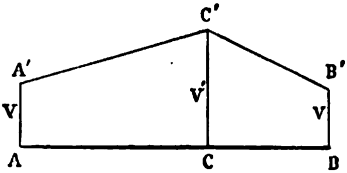

FIG. 122.

In iron, on the other hand, the effect is reversed: the potential decreases as temperature rises, so that there is generation of heat between A and C (Fig. 123), and disappearance of an equal quantity between C and B, a result which is equivalent to a transfer of heat in the opposite direction to the current.

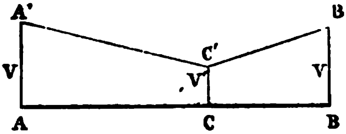

FIG. 123.

To show these effects, the ends A and B of the bar to be experimented upon may be kept at 0°, and the middle C at a higher temperature, such as 100°; then, if everything is symmetrical, the temperature at a point, M, between A and C will be the same as at M′, equidistant from C on the other side. On sending a strong current through the bar in the direction AB, the temperature will be raised throughout by the frictional development of heat, but, as the result of the effect we are considering, the temperature at M′ will be raised more than that at M in the case of copper, silver, zinc, cadmium, or antimony; while with iron, platinum, or bismuth, the temperature at M will become higher than that at M′. In metals of the former class, the Thomson-effect is said to be *positive;* in those of the latter class the Thomson-effect is *negative;* in lead it is not perceptible.

136. Thermo-Electric Currents.—In a circuit formed of different metals, if all the points where two different metals join are at the same temperature, the differences of potential to which both the Peltier- and Thomson-effects are due compensate each other all round the circuit, and there is no current. But if the joints are not all at the same temperature, this compensation no longer exists, and the circuit is traversed by a current. The production of currents under such conditions, *thermo-electric currents*, was discovered by Seebeck in 1821, therefore a good while before either of the allied phenomena that we have been discussing were known.

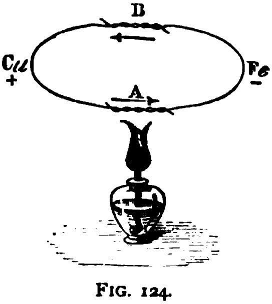

FIG. 124.

Suppose a circuit formed by twisting or soldering together at their ends wires of two different metals, as, for example, iron and copper (Fig. 124): then, if one junction, B, remaining at the ordinary temperature, the other, A, is raised to a higher temperature, a current is produced which goes from copper to iron at the heated junction, and from iron to copper at the cold one.

The metals named in the following list are arranged in such an order that if any two of them are formed into a circuit, as above described, there is a current across the heated junction from the metal whose name occurs first in the list to that whose name occurs later:

Hot. ↓

Bismuth.
Nickel.
Platinum.
Palladium.
Cobalt.
Manganese.
Silver.
Tin.
Lead.
Copper.
Gold.
Zinc.
Iron.
Arsenic.
Antimony.

↑ Cold.

137. Thermo-Electric Inversion.—The direction of the current is in accordance with the rule just given, only so long as the average temperature of the two junctions is not above or below some definite value, depending on the two metals of which the circuit is formed. Thus with iron and copper, this temperature is about 275°. If the average temperature of the junctions is above this point, or, what comes to the same thing, if the sum of the two temperatures is more than 550°, the current is from iron to copper at the hotter junction, and from copper to iron at the cooler one. If the sum of the two temperatures is equal to 550°, there is no current in the circuit. Supposing one junction kept permanently at 0°, and the temperature of the other to be gradually raised; the circuit is traversed by a current which increases in strength, at first comparatively quickly, and afterwards more slowly, till the hotter junction reaches 275°, then decreases, slowly at first, and afterwards more rapidly, till the hot junction is at 550°, when it ceases. If the temperature is raised still higher, a current reappears, but in the opposite direction. If the junction whose temperature remains constant is kept at 100° instead of at 0°, the maximum strength of current is again reached when the other junction is at 275°, but the temperature at which inversion of the current takes place is 450° instead of 550°.

An actual difference of material is not essential to the production of a thermo-electric current, a difference of physical condition, such as can be brought about by mechanical treatment, being sufficient.

For example, if the central portion of a platinum wire is wound into a rather close spiral and the ends of the wire are connected with a sensitive galvanometer, when one end of the spiral is heated (Fig. 125), the galvanometer indicates a current from the twisted to the untwisted part of the wire through the heated part.

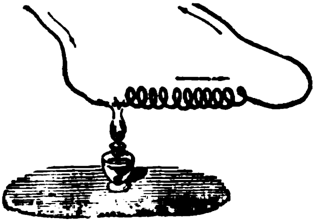

FIG. 125.

Mere dissymmetry of distribution of temperature on opposite sides of a part at which heat is applied does not produce a current if the material on both sides be chemically and physically identical. If the central part, *g g*, of a thick copper-wire (Fig. 126) is turned down to a much smaller diameter than the rest, no current is produced when the wire is made part of a closed circuit, and heat is applied at one of the points, *g*.

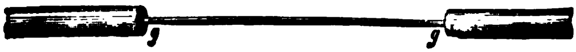

FIG. 126.

138. Law of Successive Temperatures.—*For a given couple, the electromotive force obtained by raising the junctions to the temperatures* t_1 *and* t_2 *is the sum of the electromotive forces which are obtained by raising the junctions to the temperatures* t_1 *and* θ, *and then to the temperatures* θ *and* t_2, θ *being a temperature intermediate between* t_1 *and* t_2.

This law may be expressed in the following manner :—

$$E_{t_1}^{t_2} = E_{t_1}^{\theta} + E_{\theta}^{t_2}.$$

139. Law of Intermediate Metals.—*In a thermo-electric circuit composed of two metals, A and B, with junctions at temperatures* t_1 *and* t_2 *respectively, the electromotive force is not altered if one or both the junctions are opened and one or more other metals are interposed between the metals A and B, provided that all the junctions by which the single junction at temperature* t_1 *may be replaced are kept at* t_1, *and all those by which the junction at temperature* t_2 *may be replaced are kept at* t_2.

If X is the intermediate metal, the law is expressed by the equation

$$E_{t_1}^{t_2}(AB) = E_{t_1}^{t_2}(AX) + E_{t_1}^{t_2}(XB).$$

Several important consequences result from this law.

In a thermo-electric couple it is immaterial whether the two metals are joined directly or are soldered together. If the circuit is cut at any point, and the two ends are connected with the binding screws of an electrometer or of a galvanometer, provided that the binding screws and the various parts in contact are all at the same temperature as the two ends of the circuit, no fresh electromotive force is introduced, and the electromotive forces measured are those of the original circuit.

140. Thermo-Electric Pile or Battery.—Thermo-electric forces are always very feeble, that of the bismuth-antimony pair, which is one of the strongest, and between 0° and 100° is nearly proportional to the difference of temperature of the two junctions, is 0.0000057 volt for each degree. As, however, the resistance of couples wholly metallic may be made extremely small, such couples can give currents of considerable strength. Thus the bismuth-copper couple represented in Fig. 127, when heated at the junction B by a spirit lamp, strongly deflects a magnetic needle placed inside the circuit.

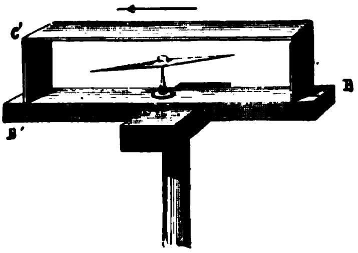

FIG. 127.

Any number of such couples may be associated together in such a way as to form a pile or battery. Melloni's pile, which has been of great service in the investigation of radiant heat, is formed of small bars of antimony, *aa*, arranged alternately with bars of bismuth, *bb*, so that all the even-numbered joints are on one side, and the odd ones on the other (Fig. 128). Several such piles are connected together, the couples being insulated by mica or asbestos, which is not altered by heat; they thus form a rectangular parallelopipedon (Fig. 129), the faces C and D being those which correspond to the junctions. These faces are covered with lamp-black, and protected by the screens T and T'. The two poles being connected with a galvanometer, a very slight difference of temperature between the two faces is sufficient to produce an appreciable deflection.

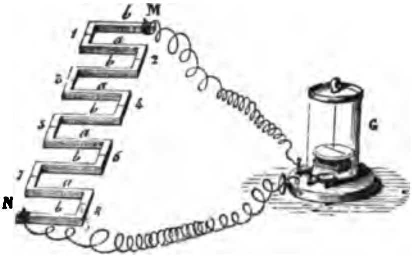

FIG. 128.

Clamond's battery, which is occasionally used in laboratories, is made up of an alloy of zinc and antimony, alternating with an alloy of zinc, nickel, and copper. The metals are arranged in series so as to form a sort of cylinder (Fig. 130); the even-numbered junctions are inside and are heated by a gas-flame, while the odd-numbered junctions are on the outside, and are arranged so that they present a large cooling surface to the air.

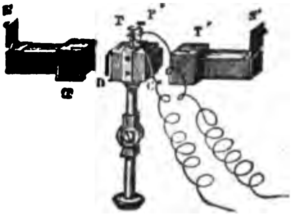

FIG. 129.

We may cite as an example a Clamond battery consisting of 120 couples, having an electromotive force of 8 volts and a resistance of 3.2 ohms, which, therefore, at the maximum, has a power of 5 watts. The couples may be grouped so as to satisfy in each case the conditions of the maximum (§ 132). To produce this current 180 litres of coal-gas are consumed per hour.

Assuming that a litre of gas gives 5200 gramme-degrees of heat, the heat expended is 260 gramme-degrees per second, which corresponds to a power of 1084 watts. The efficiency is about $\frac{1}{200}$.

141. Theory of Thermo-Electrical Phenomena.—The existence of thermo-electric currents shows clearly that the resultant electromotive force in a metallic circuit is a function of the temperature. In order to investigate the matter, let us again take the case of a circuit formed by joining at the ends a piece of iron wire and a piece of copper wire. It is known by experiment (§ 134) that heat disappears when an electric current traverses the junction of the two metals in the direction from copper to iron, and that heat is produced by a current traversing the junction from iron to copper. Hence we infer that, at ordinary temperatures, the potential of iron is higher than that of copper to an extent measured by the value of the Peltier coefficient for the two metals, that is to say, by the energy required to transfer a unit of electricity from copper to iron. It is further known (§ 136) that if, one of the junctions remaining at the temperature of the air, the other is gradually heated (Fig. 124), the circuit is traversed by a current whose direction is from iron to copper at the cold junction, and from copper to iron at the heated one. The direction of the current may also be stated as being from hot to cold in the iron wire, and from cold to hot in the copper.

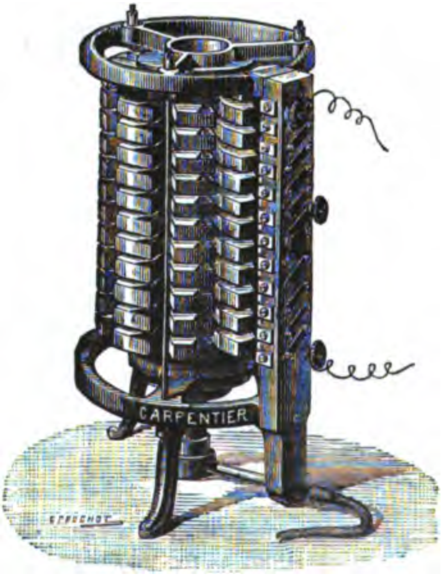

FIG. 130.

In seeking for the origin of the electromotive force in a thermo-electric circuit, we must bear in mind (§ 125) that the electromotive force is equal numerically to the work done during the passage of a unit quantity of electricity round the circuit, and that this amount of work must be equal to the energy simultaneously taken in by the circuit. A simple thermo-electric circuit consists of two homogeneous wires and two junctions, and any energy taken in by it must be taken in at one or more of these four parts.

In the case we have supposed, the direction of the current, being that of increasing temperature in the copper and that of decreasing temperature in the iron, is from lower to higher potential in both (§ 135), and therefore there is a *gain* of energy in both wires as the current passes. Energy is also *gained* at the heated joint, where the current flows from copper to iron, that is, from lower to higher potential, but there is *loss* of energy at the other joint, where the direction of the current is from iron to copper. The net gain is therefore the amount by which the sum of the energy taken in, in the continuous wires and the hot junction taken together, exceeds that given out simultaneously at the cold junction, and the electromotive force is numerically equal to this net gain of energy per unit of electricity traversing the circuit.

In order to express this relation by means of an equation, we may use the symbol σ as the coefficient of the Thomson effect; that is to say, when a current of strength C flows for t seconds in a given metal, from a part where the temperature is T to a part where the temperature is higher by the very small amount dT, the energy taken in is equal to the quantity of electricity Ct multiplied by the small difference of temperature dT and by the coefficient σ depending on the nature of the metal, or

$$Ct\,\sigma\,dT.$$

If T_1 be the temperature of the colder, and T_2 that of the hotter junction, we may suppose an infinite number of points taken one after another along each wire, so that the temperature at each point exceeds that at the preceding point by dT. Then the energy gained by the flow of a current along each elementary length of wire intervening between two consecutive points will be represented by $Ct\,\sigma\,dT$, if the direction of the current is the direction of increasing temperature (or dT positive), and by $-Ct\,\sigma\,dT$, if the current flows in the direction of decreasing temperature (dT negative). The total gain of energy due to the whole wire will be expressed by the sum of all such terms from one end to the other.

The factor Ct is common to all, but the value of σ for a given metal depends in general on the temperature; hence we cannot calculate the required sum by adding together all the intervals of temperature dT, so as to get the total difference $T_2 - T_1$, and multiplying this by $Ct\sigma$, but we must, before adding, multiply the temperature-difference dT corresponding to each successive element of the wire by the value of σ applicable to it, that is, we must take the integral $\int_{T_1}^{T_2} \sigma dT$, and multiply this by Ct.

Distinguishing the values of σ corresponding to copper and iron respectively as σ_{Cu} and σ_{Fe}, and remembering that the direction of the current is from cold to hot in the copper, and from hot to cold in the iron, we have, for the total gain of energy due to the continuous wires,

$$Ct\left(\int_{T_1}^{T_2} \sigma_{Cu} dT - \int_{T_1}^{T_2} \sigma_{Fe} dT\right).$$

We have next to consider the effect of the junctions. We have already (§ 134) seen that the coefficient of the Peltier effect for the junction of two metals measures their permanent difference of potential. Putting Π_1 for the value of this coefficient in the case of iron and copper at the temperature T_1 of the cooler junction, and Π_2 for the value of the same coefficient at the temperature T_2 of the hot junction, the gain of energy at the latter may be written $Ct\Pi_2$, and the loss at the former $Ct\Pi_1$. Hence the net gain of energy at the two junctions taken together is

$$Ct(\Pi_2 - \Pi_1).$$

If we denote the electromotive force of the circuit by E, the work done by the current in time t, or the energy given out, is ECt. Writing this as equal to the whole amount of energy taken in, we get the equation,

$$ECt = Ct\int_{T_1}^{T_2} \sigma_{Cu} dt - Ct\int_{T_1}^{T_2} \sigma_{Fe} dT + Ct\Pi_2 - Ct\Pi_1.$$

The quantity of electricity traversing the circuit, $Ct = Q$, is a factor of every term of this equation. If we divide throughout by this, or, what comes to the same thing, if we suppose the quantity of electricity to be unity, we get, as the expression for the electromotive force,

$$E = \int_{T_1}^{T_2} (\sigma_{Cu} - \sigma_{Fe}) dT + \Pi_2 - \Pi_1.$$

142. Application of Thermo-Dynamics.—The only form in which energy is supplied to a thermo-electric circuit is that of heat, but the work done by it may, in part at least, take any form we please. For example, by including in the circuit an electro-magnetic motor, a greater or less amount of mechanical energy may be obtained. If, therefore, we leave out of account the intermediate processes and attend only to final results, we may say that a thermo-electric circuit employed to maintain the motion of an electro-magnetic motor is equivalent to a steam-engine, or any similar contrivance, the function of which is to convert heat into mechanical energy.

It is thus natural to inquire how far the general conditions which have been shown to apply to the transformation of heat into mechanical energy in the steam-engine, or other forms of heat-engine, are applicable to the corresponding transformation by means of a thermo-electric circuit. The most important of these principles is that, in the case of a reversible engine,—that is to say, an engine such that an inversion of the direction of the motion is accompanied by an exact reversal of the dynamic and thermal processes going on in it,—the sum of the quantities of heat taken in by the engine, each of them divided by the temperature, reckoned from absolute zero, at which it is taken in, is equal to the sum of the quantities of heat given out, each of them divided by the temperature at which it is given out. The application of this conclusion to the case of a thermo-electric circuit leads to very important consequences.

But before this application can be made, certain conditions must be discussed. A thermo-electric circuit cannot in general be regarded as reversible. Part of the energy of the circuit, namely, the amount C^2R per second, R being the resistance of the circuit, is converted into heat in a non-reversible manner, that is to say, the production of heat is independent of the direction of the current and the application of heat to the circuit cannot be made to reproduce the equivalent quantity of electrical energy. The evolution and absorption of heat in consequence of the Peltier and Thomson effects are, however, reversible; for, being each of them proportional to the strength of the current, they are reversed when the current is reversed. The non-reversible evolution of heat being thus proportional to the *square* of the current-strength, and the reversible effects being proportional to the *first power*, it follows that, if the strength of the current is sufficiently small, the former will be very small as compared with the latter, and may, without sensible error, be neglected. To fulfil this condition, we have only to suppose that the temperatures T_1 and T_2 are very nearly the same: let them be T and $T + dT$ respectively, differing from each other by the indefinitely small amount dT. In this case the electromotive force, and therefore the strength of the current, will be very small.[1]

Going back to the equation at the end of § 141, and assuming the differences of temperatures to be very small, we may write it

$$dE = (\sigma_{Cu} - \sigma_{Fe})dT + d\Pi.$$

Where $d\Pi$ stands for the increment of the Peltier coefficient corresponding to an increment of temperature dT. Under the conditions here supposed, the processes taking place in the circuit are all reversible; we may therefore apply to them the thermo-dynamic conclusion referred to above.

The heat taken in in the continuous parts of the circuit divided by the temperature at which it is received, gives $(\sigma_{Cu} - \sigma_{Fe})\frac{dT}{JT}$ per unit of electricity, if we put J for the mechanical equivalent of heat. To this is to be added the heat absorbed at the hotter junction divided by the temperature of the junction; this may be written $\frac{\Pi + d\Pi}{J(T + dT)}$. The only part of the circuit at which heat is given out is the cooler junction. Dividing the heat here evolved by the temperature of the junction, we get $\frac{\Pi}{JT}$. Then, equating the values applicable to the heat taken in and that corresponding to the heat given out, we have

$$\frac{(\sigma_{Cu} - \sigma_{Fe})dT}{T} + \frac{\Pi + d\Pi}{T + dT} = \frac{\Pi}{T},$$

---

[1] Without making the difference of temperatures infinitely small, the current might be made as small as we please by a sufficient increase of resistance. But by writing EC instead of C^2R to express the non-reversible production of heat, we see that, so long as E is finite, this is a small quantity of the first, not of the second, order.

whence, by reduction,

$$(\sigma_{Cu} - \sigma_{Fe})\,dT = \frac{\Pi}{T}dT - d\Pi.$$

Putting this value on the right-hand side of the equation for dE above, we get

$$dE = \frac{\Pi}{T}dT.$$

If the junctions have the temperatures T_1 and T_2 respectively, the electromotive force of the circuit is represented by

$$E = \int_{T_1}^{T_2} \frac{\Pi}{T}dT.$$

143. Values of the Peltier and Thomson Coefficients.—It has been pointed out already (§ 134) that the direct experimental measurement of the coefficient of the Peltier effect presents a good deal of difficulty, but the last equation but one of § 142 contains the principle of another method of measuring this quantity. Writing this equation in the form

$$\Pi = T\frac{dE}{dT}$$

we get the Peltier coefficient expressed in terms of the rate of change of the electromotive force of the circuit with temperature.

dE in this formula stands for the infinitesimal electromotive force of a circuit whose junctions are at the absolute temperatures T and $T + dT$ respectively; but, by the law of successive temperatures (§ 138), this is the same as the increment of electromotive force in a circuit with one junction kept at a fixed temperature when the other is raised from temperature T to $T + dT$.

Now it is found by experiment that the connection between electromotive force and temperature, for a circuit formed of a given pair of metals, can be expressed by the formula

$$E = a(T_2 - T_1) + b(T_2^2 - T_1^2),$$

where a and b are constant coefficients whose values depend on the particular metals employed. They can be determined experimentally in each case by measuring the electromotive forces corresponding to various values of the temperatures T_1 and T_2.

If, in this formula, T_1 stands for the temperature of one junction which is kept constant, while T_2 varies, we get, by differentiating with respect to T_2,

$$\frac{dE}{dT} = a + 2bT_2,$$

and therefore, for the value of Π at temperature T_2,

$$\Pi = T_2 \frac{dE}{dT} = aT_2 + 2bT_2^2.$$

For copper and iron,

$$a = 3187, \text{ and } b = -2.91.$$

With these numbers the value of Π can be calculated for any given temperature: the following table gives the results for a few temperatures:—

Temperature from absolute zero .	$= T$	273°	373°	473°	547.6°	573°
Temperature centigrade	$= t$	0	100	200	274.6	300
Π (ergs per unit of electricity)		436,000	379,000	205,000	0	−85,000

The value of $\sigma_{Cu} - \sigma_{Fe}$ can also be expressed by means of the same coefficients. Referring to § 142, we see that

$$\sigma_{Cu} - \sigma_{Fe} = \frac{\Pi}{T} - \frac{d\Pi}{dT}.$$

Consequently,

$$\frac{\sigma_{Cu} - \sigma_{Fe}}{T} = -\left(\frac{1}{T}\frac{d\Pi}{dT} - \frac{\Pi}{T^2}\right) = -\frac{d\left(\frac{\Pi}{T}\right)}{dT}.$$

But

$$\frac{\Pi}{T} = \frac{dE}{dT},$$

therefore,

$$\sigma_{Cu} - \sigma_{Fe} = -T\frac{d\frac{dE}{dT}}{dT} = -T\frac{d^2E}{dT^2}.$$

Writing, as before, the electromotive force in the form

$$E = a(T - T_1) + b(T^2 - T_1^2),$$

where T_1 is the fixed temperature of the cooler junction, we have

$$\frac{dE}{dT} = a + 2bT$$

and

$$\frac{d^2E}{dT^2} = 2b.$$

Therefore, finally,

$$\sigma_{Cu} - \sigma_{Fe} = -2bT.$$

This gives at 0° C. ($T = 273$)

$$\sigma_{Cu} - \sigma_{Fe} = 1589,$$

and at 300° ($T = 573$)

$$\sigma_{Cu} - \sigma_{Fe} = 3335.$$

In the same way the difference between the values of the coefficients of the Thomson effect can be calculated for any two metals when the electromotive force of a thermo-electric circuit formed of these metals has been determined for various intervals of temperature.

It is found that the passage of an electric current through lead, between parts at different temperatures, is not accompanied by a reversible production or destruction of heat to an extent capable of being detected by experiment. Hence the coefficient σ for lead must be either accurately or nearly $= 0$.

Suppose, then, that a thermo-electric circuit is made up of lead and another metal, say iron, and that the electromotive force of the circuit has been measured for various ranges of temperature. The results enable us to determine the values of the coefficients a and b, applicable to this circuit, in the equation—

$$E = a(T - T_1) + b(T^2 - T_1^2),$$

and therefore to calculate

$$\sigma_{Pb} - \sigma_{Fe} = -2bT,$$

or, putting

$$\sigma_{Pb} = 0,$$

$$\sigma_{Fe} = 2bT.$$

In this way the value of σ_{Fe} has been found to be

at 0° C.	−1330
at 300° C.	−2791

Combining these values with those given above for $\sigma_{Fe} - \sigma_{Cu}$, we get for σ_{Cu},

at 0° C. . . . +259
at 300° C. . . . 544.

If we distinguish by suffixes the values of the coefficients a and b, in the case of circuits made up of three given metals, 1, 2, and 3, when 1 and 2, 2 and 3, or 1 and 3 are taken together, the law of intermediate metals (§ 139) gives the following relations—

$$a_{1,3} = a_{1,2} + a_{2,3}$$
$$b_{1,3} = b_{1,2} + b_{2,3},$$

by which, when the values applicable to two of the pairs have been determined, that corresponding to the third pair can be found.

144. Neutral Point.—As already stated (§ 137), when the mean temperature of the junctions of a copper-iron circuit is about 275° C. (= 548° from absolute zero) the current ceases. With circuits formed of other metals, the same thing happens at other temperatures. The formula of § 143 for the electromotive force of a circuit, when written thus—

$$E = (T_2 - T_1)\left(a + 2b \, . \, \frac{T_2 + T_1}{2}\right)$$

shows that E must vanish whenever the mean temperature of the junction satisfies the condition

$$\tfrac{1}{2}(T_2 + T_1) = -\frac{a}{2b}.$$

The temperature thus determined is called the *neutral temperature* for the given pair of metals. If we denote it by the symbol T_n, we get for the coefficient a the value

$$a = -2bT_n.$$

Putting this into the formula for electromotive force, we may write this as containing the product of two differences, namely,

$$E = 2b(T_2 - T_1)\left(\frac{T_2 + T_1}{2} - T_n\right),$$

which shows that it must vanish when either difference vanishes. The first case is when both junctions have the same temperature; the second is when the mean temperature is equal to the neutral temperature. The meaning of the neutral temperature for a given pair of metals becomes more apparent when we consider the value of the corresponding Peltier coefficient. As we have seen, this is expressed by the formula

$$\Pi = aT + 2bT^2.$$

Expressing a in terms of the neutral temperature, this becomes

$$= - 2bT \; T + 2bT^2 = 2bT(T - T_n).$$

It follows that the value of the Peltier coefficient for a given pair of metals which are at the neutral temperature is zero, or that no reversible thermal effect accompanies the passage of a current from one to the other at this temperature.

When the temperature of one junction is kept constant, the electromotive force reaches a maximum when the second junction is at the neutral point; and if one junction is kept at the neutral point, the current is in one direction or the other round the circuit, according as the temperature of the second junction is below or above the neutral point.

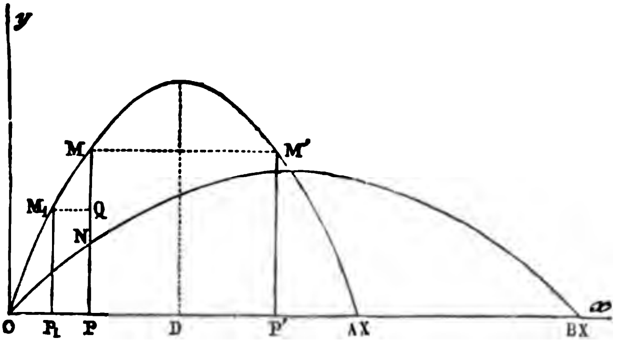

FIG. 131.

145. Geometrical Representation of the Phenomena.—Two methods may be used to represent thermo-electrical phenomena geometrically.

Suppose a couple formed of two metals, A and X, one of the junctions being kept at 0°, and the other at any temperature, t. Take the values of t as abscissæ (Fig. 131), and the corresponding electromotive forces as ordinates; a curve, AX, is obtained, which is, in fact, a parabola with a vertical axis.

If the cold junction, instead of being at 0°, is at the temperature t_1, the same curve still gives the electromotive forces, provided we measure the ordinates from the line MM′, drawn parallel to the axis of abscissæ through the point M of the curve, which corresponds to the temperature t_1. This is a consequence of the law of successive temperatures (§ 138).

The point M′, where the curve cuts the parallel MM′ to the axis of abscissæ, gives the temperature at which the electromotive force changes its sign, and gives, consequently, the temperature of inversion (§§ 137, 144). This temperature varies, as we see, with that of the cold junction.

The temperature t_m of the maximum electromotive force, on the contrary, is fixed, and is represented on the diagram by the point D. It will be seen that it is the mean of the temperatures of the cold junction and of the temperature of inversion.

In like manner, let BX be the curve corresponding to the couple formed by the metals B and X; the difference MN of the two ordinates represents, for the temperatures 0° and P of the two junctions, the electromotive force of the couple formed by the two metals A and B. We have, in fact, from the law of intermediate metals—

$$E(AX) = E(AB) + E(BX).$$

146. Thermo-Electric Height.—Another method is to plot temperatures as abscissæ, and as ordinates the values of

$$\begin{aligned} \frac{dE}{dT} &= \frac{\Pi}{T}, \\ &= a + 2bT, \end{aligned}$$

which are known as *thermo-electric heights*. The line AB in Fig. 132 represents part of a curve so drawn. The ordinate of the point where it intersects the vertical axis represents the value of a; the tangent of the angle which it makes with the axis of temperature gives the value of $2b$. The area of the vertical strip, PMM′P′, being the product of the vertical height into the base, represents the value of dE, namely, the thermo-electric height $\frac{\Pi}{T}$ multiplied by the infinitesimal difference of temperature dT of the junctions. The

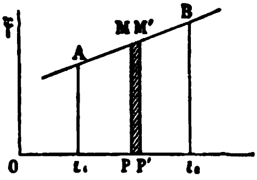

FIG. 132.

sum of all such areas between the lines t_1A and t_2B is the area of the figure t_1ABt_2, and is expressed algebraically as $\int_{t_1}^{t_2} \frac{\Pi}{T} \, dT$, and therefore represents the electromotive force of a circuit, formed of the two metals for which the figure is drawn, with junctions at the temperatures t_1 and t_2 respectively.

In Fig. 133 the area PMM′P′ represents in like manner the electromotive force of a circuit of two metals, say A and X, with junctions at the temperatures denoted by the points P and P′. Similarly, the area PNN′P′ represents the electromotive force for the same range of temperature of a circuit formed of the metals B and X.

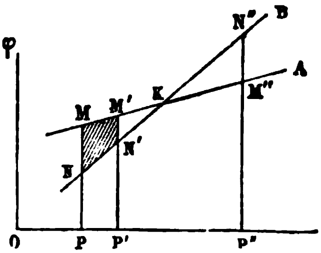

FIG. 133.

From the law of intermediate metals it follows that the difference of these areas, or NMM′N′, represents the electromotive force—still for the same range of temperature—of a circuit formed of the metals A and B. The abscissa of the point K, where the lines MM′ and NN′ intersect, represents the neutral point for the metals A and B, and the area NMM′N′ increases with rise of temperature of the hotter junction until this point is reached. An area like N″KM″ to the right of the point K is to be reckoned negative, and represents an electromotive force acting the opposite way round the circuit to one represented by an area to the left of K. If the temperature of one junction is below, and that of the other is above the neutral point, the electromotive force of the circuit is represented by the difference of the areas to the left and right of the point K respectively, and vanishes when the temperatures are such that these areas are equal, that is, as the figure indicates, when the temperatures of the two junctions are equidistant from the neutral point.

147. Thermo-Electric Diagram.—Fig. 134 gives a thermo-electric diagram, according to Professor Tait, for the principal metals. Lead is the metal taken as standard, since this does not show the Thomson effect. The abscissæ represent temperatures in centigrade degrees, and the ordinates the thermo-electric heights in microvolts (or millionths of a volt) per degree.

All the curves, excepting those for iron and nickel at high temperatures, are straight lines. In order to obtain the electromotive force of a couple AB working between the temperatures t_1 and t_2, the area to be calculated is that of a trapezium, such as MNM′N′ (Fig. 133); it is sufficient to multiply half the sum of the two parallel sides by their distance apart. This distance is the difference $t_2 - t_1$ of the two temperatures; half the sum of the parallel sides is equal to the portion intercepted by the lines A and B on the ordinate which corresponds to the mean $\frac{1}{2}(t_2 + t_1)$ of the temperatures of the two junctions.

For instance, let one of the junctions of an iron-copper couple be at 50°, and the other at 250°; the diagram gives about 10 microvolts for the difference of the ordinates corresponding to the mean temperature, 150°: the electromotive force will then be $10 \times 200 = 2000$ microvolts or 2×10^{-3} volts.

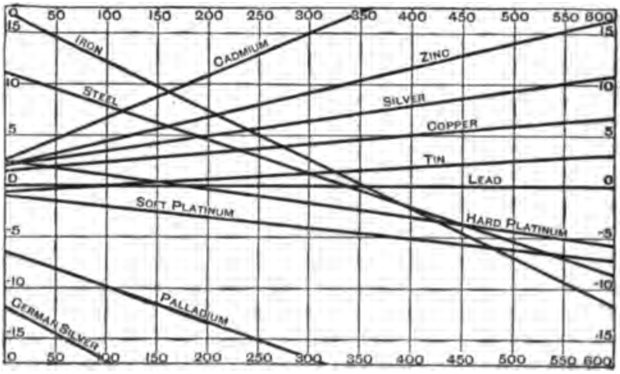

FIG. 134.

148. As the area N′M′MN (Fig. 135) represents the electromotive force in volts, it also represents in joules the work done by a coulomb traversing the circuit. In the case of the figure the current flows in the direction N′M′MN; in other words, the metal B is positive in reference to A.

The outline of the area N′M′MN may be considered as representing the cycle traversed by the electricity, and if we assume that the temperatures are measured from absolute zero, the area comprised between one of the bounding lines of the figure, the axis Oϕ, and two lines parallel to the axis of temperature, represents the work depending on the corresponding element of the cycle.

Thus NN′ represents the passage of a unit of electricity from the lower temperature T to the higher temperature T′, along the conductor B, and the area NN′Q′Q the energy taken in in this part of the circuit in consequence of the Thomson effect.

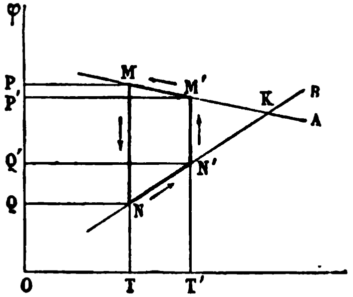

FIG. 135.

The line N′M′ denotes the passage of electricity from the conductor B to the conductor A at the temperature T′, and the area N′M′P′Q′ the energy corresponding to the Peltier effect.

The line M′M denotes the passage of electricity along the conductor A from the temperature T′ to the temperature T, and the area M′MPP′ represents the energy corresponding to the Thomson effect in the conductor A.

Finally, the line MN expresses the passage of electricity from the metal A to the metal B through the cold junction at the temperature T, and the area MNQP the energy corresponding to the Peltier effect.

The diagram is so drawn that gain of energy corresponds to the passage of a current from a lower to a higher point, and loss of energy to the passage from higher to lower. Consequently the areas QNN′Q′, Q′N′M′P′, and P′M′MP represent energy gained during the passage of a coulomb of electricity round the circuit, and the area PMNQ represents energy given out. The net gain of energy per coulomb, or the electromotive force in volts, is represented by the difference of these areas, namely, NN′M′M.

149. Measurement of Temperatures by Thermo-Electric Couples.—As the electromotive force of a thermo-electric couple is, for a fixed temperature of the cold junction, a function of the temperature of the hot junction, it can be used to determine this latter temperature.

A simple arrangement for this purpose consists in twisting together, as at O (Fig. 136), the ends of two wires, AB, forming a couple, and attaching the other two ends to a galvanometer, G, all the junctions from A and from B being at the same temperature.

It is necessary to know once for all the curve of the electromotive forces as a function of the temperature, and to be certain that the couple is comparable with itself, and is not altered by being reheated. Care must also be taken to work below the point of inversion.

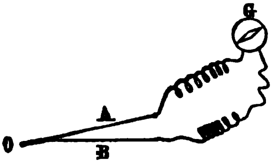

FIG. 136.

A couple of pure platinum and an alloy of rhodium with platinum is well adapted for measuring temperatures up to 1200°, and towards this limit indicates temperatures to within 10 or 20 degrees, which in most cases is sufficient.

For temperatures below 100°, by the arrangement known as Becquerel's *thermo-electric needle* (Fig. 137), the curve can be dispensed with. Two identical couples are joined in series in opposition to each other. The junction A being placed at the point whose temperature is to be determined, the junction B is placed in a bath, the temperature of which can be varied until there is no deflection of the galvanometer. The temperature of the two junctions is then the same.

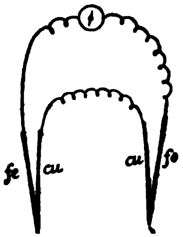

FIG. 137.

For very small differences of temperature, it is advantageous to use a bismuth-antimony couple (Fig. 138). So long as the temperature of 100° is not exceeded, the current is proportional to the difference of temperature of the two junctions. By arranging two couples in series, as shown in Fig. 138, we have Peltier's thermo-electric clip, by which the temperature of any given body may be taken, such, for instance, as that at any part of a bar, MN. This apparatus was used by Peltier in investigating the so-called "Peltier effect" at the junction of two metals.

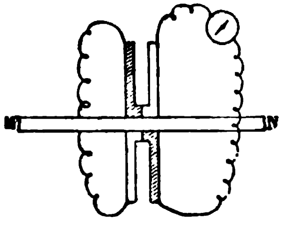

FIG. 138.

Melloni's pile, described above (Fig. 129), constitutes a differential thermometer of extreme sensitiveness. When the two faces are at the temperature of the surrounding air, and heat is allowed to radiate against one end, for instance, if the hand is held in front of it, a difference of temperature is established between them, which is proportional to the radiation, and therefore a current is produced whose intensity is proportional to this radiation.

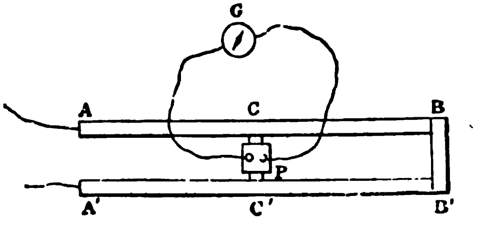

FIG. 139.

As a further example, may be cited the application of the pile by M. Leroux to measuring the Thomson effect (§ 135). If we suppose two similar bars, AB, A′B′ (Fig. 139), placed parallel to each other and joined by a cross-piece, BB′, at the heated end, a thermo-electric pile, placed with its faces in contact with two symmetrical points, C and C′, will indicate a difference of temperature between these points when a strong current is passed through the bars from A to A′ or in the opposite direction.

CHAPTER XV.

CHEMICAL ACTION OF THE CURRENT.

150. Electrolysis.—If the interpolar wire of a battery is cut, and the two ends are then placed in a liquid (Fig. 140), so that the circuit is completed by a column of liquid, two cases may present themselves. The liquid may act like air, as a perfect insulator, and then no current passes; or the current passes, and then, except in the case of mercury or any melted metal, the liquid is decomposed. A non-metallic liquid never acts like a simple conductor; it never allows any quantity of electricity to pass without a correlated decomposition.

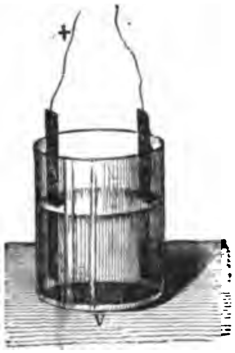

FIG. 140.

This phenomenon is called *electrolysis;* the term *electrolyte* is applied to the liquid which undergoes decomposition; and the conductors by which the current enters and leaves the liquid are called *electrodes;* that in connection with the positive terminal is called the *positive electrode*, or sometimes the *anode*, and that connected with the negative terminal is the *negative electrode* or *kathode*.

The only bodies which are susceptible of electrolysis are apparently salts liquefied either by solution or by fusion. Perfectly pure liquids, such as water, alcohol, ether, bisulphide of carbon, &c., are not electrolytes. We understand by the term salt a compound formed of a metal united either to an element such as Cl, Br, S, or to a compound radical such as SO_4, NO_3, . . . &c. The primary decomposition which takes place under the action of the current appears always to consist in the separation of the metal from the simple or compound radical with which it was combined.

The constituents into which a body is decomposed never appear in the mass of the liquid itself, but only at the electrodes: the metal at the negative and the radical at the positive electrode.

Thus if we use as electrodes two platinum plates, and immerse them in solution of sulphate of copper, copper is deposited on the negative electrode, with its characteristic colour, while at the positive electrode oxygen is given off in the form of gas, and sulphuric acid, SO_4H_2, remains in solution. This may be supposed to result from the action of the liberated SO_4 on water, thus:—

$$SO_4 + H_2O = SO_4H_2 + O.$$

151. Secondary Actions.—When the electrode is not unalterable, the body which is given off may give rise to chemical actions, which are called *secondary actions*. Thus, in the decomposition of sulphate of copper, if a plate of copper is taken as positive electrode instead of a platinum plate, no oxygen is liberated, but the radical SO_4 unites with copper and forms a quantity of sulphate of copper exactly equal to that which has been decomposed; the quantity of copper sulphate in solution remains constant, and in each unit of time the positive electrode loses just as much copper as is deposited on the negative electrode.

With an alkaline salt such as potassium sulphate, K_2SO_4, the decomposition may be supposed to take place in the same way as with copper sulphate; but the potassium set free at the negative electrode, being in contact with water, at once decomposes water, with the formation of potassium hydrate and liberation of hydrogen in the form of gas; thus:—

$$2H_2O + K_2 = 2KHO + H_2.$$

The result is that hydrogen is liberated at the negative electrode, and an equivalent quantity of oxygen at the positive; at the same time, sulphuric acid is found in solution at the positive electrode, and potassic hydrate at the other. This can be shown by means of a U tube (Fig. 141) containing a solution of potassic sulphate, coloured with infusion of red cabbage, and provided with platinum electrodes, A and B. The liquid, which is at first violet-coloured, becomes red at A where the oxygen is given off, while at B, where hydrogen is liberated, it becomes green; these changes of colour indicating the presence of free acid and base respectively.

When solution of common salt is electrolysed, the direct action is similar, but the final result is more complicated. The sodium liberated at the negative electrode decomposes water, with the formation of sodic hydrate, NaHO, and liberation of hydrogen; at the positive electrode, chlorine is liberated, partly in the form of gas, while another portion decomposes water with formation of hydrochloric acid and liberation of oxygen, and another portion still is converted into hypochlorous acid and other oxygen compounds of chlorine.

152. Decomposition of Water, of Potass, &c.—Water furnished the first instance of decomposition by the electric current.

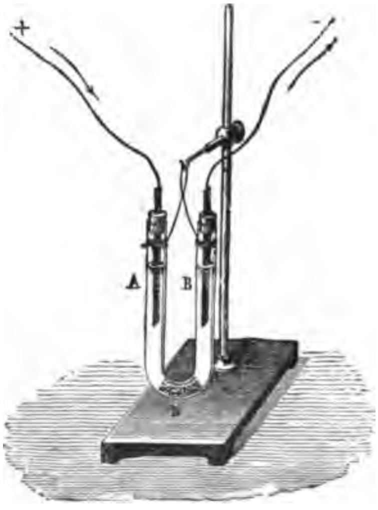

FIG. 141.

The experiment may be made with the apparatus represented in Fig. 142. This is a glass vessel, V, in the base of which are two holes, in which are cemented two wires or strips of platinum, A and B, which act as electrodes. The vessel V contains acidulated water, and two graduated glass tubes, C and D, also filled with water, are inverted over the electrodes. When a battery of three or four cells is connected with the binding screws, P and P′, the decomposition commences; the gases are liberated at the electrodes, oxygen at the positive, and hydrogen at the negative, and it is found that the volume of the latter is twice that of the former.

The addition of acid to the water is essential, as perfectly pure water does not conduct the current, and, therefore, is not decomposed. It is probable that the acid is decomposed into hydrogen and the radical SO_4, and that this acts on water, reforming sulphuric acid and liberating oxygen.

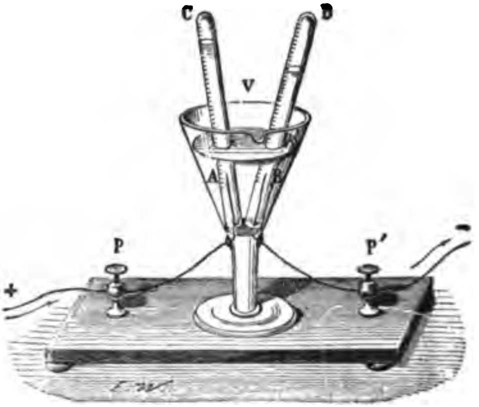

FIG. 142.

The celebrated experiment of Davy, by which he first discovered potassium, sodium, &c., may be made in the following manner. A small piece of moist caustic potash, M (Fig. 143), is placed on a platinum plate which forms the positive electrode, the negative electrode is formed by a small quantity of mercury placed in a hollow made in the upper side of the potash. When a pretty strong current is passed, oxygen is liberated on the positive plate, and the mercury swells up and becomes pasty, owing to its dissolving the potassium as it is liberated. If the resulting amalgam of mercury and potassium is heated out of contact with air, the mercury may be distilled off and potassium left behind. This decomposition may be regarded as a decomposition of 2KHO into K_2 and 2HO, which latter further breaks up into water and oxygen: $2HO = H_2O + O$.

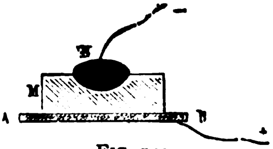

FIG. 143.

153. Electro-Chemical Equivalents.—The quantity of an electrolyte decomposed by a current depends on the nature of the electrolyte and on the quantity of electricity which passes. Thus, if the electrolyte is water, 1 gramme is decomposed by every 10,704 coulombs of electricity that are passed through it. In this case the products of decomposition are $\frac{1}{9}$ gramme of hydrogen and $\frac{8}{9}$ gramme of oxygen. To liberate 1 gramme of hydrogen requires nine times as much electricity, or 96,340 coulombs, and it was shown by Faraday that, in order to liberate the same quantity of hydrogen by the decomposition of any other substance, say hydrochloric acid, or a solution of caustic potash, the same quantity of electricity is required. He further showed that if, instead of comparing *equal* quantities of different substances, we compare quantities which are chemically equivalent to each other, the same quantity of electricity is required in every case. This result is known as *Faraday's law.*

The quantity of any substance which is liberated or decomposed, or, in general, which undergoes chemical change, during the passage of 1 coulomb of electricity, is called the *electro-chemical equivalent* of that substance. Thus the electro-chemical equivalent of hydrogen is $\frac{1}{96340} = 0.00001038$ gramme, that of oxygen is $7.98 \times 0.00001038 = 0.00008283$ gramme, and that of water is $8.98 \times 0.00001038 = 0.00009321$ gramme. Using this term, we may enunciate Faraday's law by saying that *the electro-chemical equivalents of all substances are in the same proportion as their ordinary chemical equivalents.* In fact, to get the electro-chemical equivalent of any substance, we have only to multiply the chemical equivalent of that substance by the constant factor 0.00001038. The following table gives the electro-chemical equivalents of a few important substances in addition to those named above :—

	Chemical Equivalents.	Electro-chemical Equivalents.
Silver	107.7	0.001118 gramme.
Copper	$\frac{63.2}{2}$	.000328 „
Zinc	$\frac{64.9}{2}$	.000337 „
Potassium . .	39	.000405 „
Sodium	23	.000239 „
Chlorine . . .	35.4	.000367 „

Thus, whatever be the electro-negative or acid radical in a salt, or the time taken in the operation, a coulomb liberates always 0.000328 gr. of copper, or 0.001118 gr. of silver; it decomposes 0.00009321 gr. of water, and liberates 0.00001038 gr. of hydrogen, which at 0° and 76 cm. occupies a volume of 0.1153 cubic centimetre. And as the same quantity of electricity, that is, the same number of coulombs, traverses simultaneously all parts of the circuit, all chemical actions which take place simultaneously in the circuit take place in equivalent proportions. Hence, in order to ascertain the amount of chemical action which takes place in a given time in the battery, it is only necessary to insert a *voltameter* at any part of the circuit (that is to say, an apparatus in which electrolysis is caused by the current, and which is so arranged that the products can be collected and their quantity determined), and to measure, as the case may be, the quantity of hydrogen liberated, or of copper or silver deposited, during the time in question. The action taking place simultaneously in each cell of the battery is then chemically comparable with that observed in the voltameter. This is on the understanding that the cells of the battery are connected in a single series: if they are arranged in two or more parallel series, only one half, or some smaller fraction, of the total current traverses each cell, and the resulting amount of chemical action is proportionately smaller.

154. Grotthus's Hypothesis.—In order to explain how it is that the products of decomposition always appear at the electrodes, Grotthus assumed that the elements of any electrolyte—water, for instance—are arranged in a line between the two electrodes, and that while an equivalent of oxygen and one of hydrogen are liberated at the same instant at the ends, water is being re-formed all along the line, the oxygen of each molecule combining simultaneously with the hydrogen of the following one. This conception may be represented materially by two strips of paper marked O and H respectively at equidistant intervals; if the two bands are placed side by side and drawn in opposite directions, the symbols O will be seen separately on one side, and the symbols H on the other, while H and O are in juxtaposition,

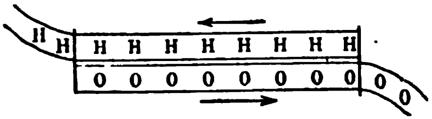

FIG. 144.

representing combination in the state of water, in the intermediate space.

In order to understand why an electrolyte never acts like a simple metallic-conductor, and at the same time to explain Faraday's law, it is sufficient to suppose that the atoms of the electrolyte are the sole carriers of electricity; for instance, in the case of water, that oxygen carries negative, and hydrogen positive electricity; and, lastly, that all the elementary atoms—atoms whose masses are proportional to the electro-chemical equivalents—always carry with them the same absolute charge of electricity, whatever be their nature.

155. Work of Electrolysis.—A coulomb always decomposes equivalent quantities of different electrolytes. On the other hand, this decomposition requires very different quantities of heat. It follows from this that the work done by a coulomb must differ greatly according to the electrolyte through which it passes. If it be assumed that the work of electricity, like that of water, is represented by the product of a quantity into a fall of potential (§§ 34, 35, 124), the conclusion follows that there must be a difference of potential between the two electrodes of a voltameter, and that this difference is such that, for each coulomb, it gives the number of joules of work required to effect the decomposition of one electro-chemical equivalent of the electrolyte.

If $1/a$ is the quantity of electricity in coulombs needed to decompose one gramme-equivalent of any substance (§ 153), and q the quantity of heat resulting from the recombination of the products of decomposition, the electrical work per coulomb is Jaq (§ 69); if H denotes the difference of potential in volts of the two electrodes, we have

$$H = Jaq = 4.17 \times 0.00001038q = 0.0000433q.$$

For water, $q = 34500$, so that $H = 1.49$ volts for the difference of potential between the electrodes of a water voltameter.

156. Polarisation of the Electrodes.—How does this difference of potential arise? Let us take as an example a water voltameter: at the beginning of the experiment, before the passage of the current, the two platinum plates are at the same potential. When, however, the current passes, the substances liberated at the electrodes change the nature of the surface and set up a difference of potential between them, which goes on increasing until it attains a constant value, from which point onwards the decomposition is a normal one. This phenomenon is known as the *polarisation of the electrodes*, and the difference of potential between the electrodes, which determines the fall in the direction of the current, is called the *electromotive force of polarisation.*

It is evident that the quantity of electricity necessary to bring about a given state of polarisation will only depend on the nature and the dimensions of the plates. This quantity represents their *capacity of polarisation.* By taking two electrodes of very unequal surface, it may happen that a quantity of electricity which is sufficient to completely polarise the smaller, only produces a very slight modification in the larger one; the final difference of potential is, however, always the same. Experiment shows, moreover, that the capacity of polarisation, in the case of unequal electrodes, is the same, whatever be the direction of the current, and therefore the sign of the electrode.

157. Polarisation Currents.—The difference of potential between the polarised electrodes may be measured by the electrometer. If the battery is disconnected from the electrodes and a galvanometer (an instrument which indicates the passage of a current by the deflection of a magnetic needle, the construction of which is described in Chapter XXVIII.) is connected with them in its place, it is found that a current traverses the galvanometer from the positive electrode to the negative, and consequently passes through the liquid from the negative electrode to the positive, or in the *opposite* direction to the battery current. This result occurs whether the electrodes are left in the original liquid, or whether they are transferred to acidulated water contained in another vessel.

This current, which is due to the electromotive force of polarisation, is called a *polarisation current;* it rapidly diminishes, and ceases when the substances formed on the electrodes have completely combined again. The quantity of electricity which corresponds to their recombination is manifestly equal to that which had caused their separation. We shall afterwards see that important applications of the polarisation current have been made. It may be added that the production of a polarisation current in the manner described above is a very delicate means of investigating whether a liquid has been decomposed by a current, and therefore whether or not it is an electrolyte.

158. Chemical Work in Batteries.—In a properly arranged battery there is no chemical action so long as the circuit is open; when it is closed the chemical action produced is solely that which results from the passage of the current in conformity with Faraday's law. As the quantity of electricity which traverses any section of a circuit is everywhere the same, the chemical action in each element of the battery is equal, equivalent for equivalent, to that which would be produced in an external voltameter.

In Volta's couple the current traverses acidulated water from the zinc to the copper plate; the hydrogen which goes along with the current is disengaged in the form of gas at the negative plate, while the acid group, SO_4, attacks the zinc plate and forms zinc sulphate. For each coulomb that traverses the circuit, 0.00034 gramme of zinc (§ 153) is dissolved and converted into zinc sulphate.

In Daniell's cell the zinc plate dissolves and gives zinc sulphate in consequence of the electrolysis of dilute sulphuric acid; the copper plate is covered with copper owing to the electrolysis of copper sulphate. Hydrogen and the group SO_4 pass through the porous diaphragm in opposite directions, and reform sulphuric acid. The formation of zinc sulphate and decomposition of copper sulphate take place in equivalent proportions, in accordance with the equation

$$Zn + CuSO_4 = Cu + SO_4.$$

In Grove's element the action is the same as regards the dilute acid and the zinc; in consequence of secondary actions it is a little more complicated for the nitric acid and the platinum. At first there is reduction of nitric acid to nitrous acid, probably as expressed by the chemical equation

$$Zn + H_2SO_4 + HNO_3 = ZnSO_4 + H_2O + HNO_2.$$

Afterwards, the nitrous acid is still further reduced, red fumes are given off, and, from this stage, the electromotive force decreases.

In like manner we may explain the reactions which take place in other couples. In all couples in which zinc is used as the positive metal, the passage of one coulomb corresponds to the same expenditure of zinc—that is, 0.00034 gramme.

159. Energy of Batteries.—The only difference which exists between the chemical work of a voltaic couple and that of a voltameter is, that the first necessarily corresponds to actions which, considered as a whole, are *exothermic*—that is to say, produce heat. The energy of the current is derived from these reactions. A voltaic battery is, in short, an agent by which chemical energy is transformed into electrical energy.

This is very clearly shown by the following experiments, due to Favre and Silbermann. The heat which was produced by the solution of a given weight of zinc in dilute acid was measured in a calorimeter. A small voltaic couple of zinc and platinum was then introduced into the calorimeter, and the circuit closed by a wire, when it was found that the same quantity of heat was produced for the same quantity of zinc dissolved. The couple was next placed in the calorimeter while the interpolar wire was outside, and it was then found that for the same quantity of zinc dissolved a smaller quantity of heat was developed in the calorimeter, and the quantity was smaller in proportion as the resistance of the wire increased. If the couple was made to do chemical work, such as the decomposition of water in a voltameter, the heat produced fell short of that corresponding to the quantity of zinc consumed by an amount equivalent to the chemical work done.

If the whole of the chemical energy is converted into electrical energy, it follows from a calculation analogous to that of § 155, that if q is the quantity of heat resulting from the whole of the reactions which correspond to a *coulomb*, the expression for the electromotive force E of the couple will be

$$E = 0.0000433q.$$

Thus in Daniell's cell the chemical work consists in the conversion of metallic zinc into zinc sulphate, and the formation of copper from copper sulphate. This process causes a production of 25,300 gramme-degrees per gramme-equivalent of the metals taking part in the action. We find, therefore, that

$$E = 25{,}300 \times 0.0000433 = 1.09 \text{ volts},$$

which agrees very closely with the results of direct measurement.

The agreement, however, is not equally good in all cases. Generally the chemical energy exceeds the electric energy, but in some instances it falls short. In the first case, more heat is developed in the cell during the passage of the current than is due, according to Joule's law, to the strength of the current and the resistance; in the second case, the production of heat in the cell is less than that calculated from Joule's law.

In any case, the work measured in joules which a couple can do for each coulomb of electricity is numerically equal to its electromotive force expressed in volts. This work is expended in heat in the circuit, and in other actions produced by the current, such as electrolysis. Hence it is not difficult to understand why a Daniell, which can only give 1.09 joules per coulomb, cannot effect the decomposition of water in a voltameter, which requires 1.49 (§ 155), while this decomposition can be effected with a single Grove, which gives 1.8. It will thus be still more readily effected by two Daniells, which give 2.18 joules per coulomb.

160. Secondary Batteries — Accumulators.—We have seen that two polarised electrodes joined by a conducting wire produce a current opposite to that which had brought about the polarisation. The secondary current thus obtained gradually becomes weaker, and soon disappears, unless the polarisation is kept up by an extraneous cause.

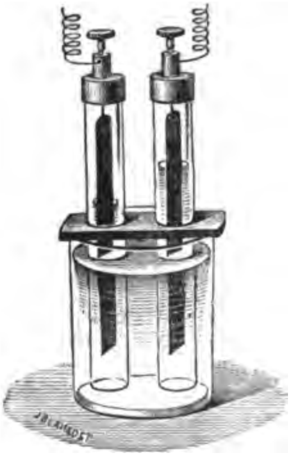

FIG. 145.

This is done in what is known as *Grove's gas battery*, a single cell of which is represented in Fig. 145. It consists of a kind of voltameter in which platinum plates acting as electrodes occupy the whole length of the tubes which are to be filled with gas. The tubes contain dilute sulphuric acid, and the current of a battery is passed until the tubes are filled with oxygen and hydrogen respectively. Contact with the battery is then broken, and the electrodes are connected either directly or through a galvanometer;

the polarisation current is then formed; the water gradually rises in the tubes, and the current only ceases when all the gas has disappeared.

Planté has shown that by using lead plates secondary couples of great capacity can be obtained. Such couples are called *accumulators.* They are formed of two lead plates placed parallel and near to each other, and immersed in dilute acid. While being charged, the plate which serves as positive electrode is transformed to a greater or less depth into an oxide of lead, while the negative plate is reduced if it had been previously oxidised. The disengagement of free hydrogen on the negative plate is an indication that the charge is complete. The plate which formed the positive electrode during the charge becomes the positive electrode during the discharge. This is called the *positive plate* of the accumulator. The discharge takes place in two periods: at first the electromotive force is pretty constant at about two volts; it then begins to fall off, and continues to do so very rapidly. It is best only to utilise the first portion of the charge, and to recharge before the potential begins to sink materially.

Planté showed that accumulators *form* by usage—that is to say, that up to a certain point their capacity is greater, the more frequently they have been charged and discharged. By repeated oxidation and deoxidation the lead acquires a spongy structure, and gradually a larger mass of metal takes part in the reaction. The *formation* is accelerated by immersing the fresh plate for a day or two in nitric acid diluted with its own volume of water.

With well-formed accumulators from 10,000 to 20,000 coulombs can in practice be stored for each kilogramme of lead. Planté obtained still higher numbers, 40,000, or even 60,000. As the discharge usually takes place under an electromotive force of about two volts, the available energy in the ordinary case is 20,000 to 40,000 joules per kilogramme of lead. Of course this energy is expended in a longer or shorter time, according to the strength of the current.

In practice the capacity of an accumulator is frequently expressed in *ampere-hours*, an ampere-hour being the quantity of electricity which passes through the circuit in an hour, when the strength of the current is 1 ampere.

$$1 \text{ ampere-hour} = 3600 \text{ coulombs.}$$

Thus, in practice, from 3 to 6 ampere-hours may be stored for each kilogramme of lead.

Experience shows that, except for very slight losses, which, of course, increase with the time which elapses between the charge and the discharge, accumulators in being discharged restore almost all the electricity which had been given to them during the charge. It must, however, be remarked that the quantity of energy restored is always necessarily less, since the charge is effected at a higher potential than the discharge. If E is the electromotive force, R the resistance of the battery, and C the strength of the current during the charge, E', R', and C' the corresponding values in the discharge, and if Q and Q' are the quantities of electricity which come into play in the two cases, the efficiency will be given by the fraction

$$u = \frac{Q'(E' - C'R')}{Q(E + CR)}$$

which is always less than unity, even if we suppose $Q = Q'$ and $E = E'$.

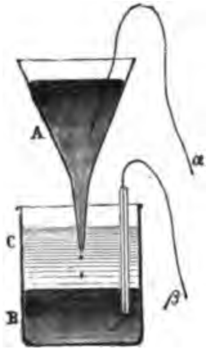

FIG. 146.

161. Electro-Capillary Phenomena.—The polarisation of a surface by changing the condition of the surface modifies those properties which, like the surface-tension, are a function of this condition.

Suppose two masses of mercury separated by acidulated water (Fig. 146), one, A, contained in a funnel with so fine a point that the mercury is in equilibrium owing to the action of the meniscus, while the other, B, forms a large surface at the bottom of the vessel C. If a platinum wire, a, dipping in the top vessel, is connected with the negative terminal of a battery, and an insulated platinum wire, β, dipping with its bared end in the mercury in the lower vessel, is connected with a point on a wire joining the terminals, the potential of the mercury A, becomes less than that of the mercury B, and the difference of potentials becomes greater and greater as the resistance of the part of the battery-circuit, between the points to which the wires a and β are attached, is increased. As this takes place, the surface-tension of the mercury in the drawn-out point increases, and the mercury recedes farther and farther from the point, until the difference of potentials has reached a value of about 0.9 volt.

If we mark the position of the mercury when the two masses are directly connected by joining the conducting wires $\alpha\beta$, and are therefore at the same potential, the pressure which must be exerted on the surface of the mercury, to bring the meniscus to the mark, might serve as a measure of the surface-tension, and at the same time of the difference of potential.

Lippmann's capillary electrometer (Fig. 147) is based on this principle. The mercury A is contained in a long tube, terminated by a finely drawn out point. By means of a caoutchouc bag the desired pressure can be exerted at the top of the tube. The pressure can be measured by means of the manometer H, and the position of the index can be read off by the microscope M. From the pressure the difference of potential is deduced by means of a table constructed once for all.

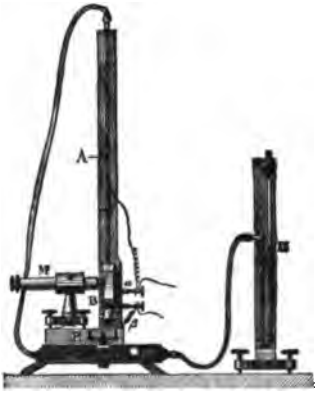

FIG. 147.

This apparatus is very sensitive, and serves to measure differences of potential from 0 to 0.9 volts. But it is of special use in what are called *zero methods*. The difference of potential to be measured is introduced into the circuit with the electrometer, and along with it a known difference of potential which can be varied at will, like that obtained between two points of a wire, AB (Fig. 148), traversed by a constant current (§§ 109, 115); the position of one of these points is varied until the mercury comes back to its mark; the two mercuries are then at the same potential, and the electromotive force is equal to the fall of potential between the two points, A and B.

The apparatus in Fig. 146 can be used to make a curious experiment on reversibility. If the height of the mercury in the funnel is just so small that there is no spontaneous flow by the point this commences as soon as the two wires, $\alpha\beta$, are connected, and at the same time the circuit is traversed by a current proceeding through the wire from the mercury B to the mercury A.

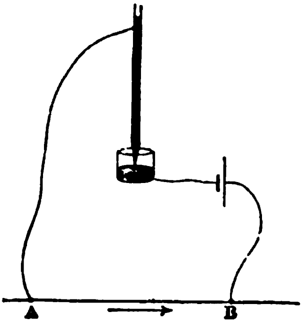

FIG. 148.

CHAPTER XVI.

MAGNETISM—GENERAL PHENOMENA.

162. Natural and Artificial Magnets.—The name *magnet* or loadstone is applied to certain specimens of natural oxide of iron (Fe_3O_4) which possess the property of attracting iron filings. All points of the loadstone do not possess this property to the same degree; when such a stone is rolled in the filings, they adhere chiefly to certain parts.

By simple friction, and without itself losing any of its properties, the stone can impart to steel the property of attracting iron. Steel magnets are also called *artificial magnets*, in opposition to the former, which are called *natural magnets.* Experiment shows, further, that the properties of the two kinds of magnets are identical, and accordingly, from their simpler and more regular form artificial magnets are the only ones used. They are ordinarily employed in the form of needles (Fig. 151) or bars (Fig. 149).

163. Poles of a Magnet.—When a bar is placed in iron filings, these adhere chiefly to the ends, as if the property of attracting iron were specially concentrated there (Fig. 149). These ends are called the *poles of the magnet.* The idea was long ago conceived that the bar consists of inert matter possessing two centres of action near its ends. Such centres of action would be poles, and owe their properties to acting masses called *magnetic masses* concentrated at them.

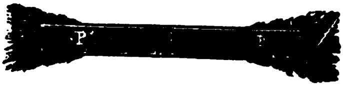

FIG. 149.

The centres of action are really disseminated over the two ends of the bar; this is seen very clearly from the manner in which the filings attach themselves to the bar, and in a still more striking manner from the way in which they arrange themselves in the following experiment.

A thin sheet of glass or cardboard is placed above a bar magnet, and iron filings are sifted uniformly over it. On giving the plate slight taps, the filings group themselves in regular curves (Fig. 150), which start from one point of the surface, and terminate at the symmetrically situated point on the other side of the centre. This experiment, the bearing of which we shall afterwards explain, shows, first of all, the existence of a *field of force* about the magnet. It demonstrates, moreover, this important fact, that magnetic action is transmitted without alteration through any substance which is not itself magnetic. This property is common to magnetism and gravity. Exception must be made for iron and a few other substances, which when they are sufficiently thick can *screen* the action of a magnet.

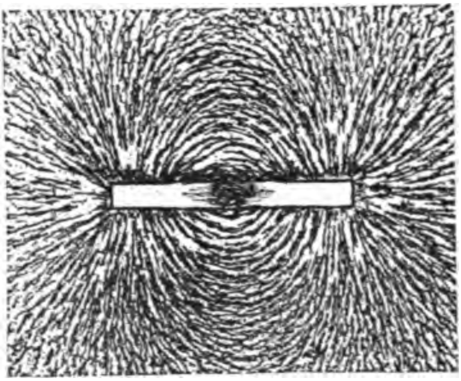

FIG. 150.

164. Distinction between the Poles—Action of the Earth.—As regards the action of a magnet on iron filings, there is no distinction between one pole and the other; the two poles are nevertheless not identical.

A second property which every magnetised bar possesses is that, when suspended horizontally, it takes up a fixed direction in space, which in this part of the world is not very far from that of north and south. The same end always turns to the north; this is called the *north pole*, and the end which turns to the south is called the *south pole*. The north and the south poles are accordingly different, and for distinction, they may be marked, once for all, with the letters N and S respectively.

Magnetic needles are generally in the form of an elongated lozenge (Fig. 151), and are provided with a metal or agate cap, so that when balanced on a point they can oscillate freely in a horizontal plane; the poles are distinguished by a mark near the north pole, whence this is sometimes spoken of as the marked end of the needle; this half is also left with the blue tint on it which the steel has acquired by tempering.

Artificial magnets are not necessarily in the shape of bars; they may have any shape—that of a sphere, for instance. In all cases the filings adhere to two regions which represent the poles, and a certain line joining these two regions, tends to take the same direction in space as the magnetic needle. This line is called the *magnetic axis* of the magnet. The axis is counted positively from the south pole to the north.

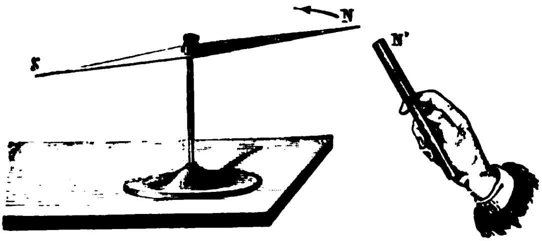

FIG. 151.

165. Reciprocal Action of Magnets.—Magnets act on each other. If a bar magnet is brought near another one suspended horizontally, it is observed that the north pole of the first repels the north pole of the second, but, on the contrary, attracts the south pole (Fig. 151). Conversely the south pole attracts the north pole and repels the south pole of the movable magnet. Hence we have this fundamental rule, which is like the corresponding one for electricity—*two poles of the same kind repel each other, and two contrary poles attract.*

166. Coulomb's Law.—Even when we conceive the magnet as reduced to two centres of force situated near the ends, there are four forces which are exerted between two adjacent magnets, and in strictness it is impossible to reduce them to a smaller number. But by taking very long magnets, formed, for instance, of steel wire, the diameter of which is very small compared with the length, $\frac{1}{50}$ for instance, they can be arranged so that the action of the distant poles may be neglected in comparison with that of the near ones, and thus we may in reality observe the action of two poles on each other. In this way Coulomb determined the law of magnetic attraction and repulsion by means of the torsion balance, and he thus discovered the following fundamental law :—

The attraction and repulsion exerted between two poles vary as the inverse square of the distance between them.

167. Strength of Magnetic Poles.—The force between two poles at a given distance varies with the magnets to which they belong. If one pole remains fixed, a second pole at a given distance will exert a definite force : if, now, another pole placed at the same distance exerts twice or three times the force previously observed, we say that its quantity of magnetism, or the *strength of the pole*, is twice or three times as great. We are thus led to take as unit quantity of magnetism, or unit magnetic pole, that which, acting on an equal pole placed at unit distance, repels it with unit force. The force, f, between two poles having strengths m and m', and placed at the distance r, is

$$f = \frac{mm'}{r^2}$$

In the C.G.S. system, the unit magnetic pole is that which, acting on an equal pole placed at a distance of a centimetre, repels it with the force of a dyne (§ 10).

If two poles, m' and m'', are united at the same point, experiment shows that the force on a given pole, m, is proportional to $m' + m''$, if the two poles are of the same kind, and to $m' - m''$ if they are opposite. Hence quantities of magnetism may be added, like algebraical quantities, and, like them, may be distinguished by the signs + and −. The sign + will be given to the magnetic mass of a north pole, and the sign − to that of a south pole. It follows from this convention that in the above formula the sign + corresponds to repulsion, and the sign − to attraction.

168. Magnetic Field.—The elementary law is thus the same for quantities of magnetism as for quantities of electricity. Confining ourselves to the case of magnetic masses which remain in definite fixed positions, the conclusions which we have deduced for quantities of electricity apply word for word to quantities of magnetism. Thus there is a magnetic potential defined and calculated like electric potential (§§ 36, 37). Moreover, it results from the choice of unit quantity in both cases, that quantities of magnetism and of electricity of the same *numerical value*, distributed in the same manner, give the *same field*, in this sense, that the force at each point has the same direction and the same intensity. Accordingly the lines of force and the equipotential surfaces are the same; the potential has everywhere the same numerical value, and represents in C.G.S. units the number of ergs necessary to bring a positive quantity equal to unity from infinity to the point in question. But this identity of the two fields only applies to the numerical values, and does not imply identity of properties. Electric force has no action on magnetic poles, and conversely; and although the properties of the two fields undoubtedly result from modifications of the same medium, the two modifications are essentially distinct, and may coexist without reacting on each other, and cannot be compounded together.

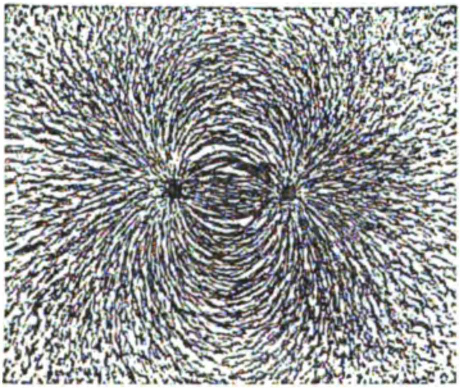

FIG. 152.

The figures formed by iron filings, as we shall afterwards see, give exact representations of magnetic fields, and the curves formed by the filings are lines of magnetic force.

The lines of force, as with electricity, always start from a positive region, and terminate at a negative one; but since the distribution is not exclusively on the surface, as in the case of electrified conductors in equilibrium, the lines of force are not in general perpendicular to the surface; two corresponding elements of surface do not necessarily possess equal and opposite quantities of magnetism; and, in conclusion, the relation between the normal component and the surface density which results from Coulomb's theorem (§ 43) is no longer generally applicable. Fig. 152 represents the field given by two equal poles of opposite signs; Fig. 153 that produced by two equal poles of the same kind. Both these figures, as well as that in Fig. 150, are from photographs of actual figures made with filings.

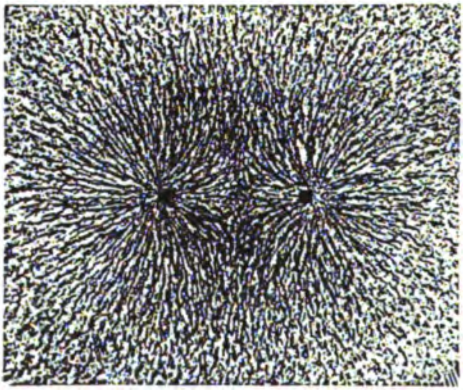

FIG. 153.

Lines of force may be used (§ 46) to represent the direction and intensity of the field. By drawing from each mass, m, a number of lines of force equal to $4\pi m$, the number of lines of force contained in a tube will be the value of the corresponding flux of force, and at each point of the field the intensity of the magnetic force will be given by the number of lines of force cut by unit surface drawn through this point normally to the lines.

169. Terrestrial Field.—The region about the earth is a magnetic field, for every magnet is there subject to force tending to make it set in a definite direction. In any given locality this direction remains the same throughout a region, of considerable extent as compared with the dimensions of artificial magnets, though small as compared with the earth,—in a room, for instance, provided there is no iron or other magnetic matter in the neighbourhood. The lines of magnetic force are consequently, throughout such a region, parallel to each other, expressing that the field is uniform (§ 42).

170. Precise Definition of Poles.—When the magnet is placed in a uniform field, all the forces exerted on the magnetic masses which it contains are parallel to each other, but in opposite directions according as these are positive or negative; all the forces which act on positive masses have a resultant equal to their sum, parallel to their direction, and applied at a point which may be called the centre of mass of the positive magnetism; and similarly for the negative magnetism. The magnet placed in a uniform field may then be considered as under the action of two forces, which are parallel and in opposite directions, applied at two fixed points in the bar. These are the points which for the future we shall call the *poles* of the magnet, giving thus a precise meaning to this term; the line which joins them is the *magnetic axis* of the magnet.

This comes to the same thing as if all the positive magnetism of the magnet were concentrated at the north pole, and the negative magnetism at the south pole, the numerical value of the quantity of magnetism concentrated at each pole being equal to that of the force applied there when the field is of unit intensity. But this applies only in the case of a uniform field; a magnet cannot in general be treated as reduced to two poles when we are dealing with its action on another magnet.

171. The Total Magnetism of a Magnet is Zero.—The action which a magnet undergoes in a uniform field like that of the earth is purely directive; it has neither vertical nor horizontal component. It has no vertical component, for the weight of a steel bar is exactly the same after being magnetised as it was before. Nor has it a horizontal component, for a bar free to move in a horizontal plane, if placed, for instance, on a cork on the surface of still water, has no tendency to motion of translation.

The action is thus reduced to a couple: accordingly, the two parallel and opposite forces, which in a uniform field act on the poles, are equal to each other. It follows, therefore, that *the sum of the quantities of positive magnetism is equal to the sum of the quantities of negative magnetism*—in other words, that *the total sum of the magnetic masses of any magnet is zero.*

172. Moment of a Magnet.—Let m be the absolute mass of each of the poles, and $2a$ their distance, the product $2am = M$ is what will be called the *magnetic moment* of the magnet.

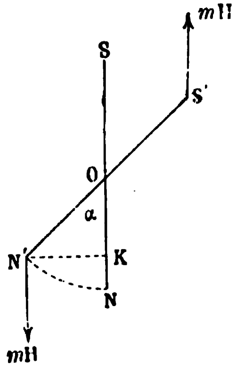

FIG. 154.

The quantity M is given directly by experiment. It is the moment of the couple, which acts on the bar when it is placed in a uniform field of unit intensity, and when its axis is perpendicular to the lines of force. In a field of intensity, H, the moment of the couple in the same conditions is MH. If the magnetic axis makes an angle, α, with the direction of the field (Fig. 154), the moment of the couple is $MH \sin \alpha$.

The positions of equilibrium are those in which the axis of the bar is parallel with the force of the field. The equilibrium is stable if the direction of the axis and that of the field are the same, and unstable if the directions are opposite.

When the bar passes from a position in which its axis makes an angle, α, with the direction of the field to the position of stable equilibrium, the work done by magnetic forces is

$$2mH\mathrm{KN} = MH(1 - \cos \alpha).$$

This is equal to $2MH$ when the magnet passes from the position of unstable equilibrium to that of stable equilibrium.

173. Moment of a System of Magnets.—Consider a system of magnets fixed rigidly to each other and placed in a uniform field. Each of them is subject to the action of a couple; all these couples, as demonstrated in mechanics, can be reduced to a single one, and therefore the system acts as if composed of a single magnet, having a definite axis and a definite moment, which may be called the *resultant magnet.*

The effect of each magnet may be represented by a line drawn in the direction of its axis and of length proportional to its moment; and in order to obtain the axis and the moment of the resultant magnet, it is sufficient to draw through any point lines equal and parallel to the straight lines which represent the component magnets of the system, and to compound them like lines representing forces. The resultant, or the line which closes the polygon, obtained by drawing in succession lines equal and parallel to the various components, represents, both in magnitude and direction, the axis and moment of the resultant magnet. If the polygon is closed of itself, the system is in neutral equilibrium in the field; it is then said to be astatic.

As a particular case we will consider that of two magnets of moments, m and m', whose axes, represented by the lines OA and OB (Fig. 155), make with each other the angle δ. The axis of the resultant magnet will be represented by the diagonal OM, and the position of equilibrium is that in which this diagonal coincides with the direction of the field. If α be the angle which the diagonal OM makes with OA, we have from the fundamental property of triangles

$$\frac{\text{OA}}{\sin(\delta-\alpha)} = \frac{\text{OB}}{\sin\alpha} = \frac{\text{OM}}{\sin\delta}$$

The first equation expanded gives

$$\tan\alpha = \frac{m'\sin\delta}{m + m'\cos\delta}$$

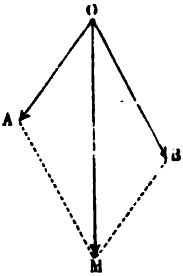

FIG. 155.

An interesting case is that in which the two axes are nearly parallel and in opposite directions—in other words, when the angle δ is near π (Fig. 156). If ω is the very small angle supplementary to δ, we have sensibly $\sin\delta = \omega$, and $\cos\delta = -1$; and the above formula becomes

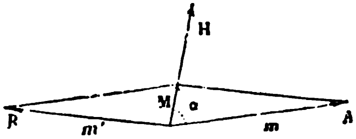

FIG. 156.

$$\tan\alpha = \frac{m'\omega}{m - m'},$$

and it will be seen that the value of α is the nearer $\frac{\pi}{2}$ the more nearly equal are m and m'; the system formed by the two magnets tends to set at right angles to the field, and the more nearly so the smaller the difference of the moments of the component magnets. It is in neutral equilibrium if $m = m'$ and simultaneously $\omega = 0$.

174. Field of a Magnet.—If we suppose the magnet reduced to its two poles, N and S (Fig. 157), the field which it produces is that of two equal masses of opposite signs placed at these points. The action PA of the pole N on a unit positive pole at P, at distances r and r' from the two poles respectively, is repulsive and equal to $\frac{m}{r^2}$; that of the pole S, PB, is attractive and equal to $\frac{m}{r'^2}$. These two forces give a resultant represented by the diagonal PM of their parallelogram.[1]

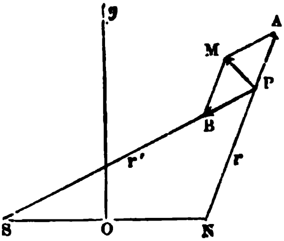

FIG. 157.

This resultant is evidently a tangent at the point P to the line of force which passes through this point.

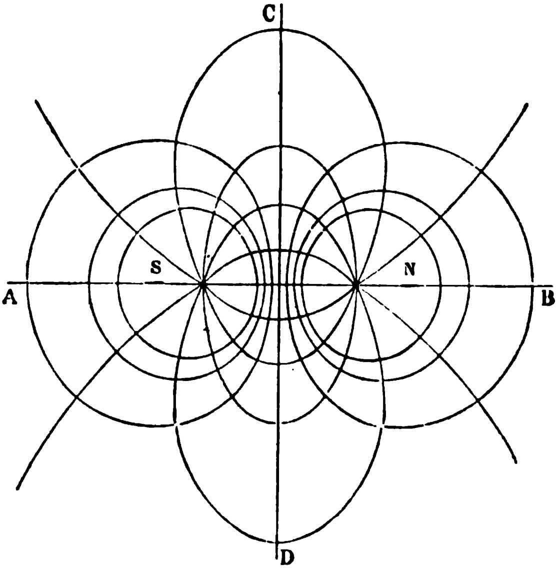

FIG. 158.

The lines of force (Fig. 158) are lines proceeding from the point N to the point S; it is easy to recognise their resemblance to the lines formed by iron filings (Fig. 152).

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[1] The diagram is not drawn to scale: PB should be of less than half the length of PA, and PM consequently more nearly at right angles to NS.

The value of the potential at the point P at distances r and r' from the two poles is

$$V = m\left(\frac{1}{r} - \frac{1}{r'}\right).$$

This equation, when V is constant, represents an equipotential surface. By giving successively to V the values 1, 2, 3, . . . we obtain the series of equipotential surfaces corresponding to the potentials 1, 2, 3. . . . These surfaces are closed ovoids, surrounding each pole; they become more and more nearly spherical as we get nearer the poles. All those which correspond to positive values of the potential surround the point N; those which correspond to negative values, the point S. They are separated by a plane of symmetry, CD, at potential zero. The lines produced by the intersection of these surfaces by a plane passing through the axis are everywhere perpendicular to the lines of force traced in this plane.

175. Properties of an Infinitely Small Magnet—Value of the Potential.—An interesting case, and one which will be useful in the sequel, is that of a magnet in which the distance of the two poles $2a$ is very small compared with the distance of the point P (Fig. 159).

If r is the distance OP, and a the angle which it makes with the axis of the magnet, we may, in the expression for the potential

$$V = m\left(\frac{1}{r'} - \frac{1}{r''}\right) = m\,\frac{r'' - r'}{r'r''}$$

replace the difference $r'' - r'$ by SC, or by its approximate value $2a \cos a$, and the product $r'r''$ by r^2, which gives

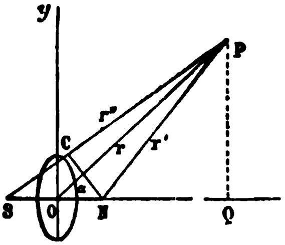

FIG. 159.

$$V = 2am\,\frac{\cos a}{r^2} = \mathfrak{M}\,\frac{\cos a}{r^2},$$

$\mathfrak{M}$ representing the moment $2am$ of the infinitely small magnet.

Through the point O, and perpendicular to the axis, draw a small plane figure—a circle, for instance—the surface of which has the same value as the moment $\mathfrak{M} = 2am$ of the magnet, and let $\bar{\omega}$ be the solid angle under which we see this surface from the point P; this solid angle is measured by the portion of the surface of a sphere of unit radius, described about the point P as centre, which is intercepted by the cone, whose base is the surface in question, and whose vertex is P. This cone intercepts on the sphere of radius r, a surface equal to $\varpi \cos a$, and on the sphere of unit radius a surface $\frac{\varpi \cos a}{r^2}$. We have thus

$$\omega = \frac{\varpi \cos a}{r^2},$$

and, therefore,

$$V = \omega.$$

Thus, *the potential at a given point due to an infinitely small magnet is numerically equal to the solid angle under which a surface, equal to the moment of the magnet and perpendicular to the axis of the magnet at its centre, is seen from this point.*

For instance, if the solid angle ω is the $\frac{1}{200}$ of the sphere, the potential at P is equal to $\frac{4\pi}{200}$ or to 0.0628 C.G.S. units of potential—in other words, it would be necessary to expend 0.0628 erg to bring unit positive pole from infinity to the point P in the presence of the very small magnet NS.

The sign of the solid angle evidently follows that of the potential; it is positive when, looking from the point in question, we see the north face, which will be called the positive face, and negative in the contrary case.

176. Magnitude and Direction of the Force.—To determine the magnetic force at the point P. From the point O as centre (Fig. 160) describe the circumference of radius r which passes through the point P: it is required to find the components along the tangent and along the radius.

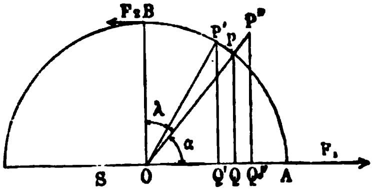

FIG. 160.

In passing from the point P to the point P′ along the circumference, the work of the magnetic force is $F_t \times$ PP′, where F is the tangential component; on the other hand, this work is equal to the change of potential $V - V'$ from the first point to the second, so that

$$F_t \times \mathrm{PP}' = V - V' = \omega - \omega' = \frac{\omega}{r^2}(\cos \alpha - \cos \alpha'),$$

or, seeing that $\cos \alpha = \dfrac{\mathrm{OQ}}{r}$.

$$F_t \times \mathrm{PP}' = \frac{\varpi}{r^3}(\mathrm{OQ} - \mathrm{OQ}') = \frac{\varpi}{r^3}\mathrm{QQ}',$$

since $\mathrm{QQ}' = \mathrm{PP}' \sin \alpha$, we get

$$F_t = \frac{\varpi}{r^3}\sin \alpha. \tag{1}$$

In like manner, in passing along the radius from the point P at the distance r to the point P″ at the distance r', calling the radial component F_n, we have

$$F_n \times \mathrm{PP}'' = V - V'' = \omega - \omega'' = \varpi \cos \alpha \left(\frac{1}{r^2} - \frac{1}{r'^2}\right);$$

as r differs very little from r', the factor

$$\frac{1}{r^2} - \frac{1}{r'^2} = \frac{r'^2 - r^2}{r^2 r'^2} = \frac{(r + r')(r - r')}{r^2 r'^2}$$

may be replaced by $\dfrac{2\mathrm{PP}''}{r^3}$, and we have therefore

$$F_n = \frac{2\varpi}{r^3}\cos \alpha. \tag{2}$$

The value of the resultant, which is the diagonal of the rectangle formed on the two forces F_t and F_n, is

$$F = \sqrt{F_n^2 + F_t^2} = \frac{\varpi}{r^3}\sqrt{3 \cos^2 \alpha + 1} = \frac{\varpi}{r^3}\sqrt{3 \sin^2 \lambda + 1} \tag{3}$$

λ being the angle which OP makes with the perpendicular bisector OB of the magnet. The direction of the resultant makes with the tangent an angle I, such that

$$\tan I = \frac{F_n}{F_t} = 2 \cot \alpha = 2 \tan \lambda. \tag{4}$$

It will be seen that the force in any given direction varies as the inverse cube of the distance.

For a point A on the axis, $a = 0$; the tangential component is zero, and we have for the total force along the axis

(5) $$F_1 = \frac{2\varpi}{r^3}.$$

At a point B in the perpendicular bisector, $a = \frac{\pi}{2}$; the force reduces to the tangential component always parallel to the axis, and we have

(6) $$F_2 = \frac{\varpi}{r^3}$$

and, therefore,

$$F_1 = 2\,F_2.$$

Two positions such as A and B, one in the prolongation of the axis of the magnet and the other on the perpendicular to it, are called *principal positions.*

Fig. 158 enables us to form an idea of the field due to an infinitely small magnet. In this case the two points N and S would coincide at the centre of the figure, and the lines of force would be closed curves tangential at this point to the axis AB. The equipotential surfaces are also all tangential at the same point to the plane of symmetry CD.

177. Relative Energy of an Infinitely Small Magnet in a Field.—The potential $V = \omega$ of the infinitely small magnet represents the work done against magnetic force in bringing a unit of positive magnetism from an infinite distance to the point P in the presence of the infinitely small magnet; or, reciprocally, in moving the infinitely small magnet from an infinite distance to its actual position in the presence of a unit magnetic pole at P. It is, therefore, the energy in the given condition of the system formed by this pole and the infinitely small magnet.

If the pole at the point P were equal to m, the energy of the system would be $m\omega$. Now this product is the flux of force from the mass m at the point P, which traverses the surface ϖ by its positive face. We shall represent by q the flux which traverses a surface, and for convenience of subsequent calculations we shall take q as *positive* when it enters by the *negative* face, and negative when it enters by the *positive* face. We shall accordingly write

$$m\omega = -q.$$

If there are other masses, m' m'', in the field, we shall have analogous expressions for each of them. The value of the energy of the infinitely small magnet in the presence of these masses is then

$$W = m\omega + m'\omega' + \dots\dots = -(q + q' + \dots) = -Q.$$

Q being the total flux which passes through the surface ϖ by its negative face. If F_n is the normal component at the point O of the field, due to the masses m, m', m'', we have $Q = \varpi F_n$.

Any variation which changes Q into Q' will correspond to an amount of positive work equal to $Q' - Q$ done by the magnetic forces.

CHAPTER XVII.

CONSTITUTION OF MAGNETS.

178. Result of Breaking a Magnetised Bar.—When a magnetised bar—a steel needle, for instance—is broken, each part forms a complete magnet, the two new poles being of the same strength as those of the original magnet, and the axis in the same direction (Fig. 161). The phenomenon can be repeated as long as the fragments obtained are large enough to admit of examination.

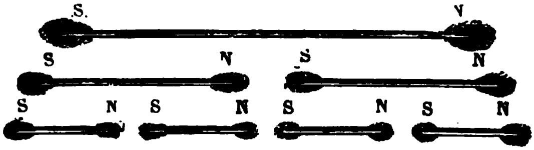

FIG. 161.

Two important consequences follow from this. The first, that it is impossible to get a detached positive or negative magnetic pole not connected with an equal pole of the opposite kind. The second is that magnetism is a phenomenon the cause of which resides in the molecules of the magnet. If we suppose a magnet split up into elementary particles, each element in its actual condition must be considered as having its two poles and a definite moment. The entire magnet is only the resultant of all these elementary magnets, and the axis, as well as the moment of the resultant magnet, may be obtained by the rule in § 173.

179. Distribution of Magnetism.—It is not possible to determine by experiment the distribution of magnetism in the interior of a magnet, any more than we can determine the distribution of electricity in the interior of a bad conductor. From Gauss's theorem (§ 41), which applies equally to the two cases, the total flux of force which traverses a surface surrounding the magnet would give the total value of the acting mass which it comprises; but we know already that, in the case of a magnet, this mass is zero (§ 171).

Whatever be the distribution of magnetic masses in the interior of a magnet, it is easy to prove that their action is equivalent to that of two equal magnetic masses of opposite signs distributed on the surface according to a certain law. For, suppose all the magnetic masses to be replaced by *fixed* electrical masses of the same numerical value (§ 167), and imagine the magnet enclosed within a conducting surface (§ 23*b*). We know that this surface will be covered on the interior with an electrical layer, of equal total mass, but of opposite sign to the sum of the interior masses, which will exert at all external points a force equal and opposite to that of the enclosed masses. Consequently, an equal and similarly distributed layer, of the same sign as the sum of the enclosed masses, would exert externally the same force as they do, and might therefore be substituted for them. Similarly in the case of a magnet. In this case, the total mass of such a layer will be zero—that is to say, it will be composed of two equal layers of opposite signs, separated by a neutral line, one covering the north end and the other the south end of the magnet. The same reasoning will apply to any surface whatever enclosing the magnet.

This shows clearly that the problem of internal distribution is altogether indeterminate.

180. Particular Cases of Magnetic Distribution—Solenoidal Filament.—Among the distributions which we may *a priori* imagine, we shall examine three which present a special interest.

Suppose, in the first case, magnetic elements identical with each other placed in file along any given curve, so that their axes having all the same sense, coincide with this line, the north pole of one being in contact with the south pole of the next. This arrangement is said to be solenoidal, and is spoken of as a *solenoidal filament*. The filament is neutral throughout its entire length, except at the ends at which there are equal magnetic masses, $+m$ and $-m$, of opposite signs. The external action of the filament is therefore reducible to that of these two masses; it is independent of the shape and length of the filament, and depends only on the position of the ends. It vanishes if the filament is closed upon itself: in this case the magnetic state of the filament is not indicated by any external effect.

The potential of a solenoidal filament at a point P situate at distances r and r' from the two ends has clearly the value

$$V = m\left(\frac{1}{r} - \frac{1}{r'}\right).$$

181. Magnetic Shells.—In the second mode of distribution we shall imagine that the magnetic elements, while still identical with each other, are arranged side by side on a given surface, and that their axes are normal to this surface, and all in the same sense. This system represents an infinitely thin plate, covered on the two faces with uniform layers of the same density, one of north, and the other of south magnetism. We shall call this a *magnetic shell;* we shall call the magnetic moment per unit of area the *strength of the shell*, and shall denote it by the letter Φ. It is equal to the product of the infinitely small thickness h of the shell by its surface density σ, or

$$\Phi = h\sigma.$$

182. Potential of a Shell.—Let dS be an element of the surface, the corresponding portion of the shell forms an infinitely small magnet with the moment $h\sigma dS$ or ΦdS, and the potential of this portion at a point P (Fig. 162) is equal to the solid angle under which a surface ΦdS tangential to the element is seen from this point (§ 175), and consequently it is equal to the angle $d\omega$, under which the surface dS itself is seen, multiplied by Φ. This potential is positive or negative according as the face of the element dS seen from the point P is positive or negative.

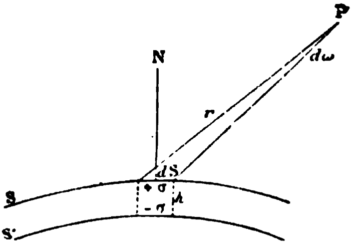

FIG. 162.

To get the potential of a shell S of any given shape bounded by the contour ABCD (Fig. 163), we must add together the solid angles corresponding to the various elements into which the surface may be subdivided. The angles corresponding to elements such as ds and ds', intercepted by a cone which cuts the surface an even number of times, cancel each other, they being equal and of opposite signs, since the faces which they present to the point P are necessarily of opposite signs. The sum is evidently the same as the solid angle under which the bounding contour ABCD of the shell is seen from the point P. This sum is therefore independent of the shape and the extent of the surface which is bounded by the contour; it is, moreover, positive or negative according as the portion of the surface seen directly from the point P through the contour is positive or negative. If ω is the solid angle, we have

$$V = \Phi\omega.$$

Hence *the potential at an external point due to a magnetic shell is equal to the product of the strength of the shell by the angle under which the bounding contour is seen from this point.*

It represents the work done in bringing unit of positive magnetism from an infinite distance to the point in question.

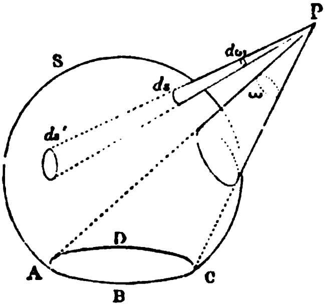

Fig. 163.

If the shell forms a closed surface, the solid angle vanishes for external points; it is equal to the angle of a complete sphere or 4π for any internal point. Hence the value of the potential is constant in both cases, and therefore the force due to a shell forming a closed surface is everywhere zero (§ 31).

If the surface is hollow, with an aperture such as AB (Fig. 164), then for a point P in the interior, at which the aperture subtends the angle ω, the solid angle of the surface should evidently be taken as $\pm(4\pi - \omega)$.

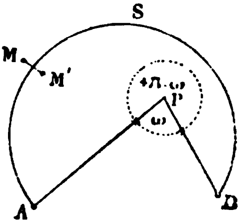

Fig. 164.

Consider two points, M and M', taken on the same perpendicular to the surface of the shell, and infinitely near the surface, one on the positive and the other on the negative side (Fig. 164), and let ω represent the solid angle under which the boundary is seen from either of these points. At M the potential is $\Phi\omega$; at M' it is $-\Phi(4\pi - \omega) = \Phi\omega - 4\pi\Phi$. Hence at the two points it has a constant difference equal to $4\pi\Phi$ in absolute value.

Thus on passing from a point of the negative face to the corresponding point of the positive face, the potential, or, in other words, the work per unit pole increases by $4\pi\Phi$, whatever be the path followed.

183. Relative Energy of a Shell in the Field.—The energy relatively to the field of the portion of the shell which corresponds to the element of surface dS, is equal to the flux of force which would traverse a surface $\varpi = \Phi dS$—in other words, to the product by Φ of the flux which traverses the element dS entering by the positive face. The total energy is obtained by multiplying by Φ the algebraical sum of these elementary fluxes,—that is to say, the total flux which passes through the contour entering by its positive face. Calling this $-Q$ (§ 177), we have

$$W = -\Phi Q.$$

Accordingly, *the potential energy of a shell is equal to the product taken with the contrary sign of the strength of the shell by the flux of force which traverses its boundary, entering by the negative face.*

The position of stable equilibrium of the shell in the field is that which corresponds to the minimum energy, and therefore to that in which the flux, entering through the negative face, is a maximum.

184. Mutual Energy of Two Shells.—Suppose that the field is that of a second shell of strength Φ'. We may put $Q = M\Phi'$, the coefficient M being the flux which there would be through the negative face of the first shell if the strength of the second were equal to unity. The expression for the energy of the first in the field of the second is then

$$W = -\Phi\Phi' M.$$

In like manner, the expression for the energy of the second in the field of the first would be $-\Phi\Phi' M'$, M' being the flux which the second would receive if the strength of the first were equal to unity. As these two quantities are the expression for the same energy, we deduce

$$M = M'.$$

From which it follows that *when two magnetic shells of unit strength are in presence of each other, the flux of force which passes from one and traverses the other by its negative face is the same for both.*

The coefficient M is called the *coefficient of mutual induction* of the two shells; its value depends only on the two contours, and on their relative positions.

185. Uniform Magnetisation.—Lastly, let us consider a third case, that of a magnet of finite dimensions and of *uniform magnetisation*,—that is to say, such that an element of volume taken at any part has the same moment and its axis in the same direction.

It is clear that a uniform magnet may be considered as formed by the juxtaposition of straight parallel solenoidal filaments of the same intensity, terminating at the surface; or, again, of a series of plane shells of the same surface-density arranged at right angles to the direction of magnetisation.

There is no apparent magnetism, except on the outer surface. For instance, with a cylinder magnetised uniformly in the direction of the axis, there will be no apparent magnetism, except on the terminal surfaces, which will be covered with uniform layers of equal density, and of opposite signs, $\pm\sigma$.

More generally, we may always represent the surface-density of a uniform magnet by two layers of displacement (Fig. 165), the displacement being in the direction of the magnetisation.

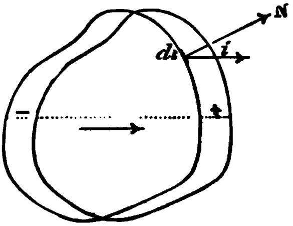

FIG. 165.

The quantity of magnetism which constitutes each of these layers is that which would cover the projection of the surface of the magnet on a plane perpendicular to the direction of magnetisation with a uniform density σ. That on an element, dS, with a projection, dS_1, is equal to σdS_1. If i is the angle which the perpendicular to the element dS makes with the direction of magnetisation, we have

$$dS_1 = dS \cos i$$

and therefore for the density σ' at dS

$$\sigma' = \sigma \cos i.$$

186. Intensity of Magnetisation.—Let M be the moment of the uniform magnet, and V its volume: put

$$A = \frac{M}{V}$$

A is the moment of unit volume. This quantity characterises the magnetisation; we shall call it ***intensity of magnetisation.***

If the magnet is not uniform, the quotient of M by V represents the mean intensity of magnetisation. In that case the ***intensity of magnetisation*** at a given point is the quotient of the moment of an element of volume cut out at this point by the volume itself. In general the intensity and direction of magnetisation vary from point to point.

Consider a cylinder of length L and of base S uniformly and longitudinally magnetised. Let A be the intensity of magnetisation, and σ the density of the layers which cover the terminal faces. The moment is equal to the product of the mass $S\sigma$ of each layer by the length L; but it is also equal to the product A by the volume SL. It follows that

$$\sigma = A.$$

This is also the value of the density at that point of the layer of displacement where the normal to the surface is parallel to the direction of magnetisation. At any point where the normal makes an angle i with the direction of magnetisation we have

$$\sigma' = A \cos i.$$

187. Magnetic Induction. — Suppose a long cylindrical bar magnetised uniformly, and let us suppose it bent so that the end faces, which, as we know, are covered with equal layers of opposite signs, are in contact. We thus obtain a ring-shaped magnet (Fig. 166), which, although it has retained its magnetisa-

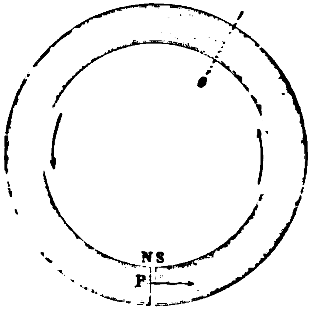

FIG. 166.

tion, is without action at external points. The magnetisation would only manifest itself if the ring were again opened, or if it were broken into pieces.

Let us now suppose that an infinitely thin layer, NS, is cut out at right angles to the magnetisation. One free face of the slit will be covered with a uniform layer of density $+A$, and the other with a layer of density $-A$. A unit positive pole placed at P in the empty space will be repelled with a force, $2\pi A$, by the positive face, and attracted with a force, $2\pi A$, by the negative face. The magnetic force in the slit is, therefore, equal to $4\pi A$. The flux, or the number of lines of force which represent it, is equal to $4\pi A$ per unit surface. It may be assumed that the lines which traverse the slit are only the continuation of the lines existing in the magnet; we shall call them *lines of magnetisation.*

If there were other centres of force, for instance, magnetic poles situate in the field, their action on the pole would be added to the action of the two faces, and would have to be compounded with the force $4\pi A$.

Let the ring be cut at any part, O, for instance (Fig. 166), and let it be stretched out so that it has again the shape of a cylinder (Fig. 167). Nothing is changed in the magnetic condition, except that the pole P, besides the force $4\pi A$, is also subject to the force due to the layers of density $\pm A$, which cover the two ends, a force which is obviously opposed to the former.

FIG. 167.

The pole P, situate in the infinitely narrow slit perpendicular to the magnetisation, is then under the action of two systems of forces, one depending on the intensity of magnetisation, which is equal to $4\pi A$; the other, which is due to the surface magnetisation and to the field, may be represented by F. These two systems produce a resultant which, as we shall afterwards see, can be directly determined by experiment. We shall call it the *magnetic induction*, and shall represent it by B.

If the two forces are in the same direction, we have

$$B = F + 4\pi A.$$

If the force F results solely from the surface magnetism, it is always in the opposite direction to $4\pi A$.

The pole which, if placed in a slit perpendicular to the magnetisation, is subject to the action of B, is only under the action of F if it is placed in a cavity which has the shape of an infinitely narrow cylinder with its axis parallel to the magnetisation (Fig. 167); in this case, in fact, the magnetisation of the medium only comes into play in so far as it is represented by two infinitely small magnetised surfaces forming the ends of the cavity, the action of which may be neglected.

188. Magnet of Uniform Induction.—In order that the induction B may be constant throughout the whole extent of the body, not only must the magnetisation A be uniform, but apart from the external field the action of the surface layer must be the same on all points of the interior.

This condition is realised only in very special cases; it is so in that of a sphere magnetised uniformly (§ 26); for an intensity of magnetisation A, the force due to the layers of displacement has the constant value

$$F = -\tfrac{4}{3}\pi A.$$

It is also realised with an ellipsoid of revolution magnetised uniformly in the direction of its axis (Fig. 168). If $2a$ is the axis of revolution, $2b$ that of the equator, the total mass of each layer is $\pi b^2 A$ (§ 185). The value of that which is comprised between the equator and a parallel plane situated at the distance x is $\pi A(b^2-y^2)$ or $\pi A b^2 \frac{x^2}{a^2}$; it is proportional to x^2, and may thence be represented by the surface of a triangle such as OAP. The centre of gravity is at two-thirds of the base, that is to say, at two-thirds of the axis counting from the centre. The moment is therefore $GG' \times \pi b^2 A = \tfrac{4}{3}\pi a b^2 A$; which is otherwise evident, seeing that $\tfrac{4}{3}\pi a b^2$ is the volume of the ellipsoid.

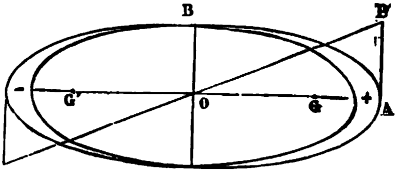

FIG. 168.

Calculation shows that the force due to the layer at a point in the interior may be expressed by

$$F = -NA,$$

N being a coefficient which only depends on the ratio of the axes, and which is smaller as the ellipsoid is more elongated. We have seen that in the case of a sphere it is equal to $\frac{4}{3}\pi$. For $a = 100\,b$, and $a = 500\,b$, calculation gives respectively $N = 0.0054$ and 0.0003.

Finally, the force F vanishes in the case of an annular magnet. This may also be assumed to be the case very approximately with a cylinder whose length is very great in proportion to its diameter. The action of the terminal masses may then be considered as negligable throughout the greater part of the length.

CHAPTER XVIII.

MAGNETIC INDUCTION.

189. Magnetisation by Induction.—Any piece of iron placed in a magnetic field becomes a magnet. The direction of the magnetisation is that of the lines of force, the south pole being at the part where they pass in, and the north pole where they pass out. This phenomenon is known as *magnetisation by induction.*

The phenomenon is molecular, and is analogous to the inductive electrification of an insulating body. It always precedes the attraction of iron by a magnet, so that attraction is, in reality, always exerted between two magnets which present to each other their opposite poles.

A filament of iron free to move, and not subject to any other action than that of the field, tends to take at each point the direction of the line of force. The figures of iron-filings are formed in this way ; each particle tends to place itself tangentially to the line of force and, the opposite poles of adjacent filings attracting each other, they arrange themselves in lines which mark out the lines of force.

190. Residual Magnetism—Coercive Force.—In a short piece of pure and unhardened iron, what is called *soft iron*, magnetisation is produced at once, and disappears when the inductive action is withdrawn. With iron which has been hardened, with impure iron, cast iron, steel, and particularly with hardened steel, the magnetisation produced by a given magnetising field is less intense, but in general it persists in greater or less degree after the cessation of the magnetising force. This is the origin of artificial magnets. The magnetisation, which lasts only as long as the inductive action, is called *temporary magnetism*, while that which continues after the inductive action has ceased is called *permanent*, or *remanent*, or *residual magnetism.*

The name *coercive force* has been applied to the property which certain materials thus possess of retaining, after the inductive action has ceased, a portion of the magnetism which this action had developed. Coercive force is, to a certain extent, a property analogous to friction, and constitutes a resistance to any change of magnetic condition. A method of estimating its numerical value will be given later (§ 197).

191. Magnetic and Diamagnetic Bodies.—Iron, including its varieties, is not the only body which is magnetised by induction. Nickel and cobalt have similar properties. These properties are also possessed, though to a less degree, by various compounds of iron, such as magnetic oxide of iron, Fe_3O_4, which constitutes loadstone, iron perchloride, and iron sulphate, whether solid or in solution. Indeed, when a more and more intense field and delicate means of observation are employed, it is found that there is probably no body, whether solid, liquid, or gaseous, which is not susceptible to the action of magnetism, and does not acquire by induction magnetic polarity. But in this respect bodies fall into two distinct categories: those of one class are acted on in the magnetic field like iron, their axis of magnetisation is along the lines of force (Fe, Fig. 169); those of the other class acquire an inverse magnetisation, that is, parallel, but in the opposite direction to the lines of force (Bi, Fig. 169). The term *magnetic* is applied to the former, and the term *diamagnetic* to the latter class of bodies. Diamagnetic properties are only shown to a very slight extent; the most strongly diamagnetic body is bismuth.

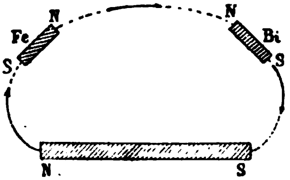

FIG. 169.

Magnetism is thus seen to be a general property of bodies, but it is extremely remarkable that while three substances—iron, cobalt, and nickel—possess it in a very high degree, it may be considered as in comparison non-existent in the others.

192. Coefficient of Magnetisation—Magnetic Susceptibility and Permeability.—The fundamental problem of magnetisation by induction is this: the magnetising force acting at a point of a piece of iron being given, to determine the intensity of the resulting magnetisation, or the value of magnetic induction at this point.

The magnetic force which is here to be considered is that resulting from the field and the reaction of the surface layer; it is that which would act on a pole placed at the point in question in a cavity in the shape of an infinitely narrow cylinder parallel to the magnetisation. We may call this the *magnetising force.* Let F be its value at the point in question, A the intensity of the magnetisation, and B the corresponding magnetic induction; we may put

$$\begin{aligned} A &= kF \\ B &= \mu F. \end{aligned} \qquad (1)$$

The coefficient k is called the *coefficient of magnetisation*, and defines the *magnetic susceptibility* of the body at the point in question; it is positive for magnetic, and negative for diamagnetic bodies. The coefficient μ is called the *coefficient of magnetic induction* and defines the *permeability.*

These two coefficients stand to each other in a very simple relation. The intensity of magnetisation, at any rate in isotropic substances, being necessarily in the same direction and the same sense as the magnetising force, we have (§ 187)

$$B = F + 4\pi A, \qquad (2)$$

and therefore from equations (1)

$$\mu = 1 + 4\pi k.$$

It will be seen that μ is greater than unity for magnetic substances, and less than unity for diamagnetic substances; since, for the latter, the absolute value of k is always very small, μ never becomes negative.

For diamagnetic substances, and for very slightly magnetic substances, k and μ may be considered as constant numerical coefficients, but for iron and similar substances they are very complicated functions. For a given substance at a given temperature they vary with the intensity of the magnetising force.

193. Influence of the Form of a Body.—Since experiment only gives the mean values of A and B for the whole mass of the body submitted to induction, in order to obtain k and μ it is necessary that these quantities, and therefore the magnetising force F, should have the same value for all points of the body. It is, therefore, not sufficient to place the body in a uniform field; it is also necessary that the surface layer developed by induction should exert either no force at all at any point within the body, or the same force at all points.

This force would be zero in the case of an anchor ring or tore acted on by a field in which the force was everywhere tangential to a circumference concentric with the ring.

We know that it is constant in the case of a sphere or of an ellipsoid of revolution with its axis parallel to the field; but then the magnetising force is no longer that of the field. Let H be the intensity of the field, A the intensity of magnetisation observed: the magnetising force F is the difference between the force of the field and the reaction of the surface layer, which we have represented by NA (§ 188), we have therefore

$$F = H - NA,$$

and as

$$A = kF,$$

it follows that

$$k = \frac{A}{H - NA}.$$

The magnetising force, expressed as a fraction of the force of the field, is given by

$$\frac{F}{H} = \frac{1}{1 + kN},$$

that is to say, it is smaller as N is larger. For a sphere $N = \frac{4}{3}\pi$ and for a value of k equal to 100, the magnetising force would not be $\frac{1}{400}$ that of the field. It would be necessary then to have recourse to very elongated ellipsoids of revolution. In an ellipsoid for which $a = 500b$, the magnetising force, with $k = 100$, would be 0.971 of that of the field (§ 188).

Most frequently very long cylinders are used, with a length 400 or 500 times the diameter, and these are placed in a uniform field parallel to the lines of force. In these conditions the action of the ends may be considered negligible.

194. Observations on Permeability.—A magnetic body placed in a uniform field, by the very fact of its magnetisation, produces a perturbation of the lines of force; near the body these cease to be parallel and equidistant; at the same time lines of magnetisation (§ 187) to the number of $4\pi A$ per unit surface traverse the body, the magnetisation of which we assume to be uniform. These lines may be regarded as the continuation of those of the field, excepting that they are packed more closely together, or are further apart, according as the body is magnetic or diamagnetic. Figs. 170 and 171 represent two spheres placed in a uniform field, and such that for the former $\mu = 2.8$, and for the latter $\mu = 0.48$. We may regard the state of the body submitted to induction as resulting from a modification of the medium which constitutes it, analogous to that which existed in the medium, air or vacuum, whose place it has taken, with this difference, that the flux of force for unit surface instead of being H has become equal to μH.

A comparison with currents will render the idea more precise.

Suppose a sphere of metal introduced into a mass of mercury traversed by a uniform current: the lines of flow, which were originally parallel, would tend to pass in greater number through the sphere if it were a better conductor than mercury, and, on the contrary, in smaller number if it were a worse conductor. Figs. 170 and 171 give a representation of the lines of flow in the two cases. The words *conductivity* for lines of flow, and *permeability* for lines of magnetic force, thus correspond to analogous ideas.

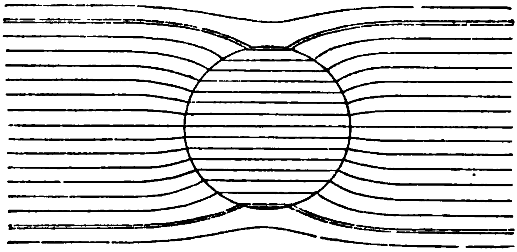

FIG. 170.

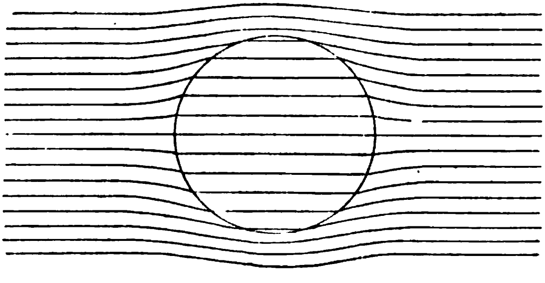

FIG. 171.

195. Curves of Magnetisation.—Suppose now that an infinitely long cylinder is placed parallel to the lines of force in a uniform field, and that then, while gradually increasing the strength of the field, we measure the intensity of magnetisation for every value of F, which, in this case, is the same as H. The results may be represented by a curve, the values of F forming the abscissæ, and those of A the ordinates. And since $B = F + 4\pi A$, or $= 4\pi A$ nearly for highly magnetic bodies, the same curve will represent the induction, provided we change the scale of the ordinates, and take the unit of length as indicating 4π times as great a value.

In the case of iron or its varieties, or cobalt, or nickel, the curve has always the same general appearance. Three parts may be distinguished: the first (Fig. 172), corresponding to very small magnetising forces, in which the intensity increases very slowly, but at first proportionally to the force; a second, where the curve rises very rapidly, and has a point of inflection to which the maximum value of k corresponds; lastly, a third part corresponding to high values of the magnetising force, and where the magnetisation increases very slowly and evidently tends to a maximum.

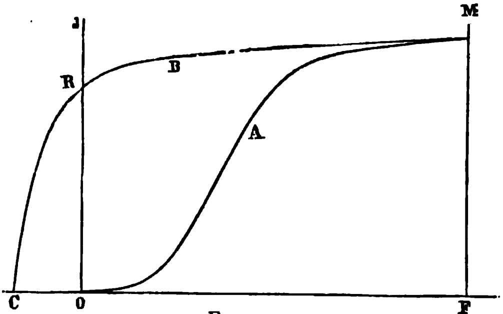

FIG. 172.

196. Remanent Magnetism.—Suppose now that after having raised the magnetising force to the value $F = $ OF, we make it gradually decrease, the magnetisation does not by any means again pass through the same values. The course of the phenomenon is represented by the curve B. When the magnetising force has become nothing, the bar has still an intensity of magnetisation represented by OR. This is called the remanent or residual magnetism. Its value is generally a large fraction of the total magnetism, and it is more or less stable according to the nature of the body.

197. Coercive Force.—If after having reduced the magnetising force to zero, it is again increased, but in the opposite direction, the magnetisation is quickly reduced to zero. The inverse force OC which just destroys the residual magnetism may be taken as a measure of the coercive force, and it is in this sense that we shall use this term in what follows.

198. Cycles of Magnetisation.—Imagine that after having exposed the bar to forces increasing from zero to F, the force is decreased from F to zero, then again raised to F, and so on successively. The results will be such as are represented by Fig. 173. The substance after having passed through the changes represented by the curve O to A in the first period, then from A to A′ along AMA′ in the second, will return from A′ to A by A′NA in the third. As the magnetising force continues to pass through the same cycle, the point which represents the state of the body at each instant will continue to describe the same loop AMA′N.

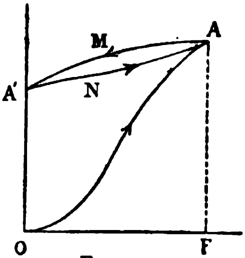

FIG. 173.

If instead of varying the magnetising force from zero to F and conversely, it is made to oscillate between two equal values of opposite sign $+F$ and $-F$, the curve representing the magnetisation is like Fig. 174. The curve OA corresponds to the initial magnetisation when the magnetising force increases for the first time from O to F; the portion ACA′ to decreasing values of the force from $+F$ to $-F$; finally the portion A′BA to increasing values from $-F$ to $+F$. The same cycle is gone through every time the same changes of magnetising force are repeated.

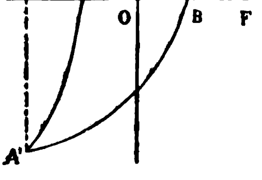

FIG. 174.

199. Retardation of Magnetisation—Hysteresis.—Thus, to the same value of the magnetising force, intensities of magnetisation correspond which depend not only on the existing conditions, but also on the previous ones. In particular, when the successive conditions form a cycle like that in Fig. 174, it may be noted that, for the same value of the magnetising force, the intensity is greater in the descending than in the ascending period. This may be expressed by saying that there is a *retardation* or lag of magnetisation in respect of the magnetising force. Professor Ewing, to whom most of these observations are due, has given the name *hysteresis* to this tendency of bodies to persist in a previous condition.

This lag may be regarded as an effect of coercive force. The two curves which form the cycle of Fig. 174 diverge more from each other as the coercive force, represented by OC, is greater. The lag is diminished if the body is subjected to vibrations while it is under the influence of the magnetising force. The general effect of the vibrations is to make the curve of increasing magnetisation rise more rapidly, and to make the curve of decreasing magnetisation fall more rapidly. One consequence of the effects of lag is that the coefficients k and μ, defined by equations (1) (§ 192), have no precise meaning, except for a body submitted to a definite magnetising force applied in a definite manner.

200. Neutral State.—If starting from any point, M, of the cycle, corresponding to the force F, we decrease the force to zero, and then increase it from zero to F, the curve will form a loop like AA′ in Fig. 173. By suitably choosing the point M, on the positive part of the ascending branch, or on the negative part of the descending branch, we may obtain zero magnetisation when the magnetising force becomes zero. Although there is no magnetisation, the condition of the bar is not identical with that which it had before being magnetised at all; it acquires more easily a magnetisation of the opposite sign to that which it has lost than a magnetisation of the same sign.

If the bar is to be restored to a symmetrical state, which is really neutral, it must be submitted to the action of forces alternately in opposite directions, gradually decreasing to zero; in other words, it must be made to pass through a series of cycles like that of Fig. 174, and corresponding to values of F gradually diminishing to zero.

For bodies with very small coercive force, the same result is obtained by repeated blows. The neutral state to which we attain by these methods is a constant one, but it is not necessarily that which the bar had before any magnetisation. A red heat also restores the bar to the neutral state.

201. Work of Magnetisation.—Let F be the magnetising force, and A the intensity of magnetisation: if the latter be increased by *dA*, the work for each unit of volume is that which must be expended in order to bring unit volume, with magnetisation *dA*, from an infinite distance to the point in question. But this work is equal to the product of the moment, *dA*, by the normal component of the force (§ 177), which is here the force *F* itself; it is thus equal to *FdA*. On the other hand, the product *FdA* is the area of the infinitely small trapezium *mnqp* (Fig. 175); it follows, therefore, that the work expended in causing each unit of volume to pass from the condition M to the condition N is represented by the curvilinear area MNQP.

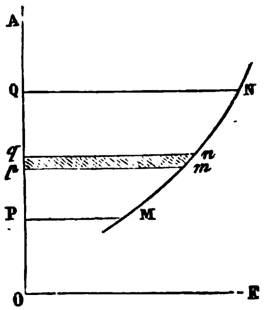

FIG. 175.

If the curve of demagnetisation coincided with the curve of magnetisation, the work corresponding to a cycle would be nothing, and we should therefore conclude that the work expended was altogether represented by the increase of the energy of the magnet. As the two curves do not coincide, owing to hysteresis, it is easy to see that the area which they enclose represents that portion of the work of magnetisation which has been expended against forces analogous to friction and can only be transformed into heat.

If the magnetising force and the intensity of magnetisation are expressed in C.G.S. units, the area measured in square centimetres will give the work transformed into heat expressed in ergs, and dividing by 4.18×10^7, we shall have the quantity of heat in gramme-degrees. If the density of iron be taken as 7.8, and its specific heat as 0.11, the thermal capacity of one cubic centimetre is 0.858. The rise of temperature for each erg will therefore be

$$\frac{1}{0.858 \times 4.18 \times 10^7} = 2.8 \times 10^{-8} \text{ degrees centigrade.}$$

In the case of Fig. 174 the area of the cycle, as readily seen, is sensibly equal to four times the product of the moment of the maximum magnetisation by the coercive force.

202. Numerical Values—Soft Iron.—Fig. 176 gives the curves of magnetisation for soft iron, steel, cast iron, cobalt, and nickel. They all show very definitely the three features mentioned above

(§ 195), but with differences in the general aspect, and particularly in the intensity.

For soft iron the curve rises with extreme rapidity; the point of inflection, which corresponds to the maximum of k, occurs for a value of the magnetising force equal to about 5 units. For ordinary iron k is then equal to about 150, but it may go up to 250 for very soft iron. For the sample represented, it is about 180.

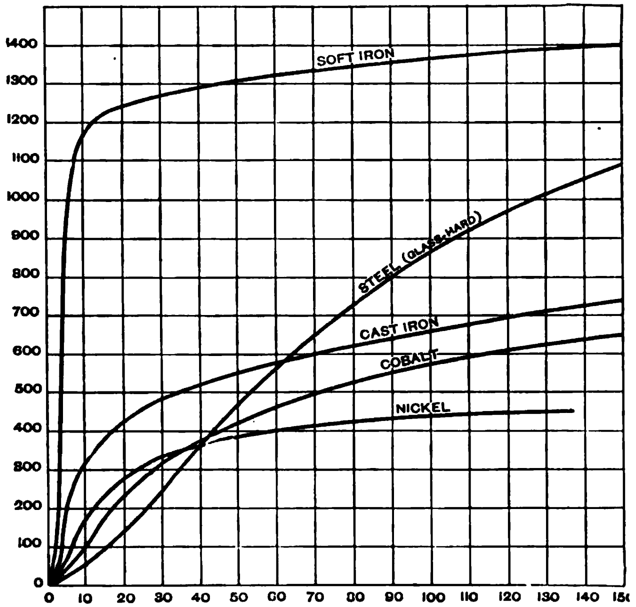

FIG. 176.

It will be seen that saturation is nearly reached for $F = 15$. This corresponds to a value of A of over 1200. It has been possible to reach 1700 by the use of very great magnetising force.

These curves show well the existence of a maximum, although they only refer to values of the field comprised between zero and 150 C.G.S. Experiment shows, moreover, that the intensity of magnetisation undergoes no appreciable variation when the field is altered from 2000 or 3000 units to the highest values which have been attained, about 20,000. The induction presents no maximum, for it consists of two parts, one $4\pi A$, which soon becomes constant, and the other, F, which increases without limit.

The residual magnetism may reach 90 per cent. of the total magnetism, but it is extremely unstable. If the reaction of the ends has a sensible value, or if the bar is submitted to vibration, the magnetism disappears to within a few hundredths. Thus practically we may say that the magnetism of soft iron is purely temporary.

The coercive force does not exceed 2.5, and the work corresponding to a complete cycle of magnetisation is about 15,000 ergs.

203. Cast-Iron and Steel.—The intensity of magnetisation of cast-iron is less than that of soft iron, and it less rapidly attains a maximum. For soft cast-iron the maximum is about 900, or about 1250 as an extreme value. The coercive force is about 12, and the work of the complete cycle of magnetisation varies from 40,000 to 60,000 ergs.

The curve for steel rises more gradually, and the numerical values vary greatly with the composition of the steel and its state as to hardening or annealing; but the maximum intensity which it can attain is not much lower than that for soft iron.

The coercive force is much greater. It increases with the proportion of carbon, but it is increased more particularly by the presence of certain metals. Chromium-steel or steel containing 1 per cent. of chromium has a coercive force of 40; steel containing 3 per cent. of tungsten a coercive force of 51. The work for a complete cycle is 40,000 to 60,000 for mild steel, 95,000 for piano-wire which has been annealed, and 116,000 for the same wire which has not been annealed. In like manner it is 65,000 for chromium-steel which has been annealed, and 167,000 for the same steel unannealed. A steel containing tungsten gave 216,000 ergs.

The presence of manganese takes from iron its magnetic properties; the alloy with 12 per cent. of manganese known as manganese-steel is not magnetic.

204. Cobalt—Nickel.—The curve for cobalt differs little from that for cast-iron. The maximum intensity of magnetisation is from 1000 to 1300; the coercive force 12, and the work corresponding to a complete cycle 40,000 to 50,000.

The maximum magnetisation for nickel is about 500; the effect of straining or annealing is much more marked with nickel than with iron. The work of a complete cycle of magnetisation, which is 25,000 for strained nickel, is only 11,000 for the same metal annealed.

205. Influence of Temperature—Critical Temperature.—Temperature has a considerable influence on magnetisation. The cardinal fact is that there is a temperature for the three magnetic metals beyond which they completely lose their exceptional properties, and become non-magnetic like ordinary bodies. This temperature is 785° for soft iron, 340° for nickel. It has not been determined exactly for cobalt, but is higher than for iron. The property disappears suddenly, and, so to speak, without any transition. Thus at 770° soft iron is at least 10,000 times more magnetic than at 785°.

At lower temperatures the permeability of iron undergoes very different variations, according to the value of the magnetising force. For very small forces, 0.3 for instance, it increases slowly with the temperature, then more rapidly, and a little below the critical temperature it attains a considerable maximum, disappearing suddenly a little above. For moderate and for great forces, it is virtually constant up to 500°, and then decreases gradually.

The critical temperature, which is 785° for soft iron, is only 735° for steel. It is lower as the proportion of carbon increases. The phenomenon is due to a profound change of state which at the same time modifies other physical properties. Thus the electrical resistance, which varies up to that point in an abnormal manner with an increasing coefficient, after passing the critical temperature has a constant coefficient which differs little from that of non-magnetic metals.

This change of state is also characterised by an absorption or a disengagement of heat, according as the temperature is rising or falling. When a piece of iron wire which has been heated to a very high temperature is allowed to cool, several points are observed at which there is an abnormal disengagement of heat corresponding to some change of state. Two such points are specially marked: one between 800° and 700° corresponds to the critical point of magnetisation and to a change of state of the iron; the other is at 660°, and has no relation to magnetism, but appears to be due to a modification of the carbon contained in the iron. At this latter point the quantity of heat disengaged is sometimes so great that a portion of iron wire which has sunk to a dull red heat again becomes incandescent.

A curious illustration of these changes of state is afforded by the alloys of nickel and iron. The alloy of 25 per cent. of nickel when raised to a temperature of 580° is no longer magnetic at lower temperatures; but when cooled to 20° below zero, it becomes magnetic, and remains so for all temperatures below 580°; so that at mean temperatures the same metal may exist in two different conditions, one magnetic and the other not so, both conditions being equally stable.

206. Bodies Slightly Magnetic or Diamagnetic.—So far as can be observed, the values of the coefficients of magnetisation are constant for bodies which are but little magnetic, or which are diamagnetic; the intensity of magnetisation is always proportional to the magnetising force. Moreover, no phenomena of hysteresis have been detected.

For diamagnetic bodies, the value of k is always very smal The highest known value is that for bismuth, for which

$$k = -\frac{1}{400,000}.$$

CHAPTER XIX.

EQUILIBRIUM AND MOTION OF MAGNETIC BODIES.

207. Equilibrium of a Magnetic Body in Field of Force.—An isotropic sphere placed in a uniform field is in neutral equilibrium in all positions. This is not the case with a sphere cut out of a crystallised body. The coefficient of magnetisation, like other physical constants, has different values in different directions, and the body has three principal coefficients, k, k', and k'', in three directions at right angles to each other. The sphere has, in this case, three positions of equilibrium, those in which one of the axes is parallel to the field. Of these three positions, one is stable; if the body is magnetic, the equilibrium is stable when the axis parallel to the field is the axis of maximum magnetisation; and if the body is diamagnetic, when the axis of minimum magnetisation is parallel to the field. If the sphere is movable about one axis, the couple which keeps it in a position of equilibrium is proportional to the difference of the coefficients relating to the other two axes.

208. Movements of very Small Bodies in the Field.—If the field is not uniform, the same conclusions apply still to a sphere so small that the field may be considered uniform throughout the space which it occupies; they apply also, at any rate very approximately, to a body of any shape provided it is very small; the mass of the surface layers is then very small, so that their action may be neglected; the magnetisation may consequently be regarded as uniform, and as due solely to the action of the field.

Let us consider an isotropic sphere of very small volume, u; let H be the intensity of the field, and k the coefficient of magnetisation; the sphere may be compared to an infinitely small magnet, with its axis parallel to the field, and of moment $M = ukH$ (§ 186). In the case of an *invariable* magnet the energy would be equal and of opposite sign to the flux of force which traverses from the negative side a surface perpendicular to the axis and numerically equal to the moment of the magnet. As the surface is hère perpendicular to the field $W = -ukH^2$. This would be the work required to bring this very small magnet from an infinite distance to the position which it occupies; but in the present case the moment is not invariable, and neglecting any effects of hysteresis, it may be assumed to be, at each point, proportional to the intensity of the field at that point. The average moment is thus the mean between o and the value corresponding to the final position; consequently, the work done, and therefore the energy acquired, is only half that which an invariable magnet would have required, and we have

$$W = -\tfrac{1}{2}ukH^2.$$

The body left to itself will remain in equilibrium in a uniform field, but in a variable field it will tend to expend the energy which it possesses, and, like a falling body, to move along the line of most rapid loss of energy. If k is positive, the energy diminishes when H increases; it will move then not in general along a line of force, as a single magnetic pole would do if that were realisable, but it will move in the direction along which the force varies most rapidly, and will ultimately come in contact with the magnet; it will be *attracted* by the magnet. If k is negative, as in the case of a diamagnetic body, it will move in the opposite direction, or in that in which the force diminishes most rapidly, and will seem to be *repelled* by the magnet. This is the true interpretation of the action exerted by magnets, or by electrified bodies, on bodies originally in the neutral condition and submitted to their influence. This special tendency of polarised elements to move, not in the direction of the lines of force, but towards a position of maximum or minimum, accounts for some peculiarities which are met with in the way in which iron filings arrange themselves under the influence of a magnet. The particles of iron outline the bar very sharply, and, except near the centre, they do not remain above the magnet itself (Fig. 150). The reason is that the strength of the field varies very rapidly near the magnet, and is a maximum at the edges of the bar. Hence, too, when the glass plate is gently tapped, the lines formed by the filings are displaced, they shrink together and move towards the magnet. Each granule of the filings moves parallel to itself, and transversely to the lines of force, towards regions of greater intensity.

The direct consideration of the forces leads, moreover, to the same result. Take a particle whose direction coincides with the line of force: the two forces applied at the ends are equal and contrary, but not directly opposite. They form with each other an obtuse angle equal to that between the tangents to the line of force at the two ends of the particle, and their resultant is normal to the line. This resultant is directed towards the concave side of the line of force if the body is magnetic, and towards the convex side if it is diamagnetic: it is evident that the first direction is that in which the force increases; the second that in which it decreases.

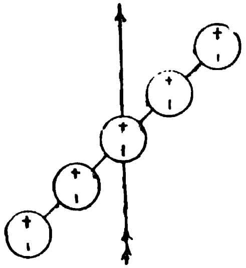

FIG. 177.

209. Equilibrium of an Elongated Body.—Let us now consider a body of an elongated shape, a cylindrical needle, for example, suspended freely in a uniform field. If each element of volume behaved as if it were alone, it would acquire a magnetisation parallel to the field, and the needle would be in neutral equilibrium in any position (Fig. 177). Experiment shows that this is the case for all feebly magnetic bodies, but that if the body is strongly magnetic the needle takes a direction parallel to the field.

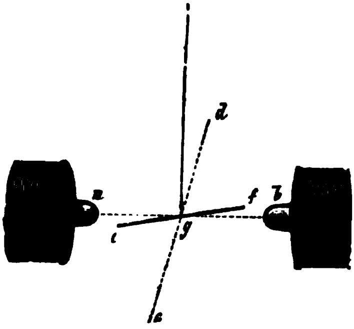

FIG. 178.

It behaves then like a needle having crystalline structure, the axis of figure corresponding with the axis of greatest magnetisation, and the reason is the same. The action which the several elements exert on each other in consequence of induced magnetism is greater when the needle is transverse to the field than when it is parallel, and the *apparent* coefficient is consequently smaller in the first case than in the second. The needle has thus a position of stable equilibrium, and the couple which maintains it there is proportional to the difference of the two apparent coefficients (§ 207).

210. Suppose now the needle is in a non-uniform field. If it is isotropic, and feebly magnetic or diamagnetic, it will merely obey the tendency of each element to move towards a maximum or minimum of force as the case may be. For instance, in a field symmetrical in reference to a centre like that between the opposite and equal poles *a* and *b* of an electro-magnet (Fig. 178), a magnetic needle, *ef*, will set along the lines of force, or *axially*, in the direction *ab*; while a diamagnetic needle will set across the lines of force, or *transversely*, in the direction *cd*; hence the term diamagnetic given to bodies which possess this property.

If the body is crystallised or strongly magnetic, it has to follow a twofold tendency, that which urges each element towards the points of maximum force, and that which tends to place the axis of greatest magnetisation parallel to the lines of the field. In the symmetrical field in question these two actions concur, and the position of equilibrium coincides with the line of the poles; but in more complex fields they may give rise to effects which are apparently very strange.

These phenomena are often known as *magneto-crystallic phenomena.*

211. Motions of Liquids and of Gases in a Non-Uniform Field.—The same effects are produced upon liquids and upon gases. Figs. 179 and 180 represent the effects produced by putting a drop of liquid in a watch-glass between two powerful equal and opposite poles. In the first case, the liquid is magnetic; it is heaped up towards the points where the force is a maximum; in the second it is diamagnetic, and moves towards points where the force is a minimum. A drop of liquid in a glass tube which is placed transversely between the two poles moves towards the centre if it is magnetic, and away from it if it is diamagnetic. In like manner the flame of a candle, which is diamagnetic, is repelled in the transverse plane (Fig. 181).

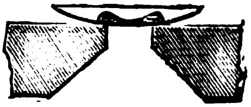

FIG. 179.

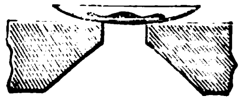

FIG. 180.

Among gases, oxygen is strongly magnetic; nitrous oxide, carbonic acid, ethylene, and cyanogen are diamagnetic; nitrogen and hydrogen seem to be neutral.

In order to demonstrate this phenomenon, a glass globe is suspended to the beam of a delicate balance over the pole of a powerful magnet. After the globe has been exhausted, it is counterpoised, and is then filled with the gas to be tested. In the case of oxygen, there is a very perceptible attraction, which is four or five times as strong as that exerted on air at the same pressure and temperature.

212. Relative Magnetisation.—A body of small dimensions and of permeability μ, placed in a liquid or gas of permeability μ', behaves like a body whose permeability is $\mu_1 = \frac{\mu}{\mu'}$, or, what comes to nearly the same thing, it acts as though the susceptibility were $k_1 = k - k'$.

This result leads to consequences analogous to those which are deduced from the principle of Archimedes for floating bodies. Three cases may present themselves: First, $k > k'$, the body behaves like a magnetic body; second, $k = k'$, the body appears neutral; and third, $k < k'$, the body behaves like a diamagnetic body. It is possible that diamagnetism might be explained by a cause of this kind.

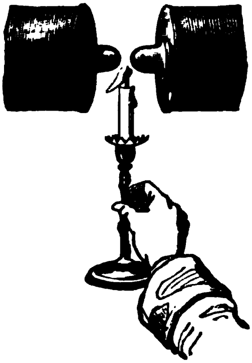

FIG. 181.

CHAPTER XX.

PERMANENT MAGNETS.

213. Permanent Magnets.—Permanent magnets are usually made in the form of straight prismatic bars, but they are also sometimes bent in the form of a horse-shoe. It is important that the metal of which they are made should have considerable coercive force. Highly tempered steel, especially such as contains tungsten, is found to answer best.

As the bars are usually short, the effect of the ends is considerable. Further, it is impossible to obtain uniform magnetisation in a prismatic or cylindrical bar. The effective magnetising force is less near the ends of the bar than it is at the middle; consequently the intensity of magnetisation diminishes towards each end, producing an apparent distribution of north magnetism throughout a greater or less portion of the bar near one end, and of south magnetism throughout a corresponding region near the other end. It has been pointed out already in § 185 that, if a cylinder were uniformly and longitudinally magnetised, there would be no apparent magnetism except on the end-faces. Consider the state of things on each side of one of the end-faces of a uniformly magnetised bar: the magnetic intensity has the uniform value A everywhere on the inner side, within the bar, and the uniform value 0 everywhere on the outside; but there is no magnetism either inside or outside the surface, only *on* the surface itself. The numerical value of the surface-density of magnetisation on the end of the bar is equal to A, the intensity of magnetisation of the bar. The condition which we express by speaking of a layer of magnetism on the end of a bar is the phenomenon by which we recognise the change of intensity of magnetisation on passing from one side of the surface to the other. If the transition from a finite magnetisation A to zero magnetisation takes place abruptly, we say that the magnetism is merely superficial; if the transition takes place gradually through a measurable distance, we say that magnetism is distributed through a corresponding portion of the mass of the magnet, the (volume-) density of magnetism being numerically equal, at any given point, to the rate of variation of magnetic intensity at that point. The result of this, in its application to an actual bar magnet, is that the poles, or centres of mass of the north and south magnetisms respectively, are within the bar at some distance from the ends.

Moreover, if the bars are thick, the metal is not homogeneous throughout; the hardening more particularly affects the superficial parts, and the magnetism seems to reside to a greater or less depth in this layer. If a bar is magnetised alternately in different directions, magnetic layers are formed which seem to superpose themselves. This fact can be shown by removing the surface layer either mechanically as by grinding, or chemically by dissolving the surface away with weak acid.

214. Distribution of Magnetism.—We have already spoken of the impossibility of ascertaining the internal constitution of a magnet from its external effects. The investigation of these effects, speaking generally, does not even determine the distribution of the surface layer by which the internal magnetism might be supposed to be replaced. This layer, in fact, is not in equilibrium according to the law of inverse squares, and consequently Coulomb's theorem cannot be applied; therefore, even if we knew the value of the normal component at each point, we could not deduce from it that of the density. Thus most discussions as to the distribution of magnetism in magnets are of little value.

Coulomb caused a very small magnetic needle to oscillate in front of various parts of the bar to be examined (Fig. 182). The needle, NS, was suspended by a cocoon fibre, and in order to make the oscillations slower it was fixed at right angles to a stout copper wire. The needle was first made to oscillate under the action of the earth alone, and then under the action of the earth and of the bar placed at a fixed distance in the plane of the meridian of the needle.

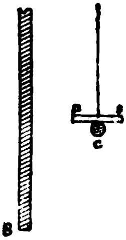

FIG. 182.

Having regard to the smallness of the needle, we may treat it as though it were suspended in a uniform field, and apply to it the formula for the pendulum. If F is the force exerted by the earth on one of the poles, and f that due to the bar, and if n and N respectively are the numbers of oscillations which the needle makes in the same time, when it oscillates under the action of the earth alone, and under that of the earth and magnet together, we have

$$\frac{N^2}{n^2}=\frac{F+f}{F}$$

and therefore

$$\frac{N^2-n^2}{n^2}=\frac{f}{F}.$$

If N' is the number of oscillations in the same time when the needle is opposite another point of the bar, we have

$$\frac{N'^2-n^2}{N^2-n^2}=\frac{f'}{f}.$$

The ratio $\frac{f'}{f}$ may thus be regarded as being that of the two normal components of the magnetic force at the corresponding points of the bar. In order to make the number obtained for the end of the bar comparable with the others, Coulomb doubled it, a correction which is certainly somewhat arbitrary.

In another series of observations Coulomb used the method of torsion. A long magnet, a knitting-needle for instance, was suspended horizontally by a metal wire, so that the wire was without torsion when the needle was in the magnetic meridian; he then measured the torsion which had to be applied to keep the end of the needle at a small fixed distance opposite various points of a long magnetised bar placed vertically in the meridian of the needle. This torsion would give an approximate measure of the normal component, if it could be assumed that the magnetism of the needle remains the same and does not vary when it is placed opposite different parts of the bar.

A third method consists in measuring the force needed to pull off a small sphere, or a small cylinder of soft iron placed in contact with different parts of the bar. To be able to assume, as is ordinarily done, that the effect measured is proportional to the square of the normal component, we must suppose that the coefficient of magnetisation of the test-sphere is independent of its magnetic intensity (§ 192), and that the presence of this sphere does not alter the magnetic state of the bar at the part which is being investigated.

We shall afterwards learn a method which gives much more correctly the value of the normal component at each point of the bar.

215. If at each point of the bar we draw ordinates proportional to the numbers found, we obtain a curve which may be called the curve of the normal components (Fig. 183); but which does not, as Coulomb thought, represent the surface densities. In particular, the points of the bar α and β which correspond to the centres of figure of the areas under these curves, are not the true poles of the magnet. A very simple example will show this: for a uniformly magnetised cylinder the method would still give a curve analogous to Fig. 183, which would fix the positions of the poles at α and β, whereas the surface layer is restricted to the terminal faces, and the true poles are therefore on the extreme ends.

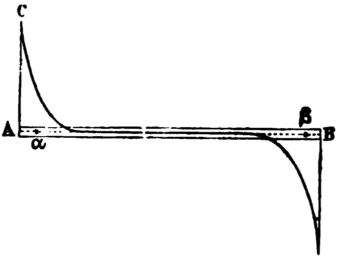

FIG. 183.

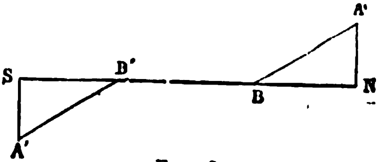

FIG. 184.

The curve resulting from experiment is approximately a triangle. In long magnets, that is to say, those in which the length is more than 50 times the diameter, the base, BN, of the representative triangle is about 25 times the diameter of the bar (Fig. 184). The centres of figure are at a fixed distance from the ends equal to 7 or 8 times the diameter. Experiment confirms this result. For bars which differ only as to length, the effective length may be expressed by $l - x$, where l is the length of the bar and x a constant.

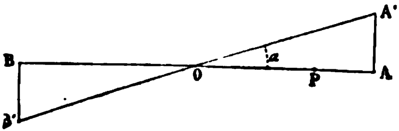

FIG. 185.

For magnets whose length is less than about 50 times the diameter, the distribution is represented approximately by the sloping line A′B′ (Fig. 185), which cuts the bar in the middle.

The points α and β are then at a distance from the ends equal to one-sixth the length.

216. Magnetic Moment of a Bar.—The moment, MH (§ 172), of the couple exerted on a bar by a field of uniform intensity, H, that of the earth, for example, is directly accessible to experiment.

One method is to suspend the magnet horizontally by placing it in a stirrup attached to a fine metal wire, which is without twist when the axis of the magnet is parallel to the field: this is the case if the direction does not change on replacing the magnet by a copper bar of the same weight. The angle through which the wire must be twisted to bring the magnet into a position at right angles to the field measures the couple, MH. If θ is the angle of twist, and C the coefficient of torsion of the wire, we have

$$MH = C\theta.$$

Instead of a metal wire, a bifilar suspension may be used (Fig. 186); in this case, if θ is the angle of torsion necessary to keep the bar in the transverse position, we have

$$MH = C' \sin \theta.$$

C' being a coefficient depending on the suspension; if $AA' = 2a$, $BB' = 2b$, $AB = l$, and if P is the weight of the suspended system, we easily find as an approximate formula

$$C' = P\frac{ab}{l}.$$

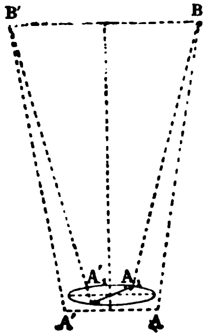

FIG. 186.

We may also deduce the couple, MH, from the duration, T, of the small oscillations of a bar suspended horizontally by a silk fibre without torsion. As the moment of the directive couple is at each instant proportional to the sine of the angle of divergence from the position of equilibrium (§ 172), and therefore, for small angles, to the angle itself, the magnet makes isochronous vibrations of which the period is

$$T = 2\pi\sqrt{\frac{K}{MH}},$$

where K is the moment of inertia of the oscillating body in respect of the axis of motion, we get thus

$$MH = 4\frac{\pi^2 K}{T^2}.$$

When once the product, MH, is found, we get the magnetic moment, M, of the bar on dividing by H, the intensity of the field.

217. Position of the Poles.—It is more difficult to determine the two factors of the product M, that is to say, the distance, $2a$, of the two poles, and the value, m, of the magnetic mass which we may suppose concentrated in each of them. The only regular method would be to determine m by applying Gauss's theorem (§ 41), that is, measuring the flux of force, $=4\pi m$, from the whole of each half of the bar, starting from the neutral line; from this the distance, $2a$, of the poles could be deduced. A knowledge of the exact position of the poles has, however, no practical importance.

218. Intensity of the Magnetisation of Steel.—If we divide the moment of a bar by its volume, we have the mean intensity of magnetisation. In ordinary bars this varies from 200 to 400 C.G.S. units; with very long thin bars it may attain 800, that is to say, almost half the maximum intensity of soft iron. Taking 7.8 as the density of steel, the *specific moment* or the moment per gramme of steel is from 25 to 50 for ordinary magnets, and it may reach a maximum of 100 C.G.S. units.

These numbers are, however, only averages; for the intensity, as has already been pointed out, is far from being uniform throughout the mass of the bar.

219. Influence of Hardening and of Annealing.—Hardening and annealing have considerable influence on the coercive force, which is greater the higher the temperature to which the steel has been raised, and the more suddenly it has been cooled. Very hard steel is magnetised with difficulty, but retains almost all the magnetism developed. Annealing lessens the effects of hardening. The steel for bar magnets is commonly tempered *blue*, that is, at the temperature at which steel acquires a blue tint in consequence of the oxidation of the surface.

Hardening and annealing have also a very marked influence on the electric resistance, and on the thermo-electric power of steel. The specific resistance at 0° varies from 15,000 C.G.S. for soft steel to 47,000 C.G.S. for glass-hard steel. The resistance thus furnishes a scale by which the condition of the steel may be defined. It can be ascertained in this way that the effects of annealing occur at relatively low temperatures, such as 100°. When a bar is kept at a given temperature, the effect of annealing increases with time, but tends towards a limit.

220. Influence of Temperature.—The moment of a magnet diminishes in general as the temperature rises. Part of the diminution is permanent, and the magnet does not regain its original moment when it returns to the original temperature. Experiment shows, however, that if a magnet is repeatedly heated to a given temperature for a sufficient length of time, and is each time remagnetised to saturation, its permanent magnetisation is ultimately unaffected by the action of any temperature below that to which it has been raised.

Steel magnets intended for instruments of observation, by being treated as above for a total time of thirty or forty hours, at the temperature of 100°, assume a condition in which the moment at a given temperature may be considered fixed. Within the limits of atmospheric variations, the effects produced are then proportional to changes of temperature. If M_0 and M are the moments of one and the same bar at the temperatures 0° and t°, we may put

$$M = M_0(1 - at),$$

a being a coefficient the value of which varies with different kinds of steel, but which is always less than 0.001.

221. Methods of Magnetisation.—In order to obtain a somewhat powerful magnetisation in steel, it is not sufficient merely to place it in a magnetic field; the particles must be shaken, as it were, so as to overcome the coercive force. Instead of causing a uniform field to act upon it, it is better to act in succession on the various parts. At the present day electric currents are almost exclusively used for magnetising large pieces of steel. Among the older methods the one in most frequent use is that of *double touch* (Fig. 187). The opposite poles NS of two strong bar magnets are applied in the middle of the bar to be magnetised, *ns*; the magnets are held at an angle of about 30°, and are moved simultaneously to the opposite ends of the bar. The operation is repeated several times on each face. A north pole, *n*, is produced at the end last touched by the south pole S, and a south pole, *s*, at the end left by the north pole N. The action is increased by allowing the ends of the bar to be magnetised to rest on the poles of two fixed magnets, these poles being of the same kind as the poles of the movable magnets acting on the same parts of the bar.

Instead of drawing the two magnets apart, they may be separated to a small fixed distance, regulated by a bit of wood put between them, and then moved together from the middle towards

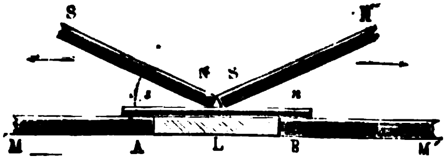

FIG. 187.

one end, then to the other, and so on backwards and forwards repeatedly, leaving off in the middle after giving each half the same number of passes.

222. Magnetic Batteries.—The magnetisation of steel being chiefly confined to the surface layers, in order to get large magnets strongly magnetised it is usual to construct them of thin plates, which are magnetised separately, and then put together. The

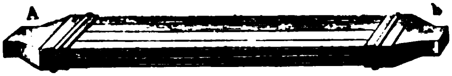

FIG. 188.

poles of the same kind all on the same side are usually placed in slits in pieces of soft iron, which become magnetised inductively, and give at the terminal faces poles of the same kind as those which they connect (Fig. 188).

The magnetic moment of a battery is far from being the sum of the moments of the plates of which it is made up, and if it is taken to pieces after some time, each of the magnets will be found to be weaker, but more especially the central ones.

This is obviously due to the action of the plates on each other, which tends to develop in each plate a magnetisation opposite to that of its neighbours.

223. Preservation of Magnets—Keepers.—In order to preserve magnets, great care must be taken not to let them be jarred, and not to expose them to their own demagnetising action when not in use. It is known that this action is nothing in a ring-shaped magnet, in which all the magnetic filaments are closed upon themselves, and are therefore without action both internally and externally (§ 180). In practice it is attempted to realise this condition as nearly as may be. Prismatic bars, as equal as possible, are placed two together parallel to each other in boxes, with opposite poles near each other, and joined by pieces of soft iron, which, so to speak, complete the magnetic circuit (Fig. 189). Such pieces of soft iron are called *Keepers*.

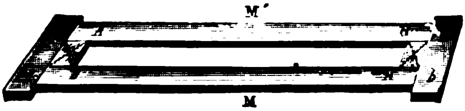

FIG. 189.

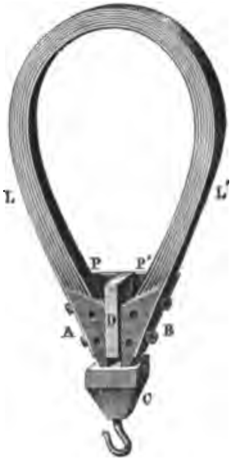

FIG. 190.

In horse-shoe magnets the two poles are joined by a single piece of soft iron, which forms a keeper (Fig. 190). The shapes of the keeper, and of the soft iron pole-pieces which terminate the *battery*, should be such as to make the magnetic intensity as nearly uniform as possible all round the circuit (§ 213).

Fig. 190 represents a horse-shoe magnet formed by Jamin's method of flexible plates of steel magnetised separately, then fitted into pole-pieces, A and B, of soft iron. A piece of soft iron, C, completes the circuit, and acts as a keeper.

It is often said that a magnet should support a permanent weight in order to retain its magnetism. By gradually increasing this charge a magnet may ultimately be made to support a weight far greater than it would do at first. When contact is broken, owing to the weight being too great, the carrying power falls most frequently below its original value.

224. Lifting Power.—The lifting power of a magnet can easily be calculated when the magnetisation is uniform. If we suppose that the two opposed surfaces have densities $\pm \sigma$ equal to A in absolute value, the attraction for each unit of surface is $2\pi A^2$ (§ 44). For the total surface of contact S, the force in dynes is $2\pi A^2 S$, and if P is the number of grammes whose weight would just pull off the keeper, we have (§ 187)—

$$gP = 2\pi A^2 S = \frac{B^2 S}{8\pi}.$$

If we take $A = 800$ for steel, we get as the greatest value for P, 4 kilogrammes per square centimetre; this is, in fact, what is given by the best magnets. With soft iron raised to saturation, 10 or 11 kilogrammes per square centimetre is easily obtained.

225. Magnetisation by the Action of the Earth.—It is very rarely that we find a piece of steel or iron, unless perhaps it is perfectly pure soft iron, which does not present magnetic polarity. Magnetisation is produced in it if it is struck, or undergoes any other mechanical disturbance while under the influence of the earth. A bar of soft iron which is placed parallel to the earth's field acquires a temporary magnetisation, which is reversed when the magnet is turned end for end. But if the bar is struck with a hammer, the magnetisation is raised to a maximum, and the iron at the same time acquires perceptible coercive force.

A fresh blow on the bar, when placed transversely to the magnetic field, takes from it its magnetisation. A strand of soft wire twisted, while being held in the direction of the dipping-needle, also acquires permanent magnetisation.

The value of k (§ 202) is about 40 for the intensity 0.47, which is that of the earth's field at London. A very long, thin, soft iron wire, for example, 1 millimetre in diameter, and 50 centimetres long, acquires a magnetic intensity of about $0.47 \times 40 = 18.8$; and, as its volume is nearly 0.40 cubic centimetre, its moment is $18.8 \times .4 = 7.5$ (§ 186).

226. Theory of Magnetisation.—It is proved by experiment that magnetisation is a molecular phenomenon (§ 178). The simplest idea is to assume that the molecules of magnetic bodies are small magnets. In the neutral state these small magnets have no external action, either because they form closed chains, or because they are arranged indiscriminately in all directions. The process of magnetisation has the effect of arranging the molecular magnets in some predominating direction, and the maximum is reached when their axes are all parallel, and their similar poles in the same direction.

It remains to establish the theory of the very complicated phenomenon of magnetism on this basis. Professor Ewing's experiments throw much light on this subject. He arranges a great number of very small magnetic needles movable on pivots, at small regular distances apart. Experiment shows that these small magnets may assume a great number of different stable configurations that do not exert external force. These configurations represent so many neutral states which are not identical with each other.

If one part of the system is disturbed it falls into a different configuration; each single magnet takes up a position of stable equilibrium after oscillations of greater or less amplitude. These oscillations represent the loss of energy which the system experiences in passing from the first configuration to the second. The change in general is not reversible.

Now, suppose the system exposed to the action of a uniform field gradually increasing from zero, the magnets are at first but slightly deflected, and if the directive force is removed, they revert to their first positions, and there is no permanent effect. As the field continues to increase, a point is reached at which the equilibrium is suddenly broken, and the system falls into a new configuration in which all the elements have approximately the same direction as the field. From this point the effect of an increase in the strength of the field is only to make the arrangement more complete.

These three phases correspond to the three parts of the curve of magnetisation (§ 195).

If now the strength of the field is decreased, the system does not pass through the same states; it tends towards whatever condition of stable equilibrium is nearest to the existing configuration. Thus without any hypothesis of a coercive force analogous to friction, we may explain the effects of hysteresis and of residual magnetism.

Experiment shows that if the system is homogeneous, that is to say, if the small magnets are uniformly distributed, the transition to quasi-parallelism is made suddenly. This is what happens in the case of soft iron. If the system is not homogeneous, and the magnets are unevenly grouped, they do not equally and simultaneously obey the external action; the magnetisation curve becomes elongated, and does not so soon attain a maximum; on the other hand the residual magnetism is more stable. This is the case of steel.

The effect of heat may be accounted for if we assign to it a double effect, that of increasing the distance of the elements, and of making them oscillate. Finally, in order to account for the disappearance of magnetic properties, Professor Ewing assumes that the oscillatory motion being gradually amplified changes into a continuous rotating one.

This theory also accounts very satisfactorily without any fresh hypothesis for the complicated relations of magnetism with the mechanical actions of stretching and torsion.

CHAPTER XXI.

TERRESTRIAL MAGNETISM.

227. Terrestrial Field.—The field of force due to the earth, though practically uniform throughout any moderate space that is not affected by the presence of magnetic bodies, varies greatly both in intensity and direction from one place to another on the earth's surface; and even in one and the same place it changes in course of time.

We know that its action on a magnet is that of a couple (§ 171). In order that a magnet may indicate the direction of the force at a given point, it must be acted upon by the earth's force independently of any other; the direction which the magnetic axis then takes is that of the field due to the earth.

In this part of the world this direction is roughly north and south, but it makes a large angle with the horizon, the north pole pointing downwards.

We may define this direction by means of two angles, the *declination*, and the *inclination* or *dip*.

The *magnetic meridian* is the vertical plane which contains the direction of the earth's force.

The *declination* is the angle which the magnetic meridian makes with the astronomical meridian.

The *inclination* or *dip* is the angle which the direction of the earth's force makes with its projection on the horizontal plane.

The declination, D, the inclination, I, and the intensity, T, of the field are three elements which characterise the earth's magnetism in a given time and place.

228. It is not practicable to construct an instrument in which a magnet is suspended freely by its centre of gravity. In practice, two instruments are employed, one in which the magnet is movable only about a vertical axis: this is called a *declination compass;* the other, in which the needle moves only about a horizontal axis passing through its centre of gravity : this is the inclination compass or *dip needle.*

Suppose the magnet in any position (Fig. 191), and let α be the angle which the vertical plane, OA, containing the axis makes with the magnetic meridian, OM ; the force, T, which acts on each unit of magnetism of the pole at O, may be resolved into two components independent of the angle α ; the one, vertical, $Z = T \sin I$; the other, horizontal, $H = T \cos I$; this latter may be again resolved into two others, also horizontal, namely, $X = H \cos \alpha$, situate in the plane of the magnet ; the other, $Y = H \sin \alpha$ perpendicular to this plane. We have thus—

$$X = T \cos I \cos \alpha,$$
$$Y = T \cos I \sin \alpha,$$
$$Z = T \sin I.$$

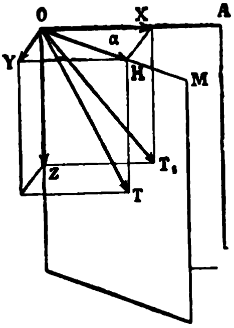

FIG. 191.

229. Declination Compass.—If the needle is movable only about a vertical axis, it is only acted on by the horizontal component ; its position of equilibrium is that in which its axis is in the magnetic meridian. When it is deflected through an angle α, the couple which acts on the needle and tends to bring it to its position of equilibrium is $MH \sin \alpha$; M being the magnetic moment of the needle. This couple is proportional to the sine of the deflection ; consequently, the law of the motion of the needle when disturbed and left to itself is that of the pendulum (§ 216).

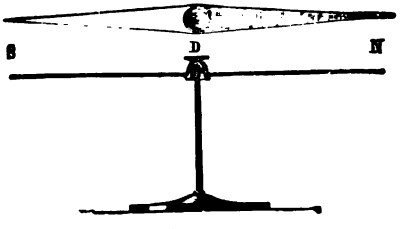

FIG. 192.

In order to realise the fundamental condition of the apparatus, it is not needful for the vertical axis of rotation of the magnet to be rigid. It may be suspended, when properly balanced, by a cocoon thread, or it may be supported by an agate cap, *D*, on a pivot (Fig. 192).

If the needle is suspended at its centre of gravity, and we load the south end, S, by a mass, *p*, placed at a distance, *d*, from the point of suspension, such that

$$pgd = MZ = MT \sin I,$$

the needle will rest horizontally in any azimuth, since the vertical component is independent of *a*.

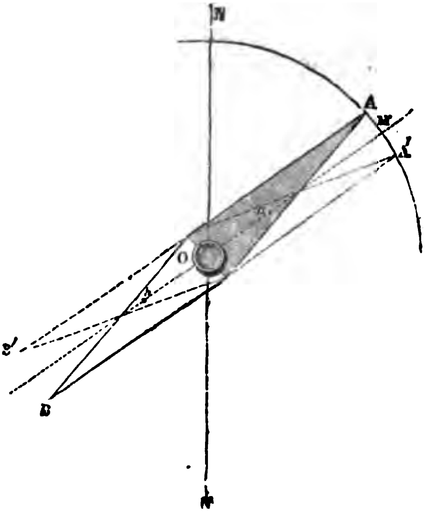

FIG. 193.

The measurement of declination consists in determining the astronomical meridian, and measuring the angle which the magnetic axis of the needle makes with this plane. In reality the angle observed is that between the meridian and the *axis of figure* of the needle, and this does not necessarily coincide with the magnetic axis. The correction for this is readily made by inverting the face of the needle (Fig. 193); the magnetic axis, *ab*, takes the same direction in each case, and the axis of figure assumes symmetrical positions, AB, A'B', with respect to this direction, first on one side and then on the other. The mean of the two angles observed, AON, A′ON, gives the required angle, MON.

Conversely, if the declination is known, it is easy to find the astronomical meridian. When the needle is at rest, the vertical plane, which makes an angle equal to the declination, east or west of the north pole, according as the declination is west or east, is the astronomical meridian.

Fig. 194 represents Brunner's declination compass. It is a theodolite which at E supports a magnet suspended horizontally by a cocoon fibre without torsion. The first thing is to determine the geographical meridian by observing the sun or a circumpolar star by means of the telescope F; the instrument is then turned about a vertical axis until the end of the magnet is bisected by the spider-line of the microscope, M. The angle turned through, which is read upon the horizontal circle A, gives the declination. In order to eliminate errors, both ends of the magnet are observed,

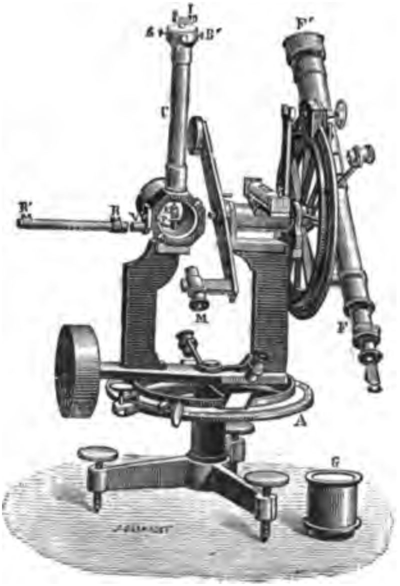

FIG. 194.

and the observations are repeated when it has been turned with the other face up.

230. Dip Circle.—For the determination of magnetic dip, a needle that can turn about a horizontal axis, therefore moving in a vertical plane, is used: a needle thus arranged is acted on only by the component of the earth's force which lies in the plane of its motion.

Let a be the angle which this plane makes with the magnetic meridian: the effective components are Z and $H\cos a$, which have a resultant

$$T_1 = \sqrt{Z^2 + H^2\cos^2 a}.$$

This makes with the horizontal an angle i given by the equation

$$\cot i = \frac{H\cos a}{Z} = \cot I \cos a.$$

This direction i is that which the needle will take; it may be called the *apparent dip* in the azimuth a. For $a = 0$ we have $i = I$, and the apparent dip is thus the true dip. If $a = \frac{\pi}{2}$ we get $\cot i = 0$, that is, the needle is vertical.

For angles a and $a \pm \frac{\pi}{2}$, which differ from each other by $\frac{\pi}{2}$, the apparent dip takes values i and i' respectively, which satisfy the equations

$$\cot i = \cot I \cos a$$
$$\cot i' = \pm \cot I \sin a$$

squaring both sides and adding, we get

$$\cot^2 i + \cot^2 i' = \cot^2 I,$$

a formula which is frequently used in determining the dip.

These equations show that in a complete rotation about a vertical axis, OP (Fig. 195), the axis of the needle will trace on the horizontal plane a circumference whose diameter is equal to OP cot I.

Fig. 196 represents *Brunner's dip circle.* The needle is a strip of steel in the shape of an elongated acute-angled lozenge, traversed in the centre by a cylindrical steel axis, which rests on agate edges, carefully adjusted to be in the same horizontal plane.

The reading is made by an arm, M, which is moved along the limb of the circle, and at each observation is set exactly parallel to the needle. With this object the arm is furnished with two small concave mirrors with their centres of curvature in the plane of the needle. The mirrors thus give in this same plane real and inverted images of the points of the needle, and the arm is moved until these reversed images are brought into contact with the points themselves.

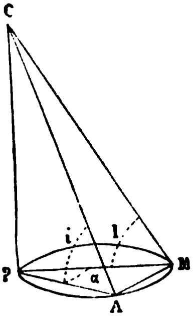

FIG. 95.

In making an observation, the vertical circle is brought into the magnetic meridian, and the angle *I* is directly measured; in order to determine the plane of the meridian, the position is found at which the needle is vertical, and the circle is then turned through 90°.

The two angles *i* and *i'*, which correspond to two positions of the circle at right angles to each other, are also often determined.

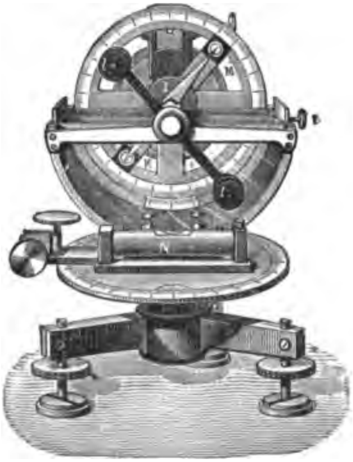

FIG. 196.

As we have seen above, the sum of the squares of the cotangents of these two angles gives the square of the cotangent of the dip.

Several causes of error must be eliminated. That due to possible eccentricity of the axis of rotation is eliminated by reading the two ends of the needle; error of the zero of graduation of the circle, by turning it through 180°; that due to obliquity of the magnetic axis, by reversing the face of the needle. There may also be an error arising from the fact that the centre of gravity is not exactly in the axis of rotation. This is eliminated by re-magnetising the needle so that the poles are reversed, and recommencing the whole series of observations. The measure of the dip in a given azimuth is thus the mean of sixteen readings.

231. Measurement of Intensity—Gauss's Method of Deflection.—The horizontal component H is measured and the total force T is deduced from it by the formula—

$$T = \frac{H}{\cos I},$$

I being the dip.

The method consists in determining for the same bar the product $MH = A$ and the quotient $\frac{M}{H} = B$; whence—

$$M = \sqrt{AB} \text{ and } H = \sqrt{\frac{A}{B}}.$$

Determination of MH.—The product MH is obtained either by the method of torsion or by that of oscillation. In the latter case we have—

$$MH = \frac{4\pi^2 K}{t^2},$$

K being the moment of inertia of the bar relatively to the axis of rotation, and t the period of a vibration of infinitely small amplitude; this is deduced from the observed period, t_1, by a well-known formula—

$$t = t_1\left(1 - \frac{\alpha^2}{16}\right),$$

α being the amplitude of oscillation in circular measure.

Measurement of $\frac{M}{H}$.—The bar is made to act on a very small magnetic needle, the deflections of which are observed by reflexion (§ 80). The bar is placed in one of the principal positions (§ 176), with its axis perpendicular to the magnetic meridian

(Fig. 197). In the first position the line joining the centres of the magnet and needle is perpendicular to the meridian; in the second position, the line joining the centres is in the meridian.

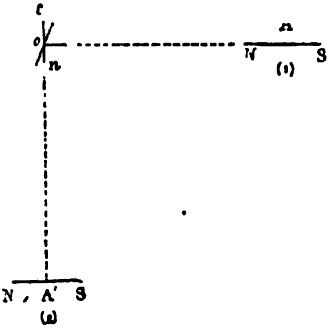

FIG. 197.

If the bar were infinitely small in comparison with the distance OA = R, the force which it would exert on unit pole at O would be (§ 176)—

$F_1 = 2\frac{M}{R^3}$, in the first position;

$F_2 = \frac{M}{R^3}$, in the second position.

If the needle itself is very small, the field may be considered uniform throughout the space which it occupies, and the moment of the couple which tends to deflect it from its position of equilibrium is, for the first position, $M'F_1 = 2M'\frac{M}{R^3}$. Let α be the deflection of the needle, the condition of equilibrium is—

$$2\frac{MM'}{R^3}\cos\alpha = M'H\sin\alpha,$$

whence

$$\frac{M}{H} = \frac{1}{2}R^3\tan\alpha.$$

If the magnet cannot be treated as though it were infinitely small, a rather more complicated formula is required—

$$F = 2\frac{MM'}{R^3}(1 + f).$$

Without entering upon a discussion of the general case, it is easy to take account of the dimensions of the magnet and needle, so far as this is required for the present purpose. Putting L for the half-length of the magnet at A (Fig. 197), m and $-m$ for the strength of its poles, and R for the distance of the point o from the middle of the magnet, we have $R - L$ and $R + L$ for the distances No and So respectively, and, therefore, for the intensity of magnetic force at o due to the magnet—

$$F = \frac{m}{(R-L)^2} - \frac{m}{(R+L)^2} = \frac{2MR}{(R^2 - L^2)^2},$$

where M is put for $2\,Lm$, the moment of the magnet. On writing out the denominator by the binomial theorem, this comes into the form—

$$F = \frac{2\,M}{R^3}\left(1 + \frac{2\,L^2}{R^2} + \frac{3\,L^4}{R^4} + \ldots\right),$$

and as L is necessarily less than R, and, in all cases employed in practice, a somewhat small fraction of it, all the terms in the bracket after the second may be neglected. It is to be observed that L in the above expression is not the half-length of the steel bar, but half the distance between its poles, half its magnetic length, a quantity which we have no means of measuring directly. Again, in a complete calculation we should find that the dimensions of the needle suspended at o would affect the numerators of the correcting terms in the value of F. For this reason, again, the above formula cannot be used directly. Reducing it to two terms and putting p^2 for the unknown numerator of the second term, we may write—

$$F = H \tan \alpha = 2\,\frac{M}{R^3}\left(1 + \frac{p^2}{R^2}\right).$$

By observing the deflection α at distance R, and also the deflection α' corresponding to a second distance R', we obtain two equations :—

$$\tan \alpha = 2\,\frac{M}{H}\left(\frac{1}{R^3} + \frac{p^2}{R^5}\right),$$

$$\tan \alpha' = 2\,\frac{M}{H}\left(\frac{1}{R'^3} + \frac{p^2}{R'^5}\right).$$

Eliminating p between these, we finally get the value—

$$\frac{M}{H} = \frac{1}{2} \cdot \frac{R^5 \tan \alpha - R'^5 \tan \alpha'}{R^2 - R'^2}.$$

It is best to take the two distances nearly in the ratio of 3 to 4. In practice, each of the angles α and α' is deduced from the mean of four observations,—with the magnet east and west of the needle, and in each case with first one pole and then the other turned towards the needle.

232. Magnetic Charts.—The terrestrial magnetic elements vary from one point of the globe to another.

The simplest plan of showing the results is to record the numbers found on a geographical map, and to draw curves so as to connect all those places where the declination, the dip, and the intensity respectively have the same value. In this way we get *isogonic*, *isoclinal*, and *isodynamic* lines.

When traced on a globe, isogonic lines resemble roughly meridional lines, and isoclinal lines roughly resemble parallels, both referred to the same axis, which is slightly inclined to the geographical axis, and cuts the surface at two points called the *magnetic poles*. At these points the dip is 90°, that is, the dipping-needle stands vertical. The term *magnetic equator* is given to the isoclinal line where the dip is zero.

An interesting isogonic is that of no declination, or, in other words, that in which the magnetic and geographical meridians coincide. It cuts the old world from the North Cape to the Persian Gulf, and passes through Western Australia. In the other hemisphere another such line crosses the eastern part of Brazil, North America, Florida, and Lake Superior; the surface of the globe is thus divided into two parts: in one, which may be called the Atlantic region, the declination is westerly; in the other, the Pacific region, it is easterly.

The *isogonic* lines traverse the British Islands approximately from S.S.W. to N.N.E. The declination is least in the extreme south-east, being about 16° at the South Foreland, and greatest on the north-west coast of Ireland, where it is about 23°. It has very nearly the same value, about 19°, at Penzance, Lancaster, and Berwick-on-Tweed.

The general direction of the *isoclinal* lines is from about 18° S. of W. to 18° N. of E. The extreme values are rather less than 67° on a line through Eastbourne and Dungeness, and nearly 73° in the Shetlands.

The lines of *equal horizontal force* run from about 16° S. of W. to 16° N. of E. The extreme values are about 0.186 at Beachy Head and 0.148 in the Shetlands.

For London (Kew Observatory, lat. 58° 28′ 6″ N., long. 0° 18′ 47″ W.) the mean values of the magnetic elements for 1894 were as follows:

		Approximate Annual Change.
Declination	17° 23′ W.	−7′
Inclination	67° 24′ N.	−2′
Horizontal force	0.1825	+0.0002
Vertical force	.4384	− .0003
Total force	.4749	− .0002

233. Law of the Distribution of Terrestrial Magnetism.—As a first approximation, and considering only the broad features, the distribution of magnetic force on the globe admits of a very simple statement, it being the same as would result from a very short magnet, *ns*, situated at the centre of the earth, and with its axis, PP′, inclined at an angle of about 15° from the axis of rotation, and with its *north* pole in the southern hemisphere (Fig. 198).

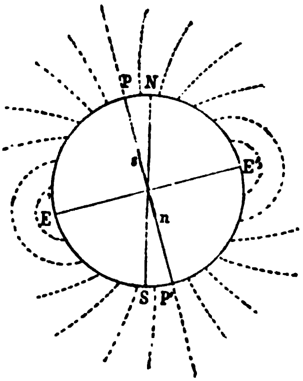

FIG. 198.

The properties of an infinitely small magnet (§§ 175, 176) enable us to trace out directly the consequences of this hypothesis.

The field is symmetrical in respect of the axis PP′ of the magnet; that is, the magnitude and direction of the force are the same at all points of any circle perpendicular to PP′; the circle which passes through the centre is the magnetic equator EE′; the others are magnetic parallels. At all points of the magnetic equator the force is horizontal and the dip zero; the inclination, for a parallel of latitude λ reckoned from the magnetic equator, is given by formula (4) (§ 176)—

$$\tan I = 2 \tan \lambda,$$

and the total intensity by the formula (3),

$$T^2 = T_e^2 (1 + 3 \sin^2 \lambda),$$

T_e being the intensity at the equator. The intensity is doubled on passing from the equator to the points P and P′, where the axis of the magnet cuts the surface of the earth. At these points, which are sometimes called the *terrestrial magnetic poles*, the force is vertical.

Any great circle passing through the axis PP′ is a *magnetic meridian*. The declination at a point M (Fig. 199) is the angle which the great circle, through this point and the axis, makes with the geographical meridian. It varies from point to point along the same meridian, except for the great circle which passes through the two axes PP′ and NS. This is both the geographical and the magnetic meridian, and at every point of it the declination is zero, and the needle points exactly north and south. This circle forms the periphery of Fig. 199. On one side of this plane the north pole deflects towards the west, and the declination is westerly; on the other side it is deflected to the east, and the declination is easterly.

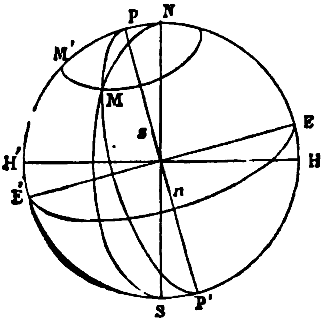

FIG. 199.

The magnetic moment ϖ of the globe is given by formula (1):

$$\varpi = T R^3.$$

Taking $T_e = 0.33$ and $R = 6.37 \times 10^8$ centimetres, we have

$$\varpi = 8.5 \times 10^{25} \text{ C.G.S.}$$

The hypothesis of an infinitely small magnet is equivalent to that of two layers of displacement (§ 26), which again is equivalent to that of a uniform magnetisation in the direction of the axis PP′ (§ 185). The intensity of magnetisation will then be

$$A = \frac{\varpi}{\frac{4}{3}\pi R^3} = \frac{T_e R^3}{\frac{4}{3}\pi R^3} = \frac{3}{4\pi} T_e = 0.079.$$

That is to say, about $\frac{1}{8500}$ the intensity of magnetisation of a magnet of average strength (§ 218).

234. Gauss's Theory.—The preceding theory only gives a rough approximation. Gauss has treated the problem more generally supposing the magnetic masses which produce the terrestrial field to be distributed in any manner.

Whatever be the distribution of these masses, they give at each point a determinate potential. Let us suppose equipotential surfaces drawn in the field corresponding to equidistant values of the potential. These surfaces cut the surface of the earth along lines which we may call ***magnetic parallels.*** These lines have the property of being everywhere perpendicular to the magnetic meridians; for, being drawn on the surface of the earth, which is supposed spherical, they are perpendicular to the vertical, and since they lie in the equipotential surfaces, they are perpendicular to the direction of the force. These lines are ***equipotential lines*** in reference to the horizontal component; at each point this component is normal to the line, and its mean value varies inversely as the distance between two adjacent lines.

The magnetic equator corresponds to the surface $V = 0$, which, unless the distribution is far from being symmetrical, will pass near the centre: the equator separates those points of the surface for which the potential is positive from those for which it is negative; the dip along this circle is not necessarily zero, nor the force constant. Thus, the magnetic parallels are not necessarily either lines of constant dip nor lines of constant force.

The points of the surface where the force is vertical, and which are erroneously called the poles, are those in which the surface of the globe is a tangent to the equipotential surface which meets it; these are the points of the surface for which the absolute value of the potential is a maximum. Thus, in order to know the distribution of magnetism, it is sufficient to draw the equipotential lines for the surface of the globe. Gauss showed that in the most general case, and within the degree of accuracy of the observations, these lines may be algebraically expressed by formulæ containing twenty-four coefficients; so that when these coefficients have been calculated once for all, by means of an equal number of observations corrected for local disturbances, it is only needful to introduce the geographical co-ordinates of any given point into the formulæ to obtain the value of the magnetic elements at this point.

The calculations of Gauss made for the year 1838 fix the following positions for those of the two so-called poles:—

North pole, 73° 35′ lat., 97° 59′ long. west from Greenwich.
South pole, 72° 35′ lat., 150° 10′ long. east from Greenwich.

It will be seen that they are a long way from corresponding to the ends of one and the same diameter.

235. Variations of Terrestrial Magnetism.—The elements of the earth's magnetism are not constant at any given place, but undergo variations in the course of time. Among these variations some appear to be accidental, while others, on the contrary, have a well-defined periodic character.

Secular Variations.—Since the date of the earliest exact observations, slow changes have been found to be going on, which may be approximately represented by supposing a continuous uniform rotation of the magnetic axis about the terrestrial axis to be taking place in the direction of the hands of a watch, for an observer placed at the north pole, the period of a complete rotation being about 900 years (Fig. 200). Thus at Paris, which is represented in the figure by P, the declination, which was easterly when first recorded, was zero in 1666; since this time it has been westerly, and went on increasing until 1824, when it amounted to 24°; at present it is decreasing, and may be expected to become nothing again about the year 2114. The magnetic pole will then be on the other side of the north pole in respect to us. As to the dip, it has diminished steadily from 1666, and, on the same hypothesis, will continue to do so until about 2114.

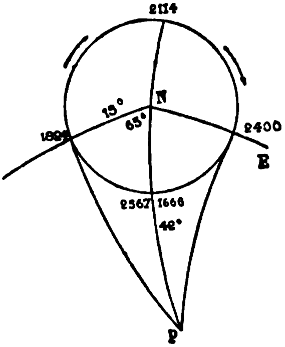

FIG. 200.

Daily Variations.—Other variations have a short period, and appear to be connected with the apparent motion of the sun, the moon, &c., and follow laws which are not yet known.

These variations affect particularly the declination, which in one and the same place has a well-marked daily oscillation with two maxima and two minima. The amplitude of the excursion of the needle is greater during the day than during the night. The time at which the greatest deflection is reached differs in different places. At Kew the declination is greatest at about 1 or 2 P.M., and least at about 8 A.M., the difference being 12′ or 14′ in summer, and rather more than half as much in winter.

Accidental Variations.—These variations are simultaneously produced over a great part of the surface of the earth, and always accompany the aurora borealis. They are called magnetic storms.

Both daily and accidental variations are observed by special apparatus, known as *registering magnetometers;* the small movements of suspended magnets, magnified by the method of reflexion, are registered continuously by photography. The variations commonly observed are those of declination and of the horizontal and vertical forces. The first is observed by means of a small bar suspended horizontally in the meridian by cocoon threads; the second is measured by means of a horizontal magnet suspended at right angles to the meridian by a twisted metal wire or by a bifilar suspension; the third, by a magnet that is movable like the beam of a balance about a knife-edge, and being exactly counterpoised for a given value of the vertical component, dips more or less as this value changes.

CHAPTER XXII.

ELECTRO-MAGNETISM.

236. Electro-Magnetism.—In studying the effects of electric currents, we have hitherto been concerned only with *internal actions*, that is to say, effects produced by the current in the conductors traversed by it. It now remains to investigate the actions produced outside the conductor, which, for this reason, are called the *external actions*. The part of Science which relates to these phenomena is called *electro-magnetism;* it originated in an experiment made by Oersted in July 1820, and it owes its principal developments to the researches of Ampère and of Faraday.

237. Oersted's Experiment—Ampère's Rule.—When a conductor through which a current is passing is brought near and parallel to a magnetic needle, the latter is deflected from its ordinary position (Fig. 201); this is the fact which Oersted observed. According to the relative positions of the needle and the circuit, the direction of the deflection is different. It may, however, be determined in each case by what is known as *Ampère's Rule.* Suppose an observer swimming in the direction of the current, so that it enters by his feet and emerges by his head : if the observer has his face turned towards the needle, *the north pole is always deflected to his left.* We shall speak of this as the left of the current.

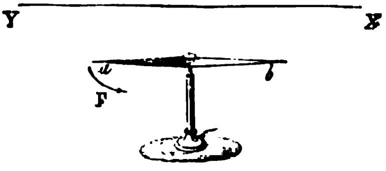

FIG. 201.

238. Ampère's Astatic Needle.—If it were not for the action of the earth, the needle would always set at right angles to the current. In order to show this, Ampère used a needle arranged like a dip-needle (§ 230), so that it moved only about a fixed axis passing through the centre of gravity, and he placed this axis parallel to the direction of the earth's force (Fig. 202). As the couple due to the earth acts in a plane containing the axis, it has no effect on the position of the needle. The needle then always sets at right angles to the current, whatever be the strength of the latter. Hence it follows that the action of the current is exerted in a direction at right angles to the conducting wire, and that it acts oppositely on the two poles.

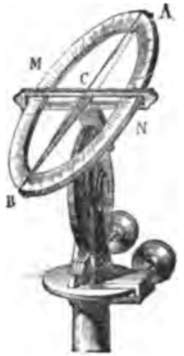

FIG. 202.

239. Galvanometer.—If we stretch the wire above a horizontal needle (Fig. 201) when it is at rest in the magnetic meridian, the needle is under the action of two systems of forces acting in planes at right angles to each other, and the needle takes up an intermediate position.

Experiment shows that, other things being equal, the extent of the deflection is independent of the degree of magnetisation of the needle, which proves that the two actions are in a constant ratio, and therefore that the force due to the current, like that due to the earth, is *proportional to the strength of the pole* upon which it acts.

The deflection increases, moreover, with the strength of the current as measured by its chemical action, and may, therefore, serve as a measure of it. This is the principle of the electro-magnetic measurement of the current, and of the apparatus which Ampère called a galvanometer (see Chapter xxviii.).

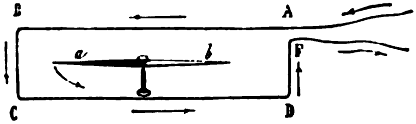

FIG. 203.

The action of the current can be increased by coiling the conductor on a frame; the needle is put at the centre of the frame (Fig. 203), and the plane of the frame is placed in the magnetic meridian. It is easily seen from Ampère's rule that all parts of the current tend to deflect the needle in the same direction. Such a frame is known as *Schweigger's Multiplier*.

240. Ampère's Movable Currents.—If in Oersted's experiment the magnet is fixed and the conductor movable, the latter ought to move so as to leave the north pole on its left. This experiment is easily made by means of Ampère's *movable currents.*

The current enters at two mercury cups, *a* and *b*, placed in the same vertical line (Fig. 204). A conducting wire suitably bent forms a frame, *d*, *e*, *f*, *g*, the ends of which are provided with points by which they dip into the mercury. Only one of these points rests on the bottom of the cup, the other simply dips in the mercury. The portion H′ of the vertical support is inside the hollow support H, but is insulated from it, and is connected by means of a wire with the mercury cup N, while H is connected with M.

If the frame is rectangular, and if we bring a magnet near it, the frame tends to set at right angles with the axis of the magnet, so that the north pole is on its left. The effect is greatest when the magnet is in the middle of the frame, all the forces then concurring with each other.

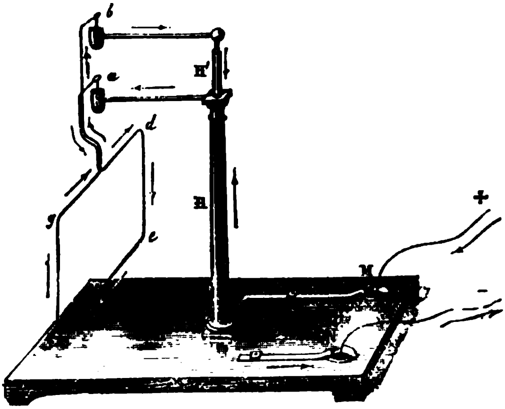

FIG. 204.

241. Directive Action of the Earth upon a Movable Current.—The earth acts on a movable circuit in the same way as a magnet, and tends to set it at right angles to the magnetic meridian. In the position of equilibrium, the current descends in the branch which is to the east, and ascends in that to the west; or, in other words, an observer looking at the circuit by the face turned towards the north sees the current circulate in the opposite direction to the hands of a watch.

Circuits may be constructed which are not acted on by the earth, or are *astatic*, by forming them of portions of equal area so arranged that the current circulates round them in opposite directions (Figs. 205 and 206).

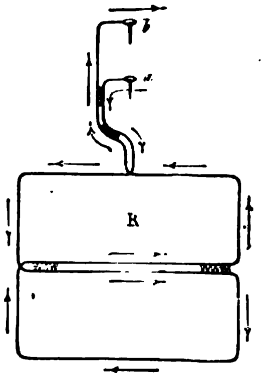

Fig. 205.

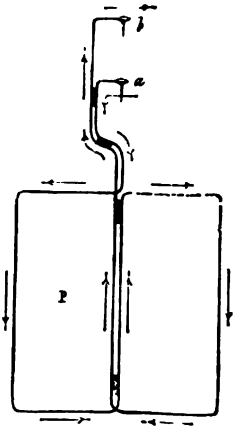

Fig. 206.

242. Action of Currents on Currents—Ampère's Law.—The use of movable currents led Ampère (in September 1820) to the important discovery of the forces exerted by currents on each other.

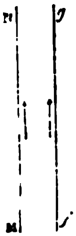

Fig. 207.

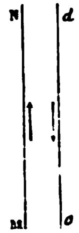

Fig. 208.

Let us take the astatic rectangular circuit in Fig. 206. If we bring a vertical conducting wire traversed by a current, which may or may not be part of the same circuit, near one of the vertical sides, we see that *two parallel currents in the same direction attract* (Fig. 207), and that *two parallel currents in opposite directions repel* (Fig. 208).

If a horizontal conducting wire is brought near the horizontal side of the astatic frame of Fig. 205, this will be seen to turn until the two currents are parallel and in the same direction (Fig. 209). This fact is frequently enunciated in the following manner: *Two currents which make an angle with each other attract and tend to set parallel if they both run towards or both away from the summit of the angle or the common perpendicular; they repel if one runs towards the common perpendicular and the other away from it.*

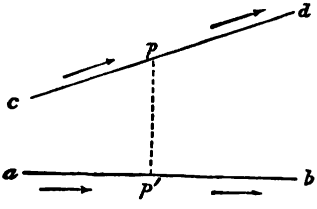

FIG. 209.

243. Currents in Opposite Directions—Sinuous Currents. —If a magnet or a movable current be brought near a conducting wire bent back on itself, as in Fig. 210, there is no action. From this it is concluded that two *equal currents in opposite directions exert equal and contrary forces.* This is also the case if a wire is bent as in Fig. 211, instead of being straight, provided that the bent wire is close to the straight wire, and is not coiled round it. Hence we have this important theorem: *provided the sinuosities are small, the effect of a sinuous current is identical with that of a straight current flowing between the same points.*

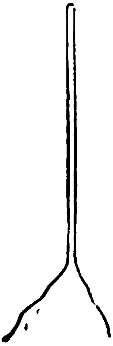

FIG. 210.

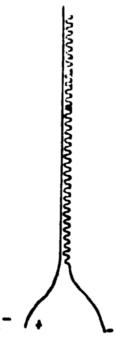

FIG. 211.

Thus if a current flows round a triangle ABC, the two elements of the current represented by the sides AC and CB have together the same effect as the third side AB at any point whose distance from the triangle is great as compared with its sides.

244. Magnetic Field due to a Current. —The essential fact which results from the experiments of Oersted and of Ampère is that an electric current creates a magnetic field, and acts on magnets or on other currents in the same sort of way as magnets themselves. Ampère showed that the field of a current is a true magnetic field, that is to say, a field of the same kind as that which is created by magnets. The identity is not merely an identity of form, like that which we have found between the electrical field and the magnetic field (§ 168), but a real and absolute identity.

The field of a current has all the characteristic properties of a magnetic field; it exerts equal and opposite forces on equal and opposite magnetic poles, and the force at each point is proportional to the strength of the magnetic pole placed at this point. The form of the magnetic field may be rendered evident in the ordinary way with iron filings.

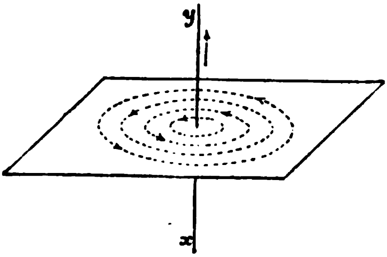

FIG. 212.

245. Field of an Infinite Rectilinear Current.—Let us consider the case of a rectilinear portion of a circuit so long that it may be treated as an infinite right line. If this passes at right angles through a sheet of cardboard on which iron filings are sifted, it is seen that the lines of force are concentric circumferences with their centre in the axis of the wire (Fig. 212).

Ampère's rule shows that they go from the right to the left of an observer placed in the current. It might be anticipated from reasons of symmetry, and it is confirmed by experiment, that the force is the same at all points of the same circumference. It follows that the equipotential surfaces are planes passing through the axis of the current, and making equal angles with each other. It can thence be directly concluded that the force varies inversely as the distance (§ 32).

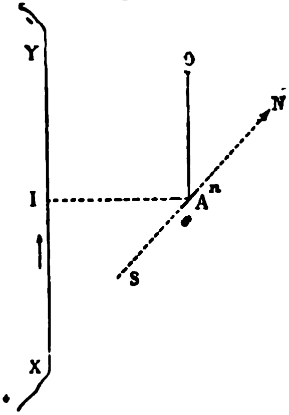

FIG. 213.

246. Biot and Savart's Experiment.—The law just mentioned was established experimentally by Biot and Savart. The experiment consists in causing a very small needle to oscillate first under the influence of the earth alone, and then under the simultaneous influence of the earth and of a very long straight current. The current being vertical, the needle is suspended horizontally in the plane which passes through the current at right angles to the magnetic meridian (Fig. 213). If the needle is very small in comparison with its distance from the current, it may be regarded as being acted upon by a uniform field, which, with the direction of the current shown by the arrows, adds itself to that of the earth. The needle retains its position of equilibrium in the magnetic meridian; if it is displaced therefrom, it vibrates according to the pendulum law.

If G is a constant which depends on the moment of inertia and on the magnetic moment of the needle, H the force due to the earth, Φ and Φ' the forces due to the current at the distances a and a', and if n, N, and N' are the numbers of oscillations made respectively in the same time under the action of the forces H, $H + \Phi$, and $H + \Phi'$, we have, from the formula for harmonic vibrations—

$$\begin{aligned} n^2 &= GH \\ N^2 &= G(H + \Phi) \\ N'^2 &= G(H + \Phi') \end{aligned}$$

As experiment gives

$$\frac{N^2 - n^2}{N'^2 - n^2} = \frac{\Phi}{\Phi'} = \frac{a'}{a},$$

it follows that the *force exerted by an infinite current on a magnetic pole varies inversely as the distance;* it is, moreover, proportional to the strength of the current; we may therefore write

$$\Phi = 2k\frac{C}{a},$$

k being a coefficient which depends on the unit in which the strength of the current is expressed, and upon the magnetic permeability of the medium.

The unit, which will be afterwards described as the *electro-magnetic unit*, is that for which $k = 1$, when (as is usually the case) the medium is air. The unit in question is equal to 10 amperes. An infinite current of 10 amperes would, therefore, act on unit pole at a distance of a centimetre with a force of 2 dynes.

If C is the strength of the current expressed in terms of the electro-magnetic unit, the law of Biot and Savart is expressed by

$$\Phi = 2\frac{C}{a}.$$

It is to be observed that $a\Phi$ is the moment, about the line of the current as axis, of the force exerted by an infinitely long current on unit pole. This moment is constant for the same strength of current whatever be the distance.

247. Laplace's Law.—In order to explain the law of Biot and Savart, Laplace showed that it was sufficient to assume that the force exerted by a pole on an element of current is defined by the formula

$$\Phi = \frac{mCds \sin \alpha}{r^2},$$

m being the strength of the pole, ds the length of the element, r the distance of the pole from the middle of the element, and α the angle which the element ds makes with the straight line r; further, that this force is applied to the middle of the element perpendicularly to the plane rds, and directed to the right of an observer placed in the element who is looking at the pole (Fig. 214). The reciprocal action of the element of the current on the pole is applied at the same point and the opposite direction.

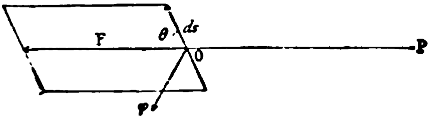

FIG. 214.

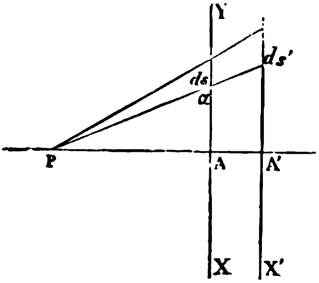

FIG. 215.

Without entering into calculations, it is easy to see that, given this law, the resultant force exerted by the infinite wire is simply the sum of the forces in the same direction exerted by its various elements, and that this sum varies inversely as the distance of the wire from the pole. For as all the elementary forces are perpendicular to the same plane, they are parallel with each other, and give a resultant equal to their sum. Further, if we take the wire at two distances, a and a', such that $a' = na$ (Fig. 215), and consider two elements, ds and ds', seen from the pole under the same angle and in the same direction, it is evident that the force exerted by the element ds', which is n times as long as the element *ds*, but acts at *n* times the distance, is simply *n* times as small as that due to *ds*. As this is the case with all the elements taken in pairs, it holds also for the sum.

If the element *ds* (Fig. 214) is turned in the plane of the figure about the point O, the force vanishes when the element is directed to the pole, and is a maximum when it is perpendicular to the straight line OP; in any given position the force is that which would be exerted between the pole and the projection of the element on a plane perpendicular to OP.

It may at first seem strange that the force exerted by the element on the pole should be applied to the element, like the force of the pole on the element; the principle of action and reaction requires, however, that the two forces should be applied at the same point, and be directly opposed. On the other hand, it is clear that we can never directly observe the force exerted by an element of a current on a pole; we can only observe the force due to the whole circuit. An elementary law like that of Laplace is thus only a simple mathematical expression, which need not correspond to any physical reality, and which is only determined by the condition of giving a result which agrees with experiment when applied to a closed circuit.

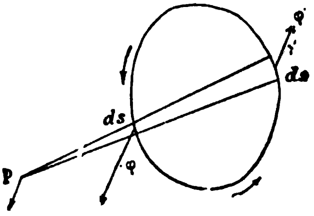

FIG. 216.

Ampère showed that the force exerted by a closed circuit on a pole passes through the pole; for there is no action on a magnet which is only movable about an axis passing through its poles. Laplace's law satisfies this condition. It is easy to apply it to the case of a plane circuit acting on a pole in its own plane (Fig. 216); the forces ϕ and ϕ', due to the two corresponding elements *ds* and *ds'*, are parallel forces in opposite directions, inversely proportional to their distance from the pole; their resultant, therefore, passes through the pole.

248. Generalisation of Laplace's Formula.—In the expression for ϕ, the quantity $\frac{m}{r^2}$ is the value F of the magnetic field at the middle point O of the element. So long as this force F remains the same, it makes no difference as regards the force exerted on an element of current, whether the field is due to a single pole or to any number of poles or electric currents. The law may then be regarded as a general one. Observing that $Fds\sin\alpha$ is the area dA of the parallelogram drawn on the element ds and the force F, we may write

$$\phi = CFds\sin\alpha = CdA,$$

and enunciate the result thus :—

The force exerted on an element of current placed in a magnetic field is equal to the product of the electro-magnetic strength of the current into the area of the parallelogram constructed on this element, and on the intensity of the field. This force is perpendicular to the plane of the parallelogram, and is directed towards the left of an observer placed in the current and looking along the lines of force of the field.

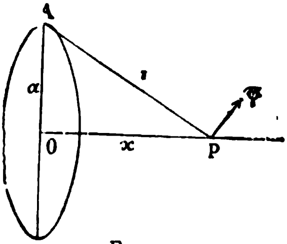

FIG. 217.

249. Force due to a Circular Current at a Point on the Axis.—As an application of Laplace's formula we will calculate the force due to a circular current at a point P of the axis (Fig. 217). Let a be the radius, and r the distance of a point of the circumference from the point P. The angle α between the direction of the current and the line r is $\frac{\pi}{2}$; the formula therefore reduces for a unit curent to

$$\phi = \frac{ds}{r^2}.$$

As the object is to get the resultant, we may (§ 247) consider the elementary force as applied to the point P itself perpendicularly to the plane rds. The force may be resolved into two others, one parallel and the other perpendicular to the axis. The components perpendicular to the axis will neutralise each other in pairs, and the resultant will be equal to the sum of the components along the axis.

The component along the axis is obtained by multiplying ϕ by the cosine of the angle PAO or $\frac{a}{r}$ which gives $\frac{ads}{r^3}$. The resultant will then be equal to the product of the constant factor $\frac{a}{r^3}$ by the sum of the elements ds, that is to say, by the circumference $2\pi a$. We have thus

$$F = \frac{2\pi a^2}{r^3} = \frac{2S}{r^3},$$

S being the area of the circle. At the centre we have $r = a$ and therefore

$$F = \frac{2\pi}{a}.$$

Thus at the centre of a circle of unit radius the force due to a unit current is 6.28 dynes.

250. Field of a Circular Current.—But for difficulties of the calculation, we might use Laplace's formula to calculate in like

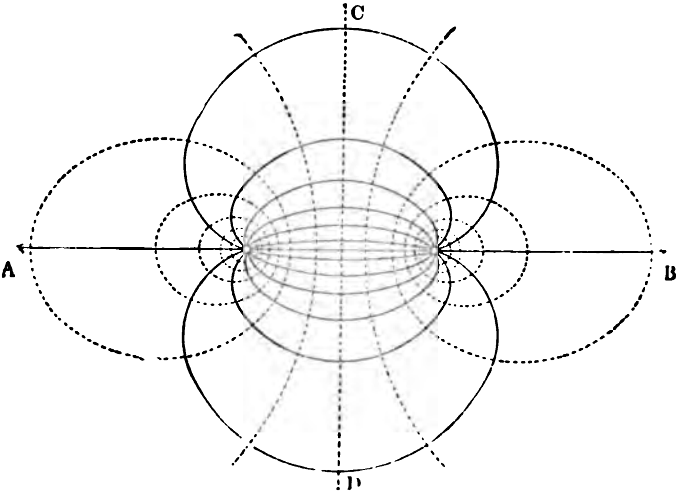

FIG. 218.

manner the strength of the field at each point. Fig. 218 represents the field of a circular current; the plane of the circle is supposed horizontal and perpendicular to the plane of the figure which it intersects along the line AB. The dotted lines represent lines of force in a plane passing through the axis CD of the current. These lines traverse the circle from the right to the left for an observer, imagined in the conductor so that the current flows from foot to head, who looks towards the middle of the circle. We shall take this as the positive direction of the axis of the current, and we shall call that face of the current which is on the left of

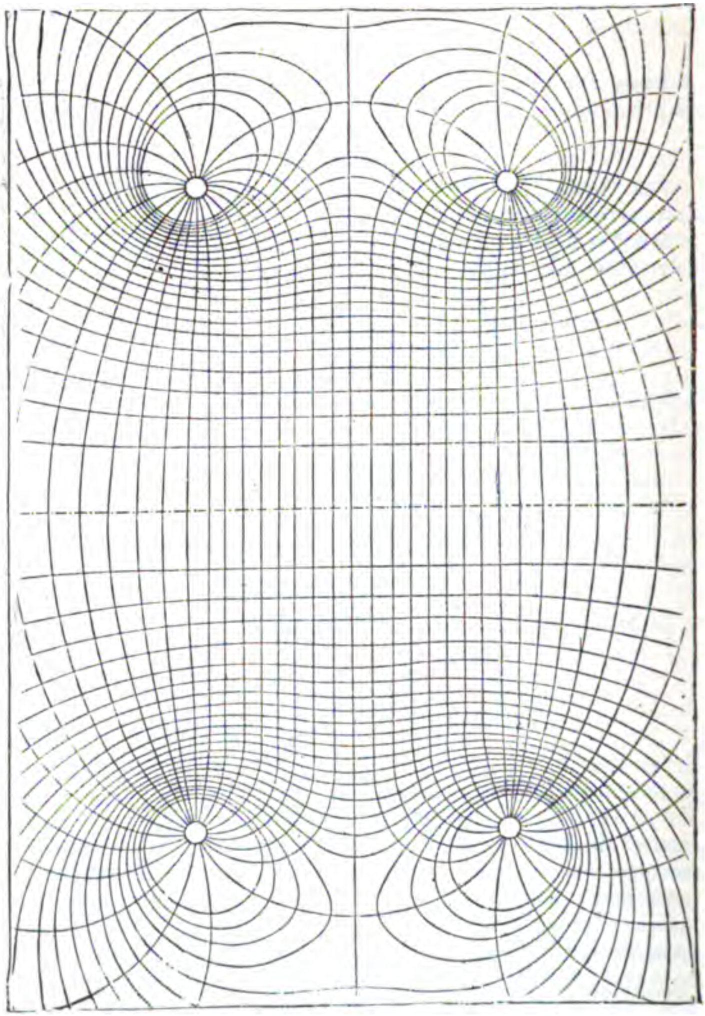

FIG. 219.

the observer the ***positive***, and that which is on the right the *negative;* hence the axis traverses the circuit from the negative to the positive face. The full lines which cut the dotted lines at right angles are the intersections of the equipotential surfaces by the plane of the figure.

251. Field due to Two Circular Currents.—Fig. 219 represents the lines of force, and the equipotential surfaces for the case of two equal and parallel circular currents with their planes at a distance equal to the radius. Throughout a considerable region the lines of force are very nearly parallel to the common axis of the two currents, and the equipotential surfaces are equidistant. The field may therefore be considered to be uniform.

252. Work done in Displacing an Element of Current.—An element of current cannot in general be displaced in a magnetic field without the force applied to it doing either negative or positive work.

This work is necessarily equal to the product of the force into the displacement resolved parallel to the direction of the force. In Fig. 220, let AB be the element considered; draw AH to represent H, the intensity of the field, and suppose the element to be displaced in any direction while remaining parallel to itself through the small distance AA′ so as to come into the position A′B′; in so doing, it sweeps out the parallelogram ABA′B′ and cuts through the lines of force which traverse this area. We shall find that the number of lines of force cut multiplied by the strength of the current gives the work done during the displacement.

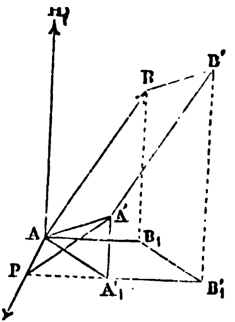

FIG. 220.

To prove this, let the parallelogram ABA′B′ be projected upon a plane perpendicular to H into the parallelogram $\mathrm{AB_1A_1'B_1'}$, and let AP be the projection, at right angles to the plane of AB and AH (the plane of the figure), of the displacement AA′. The force ϕ acting on the element of current is (§ 248)—

$$\begin{aligned}\phi &= HC\,.\,\mathrm{AB}\sin(H, \mathrm{AB}),\\ &= HC\,.\,\mathrm{AB_1},\end{aligned}$$

and is perpendicular to the plane of the figure. The work dW corresponding to the displacement is therefore

$$dW = \phi\,.\,\mathrm{AP} = HC\,.\,\mathrm{AB_1}\,.\,\mathrm{AP}.$$

But $AB_1 . AP$ is equal to the area of the parallelogram $AB_1A_1'B_1'$, and this multiplied by H gives the number of lines of force that pass through it, a number which is the same as that of the lines of force through $ABA'B'$, of which $AB_1A'_1B'_1$ is the projection.

If, during the displacement, the element of current does not remain parallel to itself, the same formula gives the value of dW on the understanding that AP represents the mean displacement, or displacement of the middle point of the element, resolved parallel to the direction of the force.

Hence *the work corresponding to the displacement of an element of current is equal to the product of the strength of the current into the number of lines of force cut during the displacement.*

The sign of the work done by electro-magnetic force changes with the sign of the current and the direction of the displacement. It is positive if the displacement is to the left of an observer in the line of the current looking in the direction of the field, negative if it is towards his right.

253. Potential of a Closed Circuit.—During the displacement of a closed circuit in a magnetic field, work is done which is evidently equal to the algebraic sum of the amounts of work due to the displacements of its different elements.

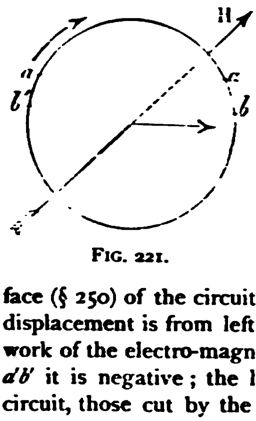

FIG. 221.

Let us consider a circuit so small that all the lines of force which traverse it are sensibly in the same direction; for instance, suppose them to pass through from front to back of the plane of the figure Fig. 221). Suppose that the current flows in the direction of the arrow, and therefore that the negative face (§ 250) of the circuit is presented to the reader; also that the displacement is from left to right. For an element such as *ab* the work of the electro-magnetic force is positive, while for the element *a'b'* it is negative; the lines of force cut by the former enter the circuit, those cut by the latter leave it; but as the work for each element is measured by the number of lines of force cut, it is clear that the resultant work will be equal in magnitude and in sign to the excess of the number of lines of force which enter over those which leave the circuit, or, in other words, to the *variation* of the flux of force through the circuit.

The small closed current consequently has a potential: that is to say, there is a function such that its variation, for any given displacement, measures the work done *against* electro-magnetic forces during this displacement. This function is the number of lines of force of the field included in the circuit. The potential increases if the number of lines of force entering by the positive face increases, or if the number of those which enter by the negative face diminishes. If then the number of lines is Q, and we call those positive which enter by the negative face, we shall have

$$V = -CQ.$$

It will be seen that this potential is the same as that of a magnetic shell which has the same contour, and the strength of which is C (§ 182).

Suppose that the small circuit is in the presence of a single pole of unit strength placed at a point P. If ω is the apparent angle under which the contour of the current is seen from the pole, the number of lines of force which enter by the positive face is ω; we have then

$$V = C\omega,$$

and therefore the potential which the closed circuit produces at P is the same as that of a shell of the same contour and of strength C.

But it is important to note that, while the potential due to a magnetic shell has only one value at each point, the potential due to a current may have an infinite number of values, which only differ from each other by a constant.

The potential, as in the case of the shell, represents the work necessary to bring a unit quantity of positive magnetism from an infinite distance to the point P; but it represents this only on the supposition that the magnetic mass has not traversed the circuit; otherwise we must add to ωC as many times $4\pi C$ as the mass has traversed the circuit from the positive to the negative face.

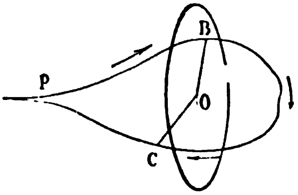

FIG. 222.

For suppose the magnetic mass to move from P, and to traverse the circuit and return to the same point by any given path (Fig. 222). This path meets the plane of the circuit in two points, one B inside, and the other C outside.

During the motion, the apparent angle of the contour varies continuously. From P to B it varies from ω to 2π; the corresponding work is $(2\pi-\omega)C$. From B to C the angle varies from 2π to 4π, and the work increases by $2\pi C$. Finally, from C to P the apparent angle increases from 4π to $4\pi+\omega$, and the corresponding work is ωC. The total work is thus equal to $4\pi C$. It would be $-4\pi C$ if the displacement had been in the opposite direction.

Accordingly, if the pole does not traverse the circuit, the potential resumes the same value when the pole returns to the point P; but if it traverses it n times, the potential is

$$V = \omega \pm 4\pi n C,$$

the positive sign corresponding to the case in which the pole has traversed the current from the positive to the negative face.

As the potential is the same, within a constant, for a small closed circuit as it is for a magnetic shell, the force at any given point is the same in both cases, as it depends only on the space-rate of variation of potential at the point.

254. Equivalence of a Closed Circuit and of a Magnetic Shell—Ampère's Theorem.—An infinitely small closed circuit *gives rise to the same field*, and in any magnetic field is *subject to the same forces*, as a magnetic shell of the same contour, provided that

$$\Phi = C,$$

that is to say, if the moment per unit of area of the shell is equal to the electro-magnetic strength of the current. The closed circuit and the magnetic shell are therefore *equivalent*. This is a general theorem, and applies to any current.

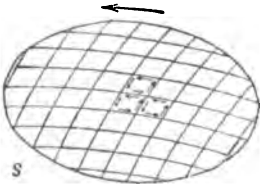

FIG. 223.

Consider a surface of any shape bounded by a contour S (Fig. 223): let us suppose this surface cut by two systems of infinitely close parallel planes, which divide it into infinitely small elements, and suppose the contours of all these elements traversed by currents of the same strength C, all flowing round in the same direction. Each line which separates two adjacent elements is traversed by two equal currents in opposite directions, which neutralise each other. Consequently only those portions of the current which correspond to the elements of the contour come into account, and the force due to these is evidently the same as that of a single continuous current of the same strength flowing round the contour S. On the other hand, each of the elementary currents may be replaced by the equivalent magnetic shell, and all such shells, taken together, form a shell having the same contour as the closed circuit considered, and a magnetic strength equal to the strength of the current. Hence it follows that *the force due to any closed current is identical with that of a magnetic shell of the same contour provided its magnetic strength is equal to the electro-magnetic strength of the current.*

This is the celebrated theorem of Ampère. It furnishes the following definition of the electro-magnetic unit of current-strength: *the electro-magnetic unit is that which gives for the strength of a current a number equal to the strength of the equivalent magnetic shell.*

255. Electro-Magnetic Solenoid.—Ampère gave the name of electro-magnetic solenoid [σωλην, *a pipe*] to a system formed of infinitely small closed currents (Fig. 224), of the same surface and the same strength, distributed at infinitely small equal distances along a curve of any form, called the *directrix*, passing through the centre of figure of each elementary circuit, and perpendicular to its plane.

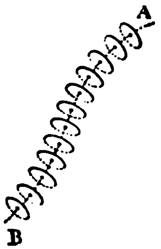

FIG. 224.

Each current may be replaced by the equivalent shell, and if each shell has a height equal to the space between two currents, the opposite faces of two consecutive shells coincide, and the system reduces to a *solenoidal filament* (§ 180).

Let λ be the surface of the currents, h their distance apart, or let $n_1 = \frac{1}{h}$ be the number of currents per unit length, C the strength, lastly σ the magnetic surface-density on the terminal faces of the equivalent magnetic filament: we have as a condition of equivalence (§ 181)—

$$\Phi = h\sigma = C,$$

and therefore

$$\sigma = \frac{C}{h} = n_1 C.$$

The force due to the solenoid is independent of the shape of the directrix; it is that of two masses, $\pm n_1 C\lambda$, situated at the two ends.

256. Electro-Magnetic Cylinder.—This was the name given by Ampère to a system of equal, parallel, and equidistant circular currents. As each current can be replaced by the equivalent magnetic shell, the system is itself equivalent to a cylinder magnetised uniformly, which has at its ends the magnetic surface-density

$$\sigma = n_1 C.$$

This system of currents is easily realised by coiling a wire in the form of a close spiral on the surface of a cylinder. Each convolution of the spiral may be replaced by its projections on the axis, and on a plane perpendicular to the axis (§ 243). For each turn of the spiral, the projection parallel to the axis is a straight line equal to the pitch of the spiral, and the projection perpendicular to the axis is a circle of the same radius as the cylinder. If the section of the cylinder is small, the combined effect of the projections parallel to the axis is neutralised by carrying the wire back along the axis. The compensation is still more perfect if the cylinder is wound with an even number of layers, in which the inclination of the turns is alternately in opposite directions.

When the coil thus formed is traversed by a current, the force exerted at any external point is the same as that due to a uniformly magnetised cylinder, the moment of which has the value

$$M = n_1 CSl = nCS,$$

l being the length of the cylinder, and n the total number of turns.

The force at a point in the interior is equal to that which would be exerted in an infinitely narrow crevasse, perpendicular to the axis of the same cylinder—that is, to the induction (§ 187). If we suppose the point to be so far from the ends that the action of the bases may be neglected, the force is reduced to that due to the two sides of the crevasse, that is

$$F = 4\pi\sigma = 4\pi n_1 C.$$

This may be proved in another way. For a unit pole which is moved along the axis through a distance e, the work is Fe; on the other hand, since the pole has traversed $n_1 e$ convolutions, the potential has changed by $4\pi C . n_1 e$. If we put the work equal to the change of potential, we arrive at the preceding expression. The force F may be considered as constant throughout the whole extent of the section. Hence, for the value of the flux through the section S, we have

$$Q = 4\pi n_1 CS.$$

Such a cylinder has all the properties of a magnetised bar.

This may be shown by means of the arrangement represented in Fig. 225. When the electro-magnetic cylinder is suspended horizontally, as in the figure, and a current is passed through it, it sets in the direction of the magnetic meridian, and when the north pole faces us, the apparent direction of the current is opposite to the hands of a watch.

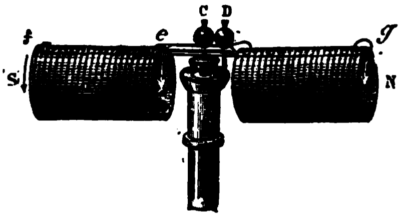

FIG. 225.

All the experiments of attraction of opposite poles and of repulsion of like poles may be repeated by acting on it either with a magnet or with another electro-magnetic cylinder.

Nevertheless, an electro-magnetic cylinder must not be confounded with a hollow magnet: in the latter all the lines of force, both inside and outside, start from the positive or north end, and terminate at the negative or south end. In the electro-magnetic cylinder the lines of force in the interior are a continuation of the external lines of force, and form closed curves which never terminate in magnetic masses.

257. Ampère's Theory of Magnetism.—The identity between the forces exerted by currents and by magnets leads us naturally to attribute the two orders of phenomena to the same cause. It is impossible to produce by means of magnets certain effects producible by currents, such as a continuous rotation (§ 262); whereas by combinations of closed currents we may produce all the effects of magnets.

It is sufficient to assume that each magnetic molecule owes its properties to a current of electricity about its axis.

It has been objected to this theory that a piece of steel or iron, being the seat of permanent currents, must also be a permanent source of heat. This objection would be fatal if, as with ordinary currents, there were a circulation through a system of molecules; but the amperian currents are supposed to circulate in the molecule itself, and as we know nothing of the nature of the molecules and of their properties, there is no contradiction in the assumption that their resistance is zero.

258. Weber's Experiments.—Weber verified experimentally the equivalence of an electro-magnetic cylinder and a magnet, and therefore also Ampère's theorem which follows from this equivalence.

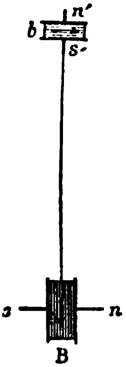

FIG. 226.

The method of experimenting was that which Gauss used to determine the ratio $\frac{M}{H}$ (§ 231). The deflecting magnet ns placed in one of the principal positions in reference to the movable needle $n's'$ (Fig. 226) is replaced by a coil, B, traversed by the current, and it is seen that the deflection of the needle is the same in both cases when

$$M = nCS,$$

M being the moment of the bar, nS, the surface enclosed by the coil, and C the strength of the current.

Similarly, a small coil, b, traversed by a current may be substituted for the deflected magnet $n's'$. If the conditions of the suspension are the same, it is found that whether the magnet ns or the coil B is used the deflection is the same.

CHAPTER XXIII.

ELECTRO-MAGNETIC FORCES.

WE shall apply the theorems of the last chapter to the investigation of a few simple cases.

259. Force exerted by a Current on a Magnetic Needle.—Returning to Oersted's experiment, let there be an infinitely long current placed above the needle parallel to the magnetic meridian (Fig. 227). If the current is in the plane of symmetry, the deflected needle remains horizontal; while it is slightly inclined in the contrary case. We will only consider the former case, and will suppose the needle reduced to its two poles. The force exerted by the current on each pole is perpendicular to the shortest distance from the pole to the current, and has a horizontal component *f*, and a vertical component acting upwards. The two vertical components act in the same direction, and tend to raise the needle; the horizontal components form a couple which, compounded with the terrestrial couple, determines the deflection of the needle.

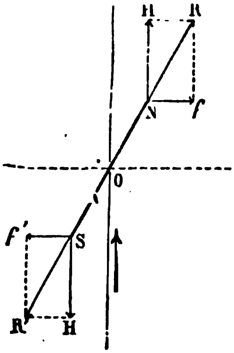

FIG. 227.

When the current is perpendicular to the direction of the needle, the resultant of the forces which it exerts always passes through the axis of the wire.

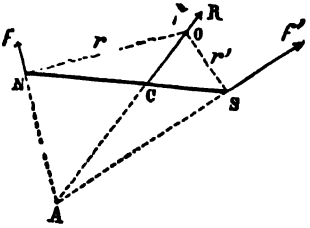

FIG. 228.

Let NS be the needle (Fig. 228), and O the section by the plane of the figure of the conductor traversed by the current, which we will suppose to be directed inwards through the paper. The forces Φ and Φ' applied at the two poles are perpendicular to the distances r and r', and satisfy the equation

$$\Phi r = \Phi' r'.$$

It follows from this that the sum of the moments of the two forces in respect of the point O is zero, and that, therefore, the resultant passes through this point. In the case of the figure it is directed towards the current; if the needle is reversed, or if the direction of the current is changed, it acts in the opposite direction.

260. Action of Parallel Currents.—Let A and B be two infinite parallel currents, of strengths C and C', placed at a distance a (Fig. 229). At the point C the magnetic force due to the infinite current A is perpendicular to the plane of the two wires, and is equal to

$$\Phi = \frac{2C}{a}.$$

The electro-magnetic force on unit length of B at C is

$$2\frac{CC'}{a},$$

it acts in the plane of the two currents, perpendicular to their direction, and tends to bring them together if they are in the same direction, and to separate them if they are in opposite directions. It varies like the force on a pole inversely as the distance.

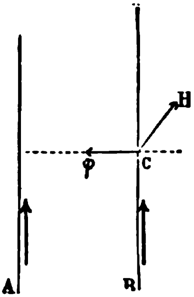

FIG. 229.

261. Closed Circuits.—A magnetic shell placed in a field of force tends towards the position in which the flux of force traversing its surface is a maximum (§ 183). This is also the case with the equivalent closed current.

If Q_1 and Q_2 are the values of the flux of force which enters the negative surface, before and after the displacement, the positive variation of the flux is $Q_2 - Q_1$, and if we suppose that during the displacement, the strength of the current has remained constant and equal to C, the work done by electro-magnetic force is

$$W = C(Q_2 - Q_1).$$

If the field is that due to a second closed current of intensity C', and if we represent by M the coefficient of mutual induction (§ 184); that is to say, the flux which, for a current of unit strength, each circuit sends through the negative face of the other, we have

$$Q_1 = M_1C' \qquad Q_2 = M_2C',$$

and consequently

$$W = CC'(M_2 - M_1).$$

For an infinitely small variation we have

$$dW = CC'dM.$$

Let us consider, for instance, a circuit moving about a vertical axis in the earth's field (Fig. 230); the position of equilibrium will be that in which the plane of the circuit is perpendicular to the magnetic meridian, the current, as seen from the north side, circulating in the opposite direction to the hands of a watch. If S is the surface of the circuit, and H the horizontal component of the earth's field, the flux through the surface in the position of stable equilibrium is SH. If the circuit is turned through a right angle, it is nothing; if it is turned through 180°, it is $-SH$. When the current is left to itself, and turns from its latter position to the former, the total variation of the flux is equal to $2SH$, and the work done by electro-magnetic force for a current of constant strength C is

$$W = 2CSH.$$

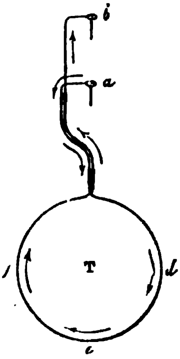

FIG. 230.

In like manner if the frame is movable about a horizontal axis, passing through the centre of gravity (Fig. 231), and at right angles to the meridian, its plane sets at right angles to the dip-needle.

Lastly, a circuit is evidently astatic if it consists of two equal areas of opposite signs (§ 241), since there is no variation of the flux whatever the displacement.

The application of the same principles shows that, under the action of its own flux, a circuit will tend to present the greatest possible surface; if the contour is flexible, it will take the form of a circle, as this comprises the greatest area within a given perimeter.

This is the true explanation of a well-known experiment of

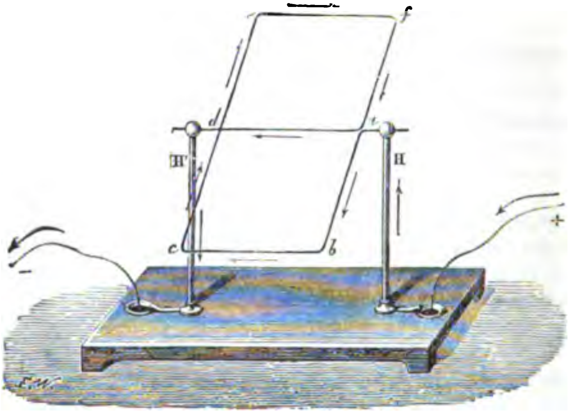

Fig. 231.

Ampère, which he regarded as proving that two consecutive elements of the same current repel one another.

Two quantities of mercury (Fig. 232), separated by an insulating

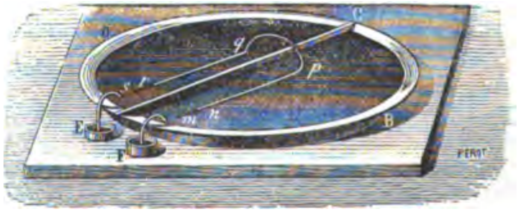

Fig. 232.

partition, are connected by an iron wire, bent so as to form two parallel horizontal branches, *np* and *rq*, connected by a cross-piece, *pq*. When connection is made with a battery by the cups E and F, the wire glides on the surface of the mercury away from the points by which the current enters, thus increasing the area of the circuit.

262. Electro-Magnetic Rotations.—It follows from the fundamental principles of mechanics, that a continuous rotatory motion in the same direction cannot arise among a system of rigid bodies which are acted on solely by the forces they exert upon each other, if these forces are functions only of the relative distances of the bodies between which they act, that is to say, if the forces have always the same values when the bodies are at the same distances. For, regarding one portion of such a system as being at rest, suppose another portion to move relatively to it from a given position along any path, and come back to the same position again. Then any change of the distance between two points of the system that takes place during one part of the motion must be undone during the remainder; for, when the moving part of the system has returned to its first position, the relative distance between any given pair of points must be the same as at first. Hence, if positive work is done during part of the motion by the forces acting between any two points, equal negative work must be done during the rest of the motion, and this will apply to every pair of points that can be taken in the system. From this we see that the total work done by the internal forces, during a complete revolution of any part of the system about an axis, must be zero. Consequently, even if rotation were started by energy supplied from without, it would die out more or less rapidly, since in every real case of motion energy is gradually dissipated by friction, and, under the conditions supposed, no energy is generated within the system to replace that which is thus lost.

This would be the case of a system of magnets or of closed circuits of invariable form, which are equivalent to magnetic shells; but it is not the case if the circuits can be deformed, as when they comprise liquid conductors or sliding contacts. Faraday was the first to show that, in this case, a continuous rotatory motion can be produced. In order that these motions may be kept up, the forces must tend to increase the velocity at every revolution; in this case, the velocity is accelerated until the work done against friction in a given time is equal to that done by the forces tending to increase the velocity, and it is then constant. The energy spent in keeping up the motion, like that spent in doing any other work by a current, is derived from the chemical energy of the battery.

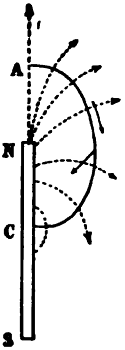

FIG. 233.

263. Rotation of a Current by a Magnet.—A conductor carrying a current is movable about an axis which coincides with the axis of the magnet. If the ends of the conductor are on either side of the same pole, as A and C (Fig. 233), the conductor turns with a continuous motion towards the left of an observer going head-first with the current, and looking in the direction of the lines of force of the magnet.

When the ends are in this position, each line of force is only cut once by the conductor during one revolution. It would always be cut twice, once positively and once negatively, if both ends of the conductor were on the same side of one of the poles outside the magnet, or one outside one pole and the other outside the other, or if both were between the poles: the total flux cut would then be nothing, and there would be no motion.

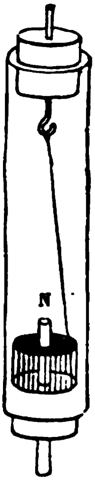

FIG. 234.

The flux cut by the arc is a maximum if the end C is at the middle of the magnet; for a complete revolution it is then the total flux from one of the poles. If m is the strength of the pole, the flux is $4\pi m$ (§ 41), and the electro-magnetic work corresponding to one revolution is $4\pi mC$. The moment of the couple about the axis of rotation is constant and equal to

$$\frac{4\pi mC}{2\pi} = 2mC;$$

it is thus independent of the size or shape of the conductor AC.

This experiment may be very simply made by means of the apparatus represented in Fig. 234; it consists of a glass tube closed by two corks; through the lower one passes the end of a magnet about which mercury is poured so that it is just below the level of the pole; a copper wire is put through the upper cork and a platinum wire hangs from it with its lower end touching the mercury. As soon as a current is passed, the platinum wire rotates continuously round the pole of the magnet, and the direction of the rotation changes with the direction of the current.

264. Rotation of a Magnet by a Current.—A cylindrical magnet is loaded by a platinum cylinder so that it floats upright in mercury, one of its poles projecting. The experiment may be made in two ways. In Fig. 236 the current enters near the edge, follows the surface of the mercury, and emerges by a rod fixed in the centre; the magnet moves round this rod as soon as a current passes. In Fig. 235 the part of the magnet which projects above the mercury conveys the current, the fixed rod dipping into a drop of mercury contained in a small hollow in the top of the magnet. The magnet rotates about its own axis. In both cases the projecting pole follows the lines of force of the current (§ 245), and for a complete turn the work of the electro-magnetic forces is equal to $4\pi mC$. With a north pole, and with the direction of the current as shown by the arrows, the rotation is opposite to that of the hands of a watch.

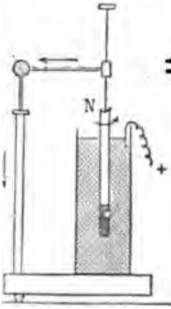

FIG. 235.

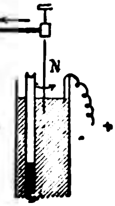

FIG. 236.

It is by virtue of the fluidity of the mercury that the magnet is able to rotate; only one of the poles traverses the closed current. If both the circuit and the magnet were rigid, the whole magnet would have to pass through the circuit, and as the amounts of work done on the two poles respectively during a complete revolution would be equal and opposite, the movement could not be maintained. With a flexible magnet, the two poles of which could move independently, the positive pole would be seen to coil itself in one direction about the current, and the negative pole in the opposite direction.

265. Rotation of a Current by a Current.—This experiment is made by means of the apparatus represented in Fig. 237. The ends *g* and *h* of the movable branches *fh* and *eg* dip in a solution of copper sulphate in connection with the negative pole of the battery. The vessel V is surrounded by a coil of copper wire through which the current passes. The experiment is quite analogous to that of Faraday (§ 263). The current, entering at X, passes by a wire under the base of the apparatus, ascends by the column P, and descends in the two branches *fh* and *eg*. If the currents flow as shown by the arrows in the figure, the part *fh* will be urged forward by the mutual forces between it and the fixed horizontal current (§ 242), and similarly *eg* will be urged backwards, so that the whole frame will rotate in the opposite direction to the hands of a watch with its face upwards. It may be said generally that a descending vertical current is urged *up-stream* by a horizontal current, and an ascending vertical current is urged *down-stream*.

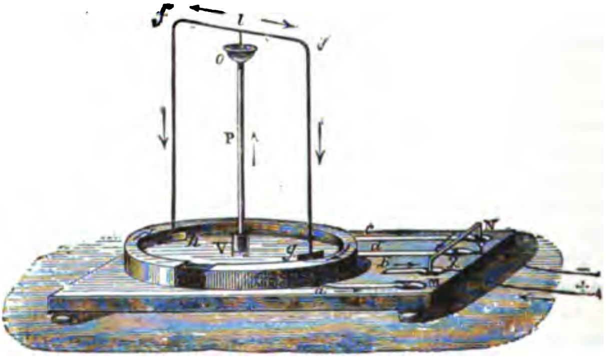

FIG. 237.

266. Rotation of Liquids and Gases.—When a current traverses a liquid, the liquid filaments, which are the seat of the current, behave like movable currents under the action of electromagnetic forces.

Davy's Experiment.—The experiment may be made in several ways: the following is due to Davy. Two platinum wires insulated, except at the ends, pass through the bottom of a vessel full of mercury, and end just below the surface. When a current is passed, the mercury rises above each wire as though each wire were a source of liquid. If a north pole is placed just above one of the little heaps thus formed, the mercury is seen to rotate very rapidly in the direction of the hands of a watch in the case of the negative pole, where the current is towards the centre, and in the opposite direction when the magnet is over the positive pole, where the current flows from the centre.

Jamin's Experiment.—The two electrodes of a voltameter are placed in the same vertical line, coincident with the line joining the poles of a horse-shoe magnet placed horizontally. The two electrodes being between the magnetic poles, if they were joined by a rigid conductor rotation would be impossible (§ 263). But as each element of the conducting liquid is independently acted on, the liquid divides into two superposed layers rotating in opposite directions. The gas bubbles in the liquid make the rotation visible.

De la Rive's Experiment.—A rapid succession of electric discharges (from a Holtz machine or an induction coil) is caused to take place between two ring-shaped electrodes, F and A, enclosed in an exhausted vessel, V (Fig. 238), and an iron rod projects upwards through the electrodes, being enclosed by the glass tube G, which is sealed air-tight into the bottom of the vessel V. The iron rod can be magnetised by means of the coil E. If the upper electrode F is positive, and the upper end of the rod is a north pole, the luminous discharge B will be seen, by an observer looking down upon the apparatus, to rotate round the magnet in the opposite direction to the hands of a watch.

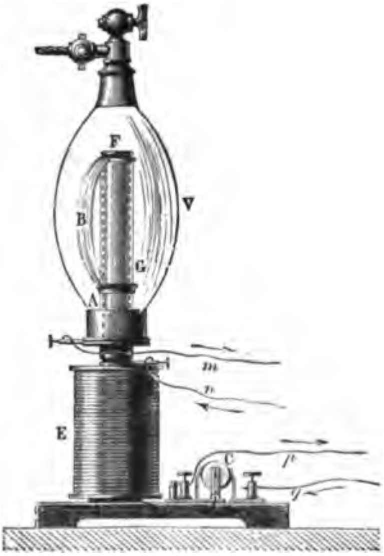

FIG. 238.

Action on the Electric Arc.—Analogous effects are produced on the electric arc formed between two carbon poles: if the pole of a magnet is brought near, the arc seems as if blown aside perpendicularly to the line which joins the pole to the arc.

267. Barlow's Wheel—Faraday's Disc.—*Barlow's Wheel* (Fig. 239) consists of a toothed metal wheel movable about a horizontal axis, so arranged that at the bottom one or more teeth dip in a trough of mercury, FD, on opposite sides of which are the two branches, AB, of a horse-shoe magnet. If the axis on the one hand, and the mercury on the other, are connected with the terminals of a battery, the wheel begins to rotate. If the current goes from the centre to the mercury, and the north pole is nearest the observer, the lower part of the wheel moves towards the right.

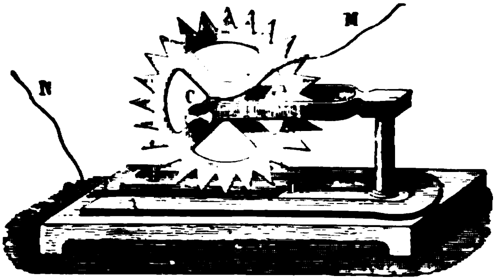

FIG. 239.

Suppose the apparatus placed in a uniform field, and let H be the component of the field parallel to the axis. Let a be the radius of the wheel, and θ the angle of two consecutive teeth, and suppose the surface of the mercury is arranged so that one tooth touches the liquid the moment the preceding tooth quits it. The current goes from the axis to the tooth in contact with the mercury; the flux which it cuts, while one tooth comes into the position of the next one, is $\frac{1}{2}a^2\theta H$; and the corresponding electro-magnetic work is $\frac{1}{2}a^2\theta HC$. For a complete turn the work is $\frac{1}{2}2\pi a^2HC$ or SHC, S being the total surface of the wheel.

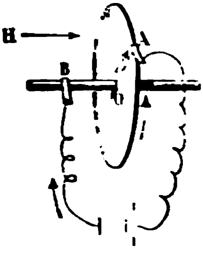

FIG. 240.

Faraday's Disc.—This apparatus (Fig. 240) works like Barlow's wheel; the current enters and leaves by two springs, one of which presses against the axis, and the other against the periphery of the wheel; the work for each turn is equal to SCH, and is independent of the velocity.

268. Rotation of a Current by the Action of the Earth.—This experiment may be made with the apparatus represented in Fig. 237, except that no current is passed through the coil. With the arrangement as shown in the figure, the circuit rotates, under the action of the earth, in the opposite direction to the hands of a watch. Resolve the earth's force into a vertical component Z, parallel to the axis of rotation, and a horizontal one H, perpendicular to this axis. The apparatus is astatic (§ 241) so far as the component H is concerned, but the horizontal part of the current is acted on by the vertical magnetic force Z, in the same way as the current in Barlow's wheel or Faraday's disc is acted on by the component of magnetic force parallel to the axis. If $2a$ is the distance between the vertical branches where they dip into the liquid, the work done during one complete revolution of the apparatus is $\pi a^2 ZC$, where C is the strength of the current in both branches taken together.

CHAPTER XXIV.

MAGNETISATION BY CURRENTS.

269. Arago's Experiment.—As the field of a current is of the same nature as that of a magnet, it should produce the same magnetising effects. The fact of the magnetisation of iron by a current was discovered by Arago in 1820. He observed that a copper wire traversed by a current and dipped in iron filings attracts them, and is uniformly covered with them. Each particle of iron becomes a magnet, and sets at right angles to the wire along the line of force, the north pole being to the left of the current. The action exerted on the two poles has a resultant directed towards the axis of the wire (§ 259).

In like manner a bar is magnetised when it is set crosswise with the current. The action is increased, as in the multiplier, by coiling the wire round and round in a plane perpendicular to the axis of the bar. The direction of the magnetisation is that of the lines of force in the interior of the coil. It is to be observed that this direction is independent of the direction of the winding, and only depends on the direction in which the current circulates about the axis. These two directions are related to each other in the same way as the lengthways movement of a right-hand screw is related to its motion of rotation. Several formulæ have been used to connect the position of the poles with the direction of the current. The simplest is that of Ampère: *the north pole is to the left of the*

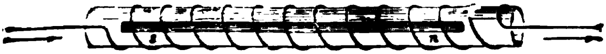

FIG. 241.

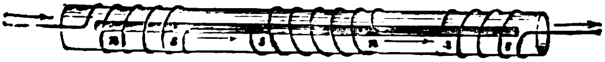

FIG. 242.

current. It is also often said that the north pole is at that end of the helix where the apparent direction of the current is opposite to that of the hands of a watch.

If the direction of the winding is abruptly changed by bending the wire back on itself, the direction of the rotation is changed, and therefore that of the magnetisation. The experiment is easily made by coiling a copper wire on a glass tube, and placing a knitting-needle inside the tube (Figs. 241 and 242). On passing a current through the wire, the needle is found to be strongly magnetised, and there are as many reversals of poles, or *consequent poles*, as there are reversals of the current. These poles are alternately of opposite signs. Those of the ends are of the same kind, or of opposite kinds, according as the number of reversals is odd or even.

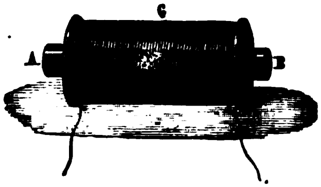

FIG. 243.

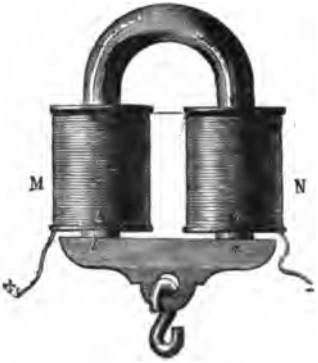

FIG. 244.

270. Electro-Magnets.—The magnetisation of soft iron by the current is of extreme importance in the applications of electricity. Not only can the magnets thus obtained be made much more powerful than steel magnets, but the property they possess, more especially if the iron is quite soft and the bars short, of being essentially temporary, of only existing during the passage of the current, of being created and destroyed, so to speak, with it, renders them suitable for a host of applications.

The temporary magnets thus obtained by the action of the current on soft iron are called *electro-magnets.* The conducting wire, insulated by silk or cotton, is wound in one or more layers round the core of soft iron.

The soft iron core may be straight, as in Fig. 243, or in the shape of a horse-shoe, as in Fig. 244. In the latter case the windings are not generally continued round the bend. In order that the poles at the two ends may be of opposite kinds, the wire should go round each branch in the same direction as it would do if the core had been bent after the winding had been finished. The windings ought to appear in opposite directions on the two legs to an observer who is looking at the two ends, the current going like the hands of a watch round the south pole, and in the opposite direction round the north pole (Fig. 245).

Owing mainly to the difficulty of bending massive iron bars without imparting to them a coercive force, large horse-shoe magnets are usually formed with parallel coils, the cores of which are connected by a cross-piece or *yoke*, T, of soft iron (Fig. 246).

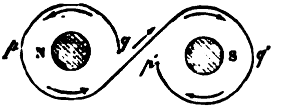

FIG. 245.

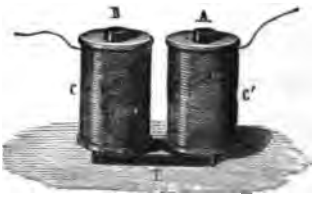

FIG. 246.

271. General Observations on Electro-Magnets.—The direction of the force at a given point in the magnetic field of a coil, and its *relative* intensity as compared with that at any other given point in the field, are independent of the strength of the current, and depend only on the geometry of the coil—that is, upon its size and shape, and on the number of turns of wire; but the *absolute* intensity of the field is proportional to the strength of the current. If, however, a piece of soft iron is placed inside the coil, its magnetisation varies with the strength of the current in a very complicated way, which it is not possible to include under any general law. In the following remarks, therefore, we shall confine ourselves to a few general considerations and the discussion of some special cases.

We may suppose the cross-section of the channel divided into any number of equal parts, and the whole mass of metal split into the same number of rings; whatever this number may be, the magnetic force is always the same, and so also is the production of heat. But this system of rings traversed by equal and parallel currents is equivalent to a coil of the same number of turns, which would give in the section of the channel the same mean density of current. From this the remarkable consequence follows, that whatever be the size of the wire coiled on the bobbin, for the same volume of wire, and therefore the same weight of copper, the magnetising force is the same, as well as the production of heat, provided the mean density of the current remains constant.

Let d be the diameter of the bare wire, $D = d(1 + \delta)$ that of the covered wire, l the length of wire, ρ its specific resistance in ohms, and a the density of the current in amperes per square centimetre: we have

$$R = \frac{4\rho l}{\pi d^2} \text{ and } C = \tfrac{1}{4}\pi d^2 a,$$

and therefore for the energy spent per second in generating heat

$$W = RC^2 = \tfrac{1}{4}\pi d^2 l \rho a^2.$$

As the space filled by the wire is

$$V = \tfrac{1}{4}\pi D^2 l = \tfrac{1}{4}\pi d^2(1 + \delta)^2 l,$$

the heat produced per unit of this space is

$$w = \frac{W}{V} = \frac{\rho a^2}{(1 + \delta)^2}.$$

Taking $\rho = 1600 \times 10^{-9}$ and $\delta = 0.035$, a number found experimentally for the thickness of the double layer of cotton which serves as insulator, we get $w = 1.5 \times 10^{-6} a^2$.

It is to be observed that the production of heat in the coil is the same with or without the soft iron core. Although energy is expended in producing magnetisation, there is no expenditure of energy in maintaining it, any more than there is in keeping a spring in a fixed position after it has once been compressed. As all the useful expenditure reduces to the heating of the wire of the coil, the conditions of maximum work are satisfied when the resistance of the wire of the electro-magnet is equal to that of the rest of the circuit (§ 131).

272. Case of a Long Coil.—The simplest case is that of a long cylindrical coil, uniformly wound, surrounding a thin cylindrical bar parallel to the axis (Fig. 247). The field inside the coil is uniform, and its value for a current of strength C, and a number of turns n_1 for unit length is (§ 256)—

$$F = 4\pi n_1 C.$$

The magnetisation of the cylinder, if it is so long that the action of the ends may be neglected, is also uniform, and the intensity of magnetisation is

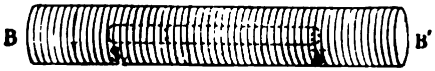

FIG. 247.

$$A = kF = 4\pi n_1 kC.$$

The value of the induction (§ 192) is

$$B = \mu F = 4\pi n_1(1 + 4\pi k)C,$$

and the flux or number of lines of force for the entire section S of the bar is

$$Q = BS.$$

For a given current, the magnetising force thus depends only on the number n_1 of turns per unit length, and not at all on their shape or size. In order to economise wire and reduce the expenditure of energy in generating heat, it is desirable to wind the wire directly on the core. The effect of unit length of the wire is then made as great as possible.

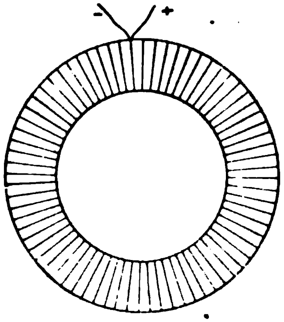

FIG. 248.

273. Ring Magnet.—If the core has the shape of a *tore* or anchor ring covered with equi-distant windings in planes passing through the axis (Fig. 248), the system may be regarded as formed of concentric closed solenoids having no external action. The elementary solenoids are not, however, all of equal strength, seeing that those near the outside of the ring are longer than those near the inside circumference, although the current passes round all of them the same number of times.

But if the section S is small compared with the radius of the ring, it may be assumed, without appreciable error, that the strength of each of the filaments is the same as that of the central filament; if n is the number of turns of wire for unit length of the mean circumference, we have as the approximate value of the flux of induction through a cross-section of the ring, the strength of the current being C,—

$$Q = BS = \mu FS = \mu 4\pi n_1 CS.$$

Multiplying both sides by the length l of the mean circumference, and calling n the total number of turns of wire, we have

$$Ql = \mu 4\pi nCS;$$

or, again,

$$4\pi nC = \frac{1}{\mu}\frac{l}{S}Q \quad . \quad . \quad . \quad (1).$$

274. The reasoning of § 256 gives this expression directly: let a unit pole travel along an infinitely thin channel coinciding with the mean circumference, F being the value of the force (§ 187), and l the length of the path traversed, the work done is Fl; on the other hand, it is equal to $4\pi C$, multiplied by the number of turns of wire passed through (§ 253); hence

$$Fl = 4\pi Cn_1 l.$$

Replacing F by its value $\frac{1}{\mu}\frac{Q}{S}$ (§ 192), we arrive at the expression (1).

The quantity Q may, as we shall see (§ 307), be determined by experiment.

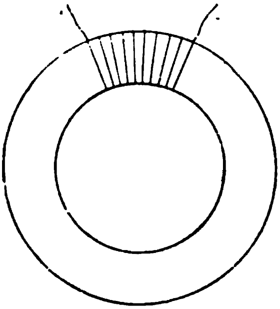

FIG. 249.

275. If the core is only covered for a part of its length (Fig. 249), some lines of force will escape at the ends of the coil, but generally in so small a number that we can assume the flux constant and still apply the formula (1).

In this case the magnetisation of the ring only depends on the product nC of the strength of the current into the number of turns of wire, and not on the manner in which these turns are arranged. This product is called the number of *ampere-turns*, when the current is expressed in amperes, in which case the second member of equation (1) should be multiplied by 10. Dividing by the length l of the circumference, we get the number of ampere-turns per centimetre of the ring. The same degree of magnetisation corresponds to the same number of ampere-turns per centimetre of length, whatever be the section and length of the ring. The two terms of the product may be varied at will; ten turns of a wire which conveys one ampere produces the same effect as a single turn which conveys ten amperes.

These consequences are not confined to the special case of an anchor ring; they apply to any closed magnetic circuit from which no lines of force escape at the surface, and which may therefore be considered as identical with a tube of force. They are not applicable to the case of a circuit which is not closed—a straight bar, for instance, which allows lines of force to escape all over its surface.

276. Magnetic Resistance or Reluctance.—Formula (1) of § 273,

$$4\pi nC = \frac{1}{\mu}\frac{l}{S}Q,$$

may be compared with that of Ohm,

$$E = \rho\frac{l}{S}C.$$

Ohm's law, when expressed in the form of the last equation, may be taken as a statement that the difference of electric potentials, E, between the ends of a conductor is equal to the electric flux, or strength of current, multiplied by a factor depending on the material of the conductor and by a geometrical factor l/S. The corresponding magnetic equation states that the difference of magnetic potentials $4\pi nC$ is equal to the magnetic flux multiplied by a factor characteristic of the material, and by the same geometrical factor as in the other case.

This parallelism of formulæ has led to the somewhat general use of similar terminology in reference to the two classes of phenomena. The *magnetic circuit* is compared with the electric circuit: as E in the case of the latter is called electro-motive force, so $4\pi nC$ in the former is called *magneto-motive force*, and, by analogy with electric resistance, $\frac{l}{\mu S}$ is called *magnetic resistance*. It may be remarked in passing, that the last analogy implies the comparison of μ with the reciprocal of specific resistance: this would be expressed by calling it specific magnetic conductivity, an idea already embodied in the accepted name *permeability*.

With regard to these comparisons, it is to be observed that there is no known magnetic phenomenon which is physically analogous to electric conduction, whether in the form of disruptive discharge through a dielectric, or in that of conduction through metals or electrolytes. Physically, the analogy between magnetic permeability and specific inductive capacity is much closer than that between permeability and metallic conductivity, and the true electric analogue of a magnetised piece of iron is rather a polarised dielectric than a conductor carrying a current. Just as every terminated magnet has two equal opposite poles, and as new poles of the same strength are generated by dividing the magnet and interposing a stratum of air, so any portion of a polarised dielectric, bounded by conducting surfaces, has equal opposite charges on its two ends, and new charges of the same magnitude make their appearance whenever the dielectric is divided by an interposed conductor. If we were acquainted with liquid or gaseous bodies capable of assuming a high magnetic intensity, and thus comparable with liquid and gaseous dielectrics, the above analogy would be much more obvious and striking.

A further failure in the analogy between the electric circuit and the magnetic circuit arises from the fact that whereas the electric resistance of a conductor is independent of the strength of the current, the so-called magnetic resistance depends essentially upon the magnetic induction. It is, therefore, undesirable to use the same word in reference to both properties. In order to retain the advantage of the real mathematical parallelism between the laws of the electric and magnetic circuits, without implying a closer physical analogy than really exists, the term *reluctance* has been employed instead of resistance in the magnetic case, and we shall adopt it in what follows.

277. Calculations relating to the Magnetic Circuit.—The analogies pointed out above may be employed for the discussion of the conditions existing in a magnetic circuit, and lead to results which are in close accord with experiment. In making such applications we have to treat the lines of magnetic force as always forming closed curves, whether they traverse magnetic substances, or air, or other non-magnetic bodies. In circuits of complicated shapes there are difficulties in the way of precise numerical calculation of exactly the same kind as those which would present themselves in the application of Ohm's law to similar cases; and if a circuit includes relatively large portions of non-magnetic material, further difficulties arise in determining the course of the lines of force. Allowing for such considerations, we may calculate with magneto-motive forces and reluctances in exactly the same way as with electro-motive forces and resistances.

Kirchoff's rules (§ 116) apply to the case of divided circuits. Thus the fluxes Q, Q', Q'' . . . of the branches which terminate at the same point satisfy the equation

$$Q + Q' + Q'' \dots = 0 \quad . \quad . \quad . \quad (2).$$

In a closed contour the magneto-motive force is equal to the sum of the products obtained by multiplying the flux of each branch by the corresponding reluctance. Thus if n is the algebraic sum of the turns, C the strength of current, while $l\ l'\ l''$ . . . represent the lengths of the various segments, $\mu\ \mu'\ \mu''$ . . . the permeabilities, $S\ S'\ S''$ . . . the sections, and $Q\ Q'\ Q''$ . . . the fluxes which traverse them—

$$4\pi n C = \frac{1}{\mu}\,\frac{l}{S}Q + \frac{1}{\mu'}\,\frac{l'}{S'}Q' + \frac{1}{\mu''}\,\frac{l''}{S''}Q'' \quad . \quad . \quad . \quad (3).$$

278. Construction of Electro-Magnets.—The theory of the magnetic circuit accounts for all the facts presented by electro-magnets, and enables us to establish exact rules for their construction. It leads to the same conclusions as the consideration of the demagnetising action of the ends. It shows why the force exerted by an electro-magnet upon its armature diminishes so rapidly when the distance between them increases. As the permeability of air is several hundred times less than that of iron, the interposition of a layer of air is equivalent to a great increase in the length of the core; the number of ampere-turns remaining the same, the induction is greatly decreased. Hence the necessity of having the surfaces in contact well worked and smooth; even then a joint always introduces an appreciable reluctance, and, other things being equal, an electro-magnet with a yoke (Fig. 246) is not so good as a bent electro-magnet (Fig. 244).

In planning an electro-magnet, it is generally assumed that a particular value of the induction B is to be obtained with the wire used. Too high a value must not be taken, since, beyond a certain limit, the increased expenditure is out of proportion to the gain of magnetisation. In the case of an electro-magnet which is intended to support a weight and to act in closed circuit, the weight to be lifted is given. The polar surface is then determined by the equation (§ 224)—

$$Pg = \frac{B^2 S}{8\pi},$$

and the number of *ampere-turns* per centimetre by the formula

$$\frac{4\pi n C_a}{10 l} = \frac{B}{\mu}.$$

The value of B being given, that of μ is also defined by the curve of magnetisation (§ 202) for the iron in question.

The length, l, of the circuit of soft iron is arbitrary; it is economical to take it as small as possible, and give the electro-magnet a compact form like that in Fig. 250. There must be space enough to contain the wire necessary. The volume of the wire is determined by the condition of not exceeding a certain temperature. The thickness of the wire depends solely on the current to be used.

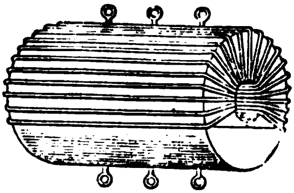

FIG. 250.

The rule to be followed is that the coil should have such a surface that for a given excess of temperature over the surrounding medium, the rate of loss of heat by radiation may be equal to the rate of production of heat by the current. Experiment shows that, with ordinary coils, the radiation per square centimetre per degree may be taken as about 0.001 watt. If we know (§ 271) the heat produced per cubic centimetre, it is easy to calculate the necessary surface, and therefore the number of layers of wire for any value of the density of the current.

For a straight electro-magnet, the simplest course is to consider the core as an ellipsoid, and calculate the value of the coefficient N (§ 193). Since k is given, we know the ratio of the effective force to the field, and, multiplying this by 0.4π times the number of ampere-turns, and by μ, we get the induction.

279. Ruhmkorff's Electro-Magnet.—In order to obtain an intense field, the two polar surfaces are placed opposite each other, so as to prevent as much as possible dispersion of lines of force in the intervening space. This is the arrangement which Ruhmkorff used in his electro-magnet (Fig. 251). The two coils, S and S′, are placed opposite each other with their axes in the same line, and they are connected by a sliding yoke, E E′, by which they can be placed at various distances, and clamped where required. In most cases the section of the yoke is made too small, and many lines of force escape at P and E.

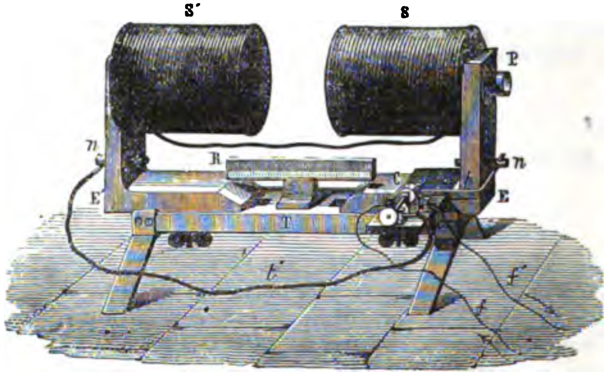

FIG. 251.

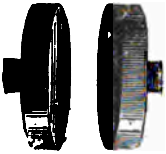

FIG. 252.

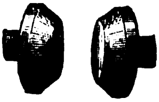

FIG. 253.

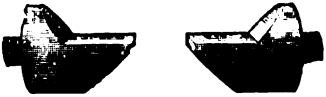

FIG. 254.

In order to get fields either uniform or variable, the ends of the cores are provided with flat discs (Fig. 252), or conical (Fig. 253) or bevelled pieces (Fig. 254).

The wire makes about 1500 turns, and can bear a current of 25 amperes without being too much heated, which gives 37,500 ampere-turns as a maximum. In these conditions experiment shows that a field of about 14,000 units is obtained between two poles in the shape of truncated cones (Fig. 253), 1 centimetre apart.

280. Polarised Electro-Magnets.—This term is given to magnets, the cores of which are already magnetised by a permanent magnet (Fig. 255). This arrangement is used whenever very sensitive electro-magnets are wanted, which rapidly obey feeble currents. The magnetisation curve (§ 202) rises very slowly for weak currents; but if the soft iron is so arranged that it has a magnetisation nearly corresponding with the point of inflection, the least current produces a considerable variation in the magnetisation.

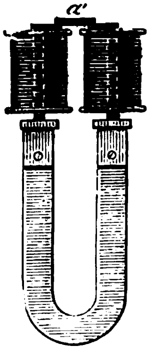

FIG. 255.

281. Transverse Magnetisation.—Suppose that an iron or steel wire is wound spirally on a glass tube, and that a current is passed through a copper wire in the axis of the tube; if C is the strength of the current, a the external radius of the tube, the magnetic force exerted by the infinite current on any element of the iron wire in the direction of its axis is equal to $\frac{2C}{a}$ (§ 246). The iron wire becomes a solenoidal filament, and exerts no force at an external point; but if it is uncoiled, its ends have poles of opposite kinds.

The effect is the same on an iron wire traversed by a current. The surface is transversely magnetised, and may be regarded as formed of circular solenoidal filaments perpendicular to the axis. As all these solenoids are closed, there is no external force. But if the wire were sawn longitudinally along its axis, the two edges forming the limit of each section would present the properties of two polar lines of opposite signs.

282. Magnetisation of Steel.—Currents are ordinarily used instead of permanent magnets for magnetising steel bars. A ring is formed of several turns of stout wire, through which a strong current is passed. The ring is placed round the middle of the bar, and then when the circuit is closed, it is passed from one end of the bar to the other; after having been carried the same number of times over each half of the bar, the ring is brought to the middle again and the current is stopped.

With horse-shoe magnets a double ring is used coiled in the form 8; the limbs of the horse-shoe are placed one in each loop, and the ring is moved backwards and forwards several times along the limbs. The magnetisation thus obtained is both strong and regular.

283. Rotatory Magnetic Power.—Any substance, whether solid, liquid, or gaseous, when placed in a magnetic field acquires the property of rotating the plane of polarisation of a ray of light which traverses it. The effect is greatest when the direction of the ray is the same as that of the lines of force of the field; it vanishes when these two directions are at right angles to each other. The action is less with doubly refracting than with singly refracting substances. Faraday's heavy glass (borosilicate of lead) among solids, and bisulphide of carbon among liquids, show this effect in an especial degree.

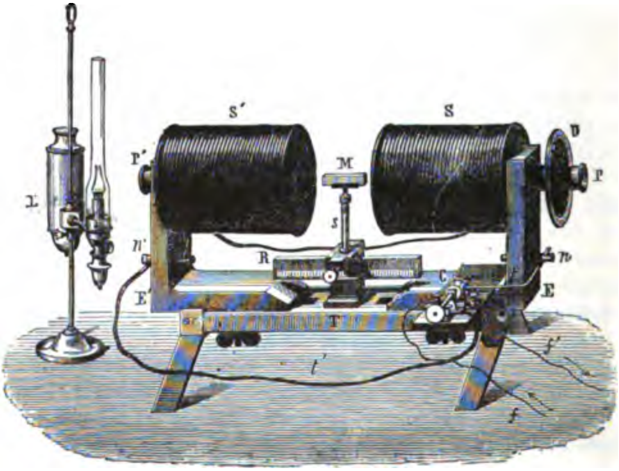

FIG. 256.

The experiment is easily made with a Ruhmkorff's electro-magnet (Fig. 256); the iron cores are hollow, and rays of light from a lamp, L, travel along the axis P′ P; at P′ is the polariser, while P is the analyser. Suppose the light is homogeneous; the analyser being set so as to extinguish the light, when the current is passed the light reappears, and in order to extinguish it again the analyser must be turned through an angle which can be read off from the limb D, and is a measure of the rotation. By reversing the current, an equal rotation is obtained in the opposite direction.

For a given substance the direction of the rotation is independent of the direction in which the rays travel; it is the same whether the propagation is in the direction of the lines of force or in the opposite direction. It follows from this that if the ray turns back again, and traverses the substance a second time in the opposite direction, the rotation is doubled. Thus, by increasing the length of the path of the ray by successive reflections (Fig. 257), the rotation can be increased in the same proportion.

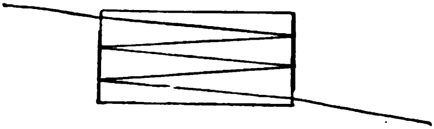

FIG. 257.

For all diamagnetic substances the direction of the rotation is that of the current which produces the field. This direction we shall take as *positive*. For most magnetic substances, however, the rotation is negative.

284. Verdet's Law.—*The rotation of the plane of polarisation between two points is proportional to the difference of magnetic potential between these two points*, or, in other words, *to the work corresponding to the displacement of unit pole from the first point to the second.*

If V and V' are the magnetic potentials at the two points in question taken on the path of the ray, the angle θ through which the plane of polarisation is turned is expressed by the equation

$$\theta = \omega(V - V'),$$

ω being the rotation which, for the body in question, corresponds with unit difference of potential. This quantity has been called *Verdet's constant;* it defines the rotatory power of the body. For the different rays of the spectrum it varies *nearly* as the inverse square of the wave length.

For the ray D and the temperature 0°, the value of the constant is $0'.043$ for carbon bisulphide, and $0'.013$ for water. It is less as the temperature is higher; for carbon bisulphide we have

$$\omega_t = \omega_0(1 - 0.00104t - 0.0000014t^2).$$

285. Optical Galvanometer.—The rotation of the plane of polarisation furnishes a convenient means of measuring the strength of a current in absolute value.

Let a length e of the substance be placed in a uniform field, for instance along the axis of a long coil with n_1 turns for unit length. The difference of potential per centimetre is $F = 4\pi n_1 C$ (§ 256), and if θ is the observed rotation we have

$$\theta = \omega 4\pi n_1 Ce,$$

an expression which only requires two measurements, those of e and θ.

Or, more simply still, let the substance (carbon bisulphide, for instance) be placed in a long tube closed at both ends by glass plates, and round the middle let there be a coil formed of n turns of any given shape and size (Fig. 258). If the tube is so long that the force due to the coil at the ends may be neglected, the difference of potential between the ends is $4\pi nC$, and the rotation is therefore

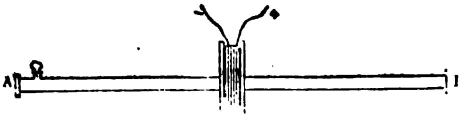

FIG. 258.

$$\theta = \omega 4\pi nC.$$

The method requires no measurement of the coil, nor accurate placing of it; all that is needed is the number of turns of wire.

The above formula assumes that the tube is infinitely long in both directions. It is easy to find the required correction: the coil remaining fixed, the tube is moved so that the end A comes into the place of the end B; the correction will be twice the rotation measured in this new position.

286. Hall's Phenomenon.—Suppose a very thin metal plate (Fig. 259) provided with four electrodes, A, B, *a*, *b*, the two first of which can be connected with a battery, and the two others with a galvanometer. When the current is made between A and B, the electrodes *a* and *b* can be moved about, and points found by trial such that no current passes in the galvanometer. These two points are then at the same potential.

When this adjustment has been made, if the plate is placed in a strong magnetic field at right angles to the lines of force, the deflection of the galvanometer shows that a portion of the current AB passes through the galvanometer. It is also found that an instantaneous discharge, other things being equal, divides in the same way and in the same proportion as a continuous current.

If the current is from A to B, the displacement is in the direction of the electro-magnetic force ϕ, that is, from *a* through the galvanometer to *b* in the case of iron, cobalt, and zinc; and in the contrary direction, that is to say, from *b* through the galvanometer to *a*, with gold, nickel, and bismuth. With platinum and lead there seems to be no effect.

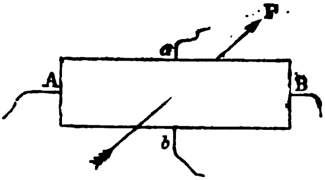

FIG. 259.

Let C be the strength of the original current, c the strength of the current in the galvanometer, F the intensity of the field: if V_a and V_b are the potentials at the points a and b, and R the resistance between these two points, we have from Ohm's law—

$$E = V - V_b = cR.$$

Experiment shows that if e is the thickness of the plate, and G a constant, the law of the phenomenon is given by the formula

$$E = G\frac{CF}{e}.$$

The following are the values of the constant G:—

Bismuth	− 8580
Nickel	− 14
Gold	− 0.66
Zinc	+ 0.83
Cobalt	+ 2.40
Iron	+ 7.85
Antimony	+ 14.00

The sign + represents the case in which the current is in the direction of the electro-magnetic force.

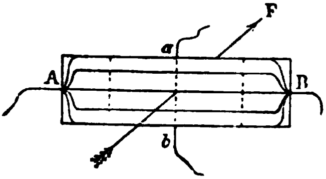

FIG. 260.

The phenomenon is due to a modification of the lines of flow of the current under the influence of the magnetic field. Outside this field, if the plate is homogeneous, and has the same thickness throughout, the current at a small distance from the electrodes A and B is uniformly distributed; the lines of flow are parallel to AB, and the equipotential lines are perpendicular to them (Fig. 260).

But when the field is set up, the lines of flow and the equipotential lines are deformed in the case of antimony, as shown in Figs. 261 and 262; the potential of the point *a* is higher than that of *b* in the former case, and lower in the latter.

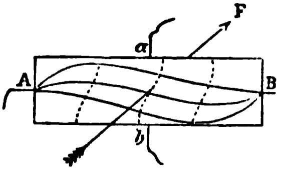

FIG. 261.

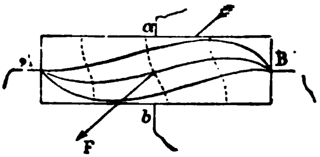

FIG. 262.

Under the same circumstances the resistance of bismuth increases. This increase of resistance may serve to measure the intensity of the magnetic field.

CHAPTER XXV.

INDUCTION.

287. Induction Currents.—Whenever the flux of magnetic force through a closed circuit is changed, the circuit becomes the seat of a temporary current which lasts as long as the variation of the flux. Such a current is called an *induction current* or *induced current.* Induction currents were discovered by Faraday in 1831, and we shall here describe some of his fundamental experiments.

288. Induction by Currents.—A closed circuit, AB (Fig. 263), contains a galvanometer, G; a second circuit, parallel to the first for a great part of its length, is connected with a battery, and can be closed or broken, or the current can be reversed at pleasure. The former will be called the *induced* or *secondary* circuit, and the latter the *inducing* or *primary* circuit.

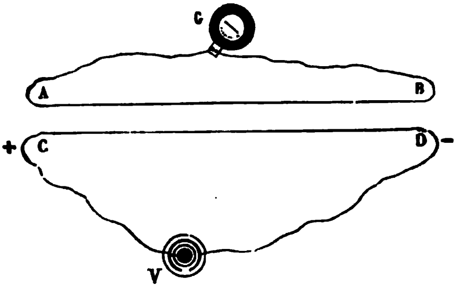

FIG. 263.

Every time the battery circuit is closed, the induced circuit AB is traversed by a momentary current in a contrary or *inverse* direction to that in the parallel portion CD of the inducing circuit. Every time the inducing circuit is broken, the secondary is traversed by a momentary current in the same direction as the inducing current, that is, by a *direct* current.

Nothing is observable in the induced circuit so long as a steady current passes through the inducing circuit, and so long as it remains at the same distance. But when the wire CD is brought near AB, or when the current in CD is increased, an *inverse* induced current is produced in AB; while if the wire CD is moved away or the strength of its current is decreased, a *direct* induced current is produced.

In short, a current which begins to flow, or increases in strength, or is brought nearer, produces in an adjacent circuit an *inverse* induced current; a current which ceases, or diminishes in strength, or is moved away, produces a *direct* induced current. These currents, which are produced by the influence of other currents, were spoken of by Faraday as currents due to *volta-electric* induction.

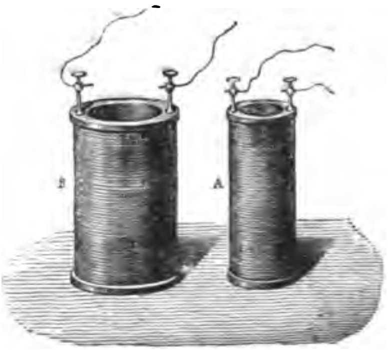

FIG. 264.

The effects are stronger the nearer are the two wires, and the greater the length of the parallel portions. The experiment is ordinarily made with two coils of insulated wire, one of which (Fig. 264) can be placed inside the other, like A and B.

289. Induction by Magnets.—Suppose the coil B (Fig. 264) to be connected with a galvanometer: if a bar magnet is inserted in it, the effects are the same as though a second coil, A, carrying a current, had been put in in place of the magnet. In order to find the direction of the induced current, we may suppose the magnet replaced by the equivalent electro-magnetic cylinder (§ 256). In like manner any increase in the strength of the magnet produces an inverse current, and any diminution a direct current. This action Faraday called *magneto-electric induction.*

Both effects are obtained *simultaneously* and with far greater strength by placing a *core* of soft iron, D, in the inducing coil (Fig. 265). When the current is made, the cylinder of soft iron is magnetised, and the two actions, of the coil and of the magnet, which are evidently in the same direction, are added together. They are also added when the current is broken.

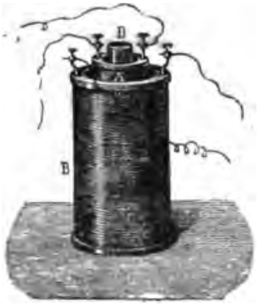

Fig. 265.

290. Induction by the Action of the Earth.—If a closed circuit be displaced or deformed in the magnetic field of the earth so that the number of lines of force passing through it is altered, an induced current is produced, the direction of the current depending upon whether the change of the number of lines of force through the circuit is positive or negative. Thus if a coil, held with its axis in the line of dip, and connected by long wires with a galvanometer, is quickly turned through an angle of 180°, the galvanometer needle is deflected.

291. Self-Induction—Extra Current.—Lastly, Faraday showed that any variation in the strength of a current produces in the circuit itself, in which the current passes, an induction current, which is superposed on the principal current, and always opposes the actual variation of strength, tending to weaken a current which is increasing, and to strengthen one that is decreasing. This phenomenon is called the *induction of a current on itself*, or, more briefly, *self-induction*, and the resulting current is called an *extra current*. The effect is especially marked in circuits which contain coils or electro-magnets. In this case the spark, on breaking the circuit, is much stronger and louder than the spark on closing the circuit, and if the body is interposed in the circuit, the extra current may produce strong physiological effects. Faraday used the following method for showing the extra current produced on making and breaking the circuit :—

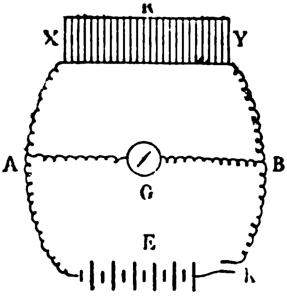

Fig. 266.

A coil, R (Fig. 266), is connected with a battery, E, and a key, K, by which the circuit can be closed or opened, and a galvanometer, G, is connected in multiple arc with the coil, so that the battery-current divides between it and the coil. Let *a* be the position of equilibrium of the needle under the influence of the current which passes through the galvanometer when the circuit is closed at K, and a stationary condition is established. The needle is kept at the angle *a* by means of a stop (Fig. 267), which prevents it from going back to zero when the circuit is interrupted. On again closing the circuit, the needle is deflected beyond *a*, because the "extra current" delays the growth of the current in R, and at first more than the normal proportion flows along AGB.

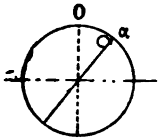

FIG. 267.

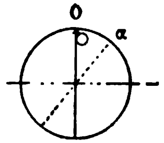

FIG. 268.

Next, the stop is placed at O (Fig. 268), so as to keep the needle at 0°, while a steady current is again made to traverse the circuit; when the circuit is now broken at K, the needle is deflected in the opposite direction, because the current in the coil continues for an instant to flow after the battery-circuit has been broken. If the original directions were AXYB and AGB, the current continuing along XY, after K is open, causes a current through the galvanometer from B to A.

292. Characteristics of Induction Currents.—All the phenomenon of induction have the common characteristic that they correspond to a modification of the magnetic field enclosed by the induced circuit, whether this field be due to currents or to magnets, or to a current in the circuit itself. It remains to establish the numerical laws of these currents.

We will first consider currents due to a displacement of the circuit.

Almost immediately after the discovery of induced currents, Lenz gave a very simple rule for their direction, which is this: *the direction of an induced current is always such that by its electro-magnetic action it tends to oppose the displacement.*

In investigating the conditions which determine the *strength* of an induction current, we may remark, in the first place, that the existence of such a current implies the existence of an induced *electromotive force*, on which the strength of the current depends in the way expressed by Ohm's law. As the conditions determining the magnitude of the electromotive force are decidedly simpler, we shall examine them first.

Going back to § 130, we know that, when work is done at the expense of the energy of an electric current, an inverse electromotive force is generated, the magnitude of which is expressed by the equation

$$e = \frac{W}{C},$$

where W is the work done in unit of time as the result of a current of strength C. If W is measured in watts, and C in amperes, e is given by this equation in volts.

We have seen further (§ 252), that when a conductor carrying a current moves in a magnetic field so as to cut lines of force, work is done equal to the product of the strength of the current into the number of lines of force cut through; or, more generally (§ 261), that when the flux of magnetic force through the area of a circuit alters, work is done equal to the product of the strength of the current into the increment of the magnetic flux.

Apply these considerations to a case such as that represented in Fig. 269, where AB and A′B′ are parallel metal rails connected by a cross-bar CC′, which can slide along them at right angles to its own length, and P is a battery by which a current can be sent through the rails and cross-bar. Suppose the plane of the rails and bar to be perpendicular to the direction of a magnetic field of intensity H, let the direction of the current be from A to C and C′ to A′ in the figure, and the direction of the magnetic force to be as indicated by the arrow Z. Then the electro-magnetic forces will tend to enlarge the area of the figure, so as to make it enclose a greater number of lines of force, and the sliding bar will move towards BB′. Let ϕ be the force tending to displace the bar: while it moves through a small distance ds, the work done will be $dW = \phi ds$. But (§§ 252 and 261) we have also $dW = CdQ$, if C is the strength of current and dQ the increment of magnetic flux through the circuit caused by the displacement. Repeating the reasoning of § 130, we may write the following equation, which

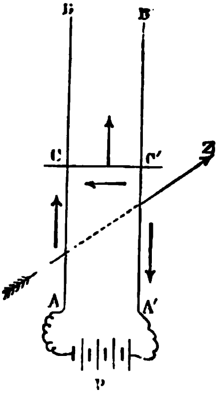

FIG. 269.

states that the energy received by the circuit in an element of time, dt, in consequence of the electromotive force E of the battery, is equal to the energy expended in generating heat in and doing work:

$$ECdt = C^2Rdt + CdQ.$$

If the sliding bar were not allowed to move, that is, if no work were done, the strength of the current would be C_0, and the above equation would become

$$EC_0dt = C_0^2Rdt.$$

It is evident that C is not the same as C_0, and, as the resistance is the same in both cases, a change in the strength of the current can only be due to a change in the effective electromotive force of the circuit—in other words, to the generation of an additional electromotive force e. The effective electromotive force is the algebraic sum of the electromotive forces of the circuit, and by Ohm's law we may write

$$E + e = CR.$$

But if we divide the first of the above equations by C and by dt, we get

$$E - \frac{dQ}{dt} = CR,$$

and therefore

$$e = -\frac{dQ}{dt} = -Hl\frac{ds}{dt} = -Hlv.$$

It is to be noted that the induced electromotive force e is independent of the strength of the current, and therefore has the same value even if there is no other electromotive force, and therefore no current except that due to induction. It depends only on the rate of increase of the magnetic flux through the circuit. This, in the case supposed, is equal to the strength of the magnetic field H multiplied by the distance l between the rails and by the velocity, $v = \frac{ds}{dt}$, with which the bar moves.

The negative sign in the expression for e receives its interpretation from Lenz's law, of which, in fact, it is the expression. It indicates that the electromotive force, due to an increase of the magnetic flux in a given direction, would, if it acted alone in a circuit, produce a current causing a magnetic flux in the opposite direction. Thus if, in the case already considered, the battery is replaced by a conducting wire R (Fig. 270), motion of the cross-bar from A towards B will produce a current in the direction of the arrows, that is, opposite to the battery current in the previous case. The current is inverted if the motion is from B towards A. These effects can be rendered visible by including a galvanometer between A and A′.

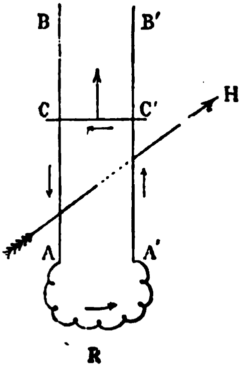

FIG. 270.

293. General Law of Induction—Electromotive Force.—The general law of induction is only a generalisation of the foregoing. Whether the circuit moves across the lines of force, or whether the lines of force move in reference to the circuit, whether or not the electro-magnetic forces do work, that is to say, whether there is or is not a displacement of their points of application, each element of the circuit which in the time dt cuts a number of lines of force dq, is the seat of an electromotive force,

$$de = -\frac{dq}{dt},$$

whose direction is from foot to head of an observer who, looking along the lines of force, sees them pass from right to left.

All these electromotive forces add up algebraically, like those of a number of galvanic cells connected in the same circuit.

If the conductor does not form a closed circuit, the two electricities are accumulated at the opposite ends until a difference of potential is produced equal to the algebraic sum of the elementary electromotive forces.

If the circuit is closed, the total electromotive force is likewise the algebraic sum of the electromotive forces corresponding to the different elements: and since the algebraic sum of the lines of force cut by all the elements is equal to the variation dQ of the flux of force through the circuit, the electromotive force at the time t is

$$e = -\frac{dQ}{dt}.$$

If R is the resistance of the circuit, this electromotive force produces at the moment in question a current of strength

$$C = \frac{e}{R} = -\frac{1}{R}\frac{dQ}{dt}.$$

The direction of the current is such that its axis (§ 250) is in the opposite direction to the flux if dQ is positive, and in the same direction if dQ is negative.

294. Coefficient of Self-Induction.—It has been shown (§§ 244 *et seq.*) that the existence of an electric current in a conductor implies the existence of magnetic force in the neighbouring space. Indeed, in order to ascertain the existence, direction, and strength of a current, we usually examine, not the conductor itself, but the magnetic properties of the space near it. Whatever the ultimate nature of the phenomenon which we recognise and speak of as an electric current may be, it is an incomplete view of it to confine our attention to the process going on in the conducting circuit; it involves also, as an essential part, the existence of the correlative magnetic field.

Every complete circuit forms a closed curve, and the corresponding lines of magnetic force, which also form closed curves, are linked through it (see, for example, Figs. 218, 219). Hence, irrespective of any other magnet or circuit, there is, through the area of every circuit carrying a current, a flux of magnetic force depending on the current in the circuit. As already mentioned (§ 271), the magnetic field due to a current remains similar to itself whatever the strength of the current, but the force at any point of the field is proportional to the current. Hence the total flux through the circuit is proportional to the current, and may be represented by

$$Q = LC,$$

where L is a factor depending on the geometrical characters of the circuit,—its shape, the area enclosed, and the number of times the current goes round it,—and on the magnetic permeability of the surrounding medium.

The coefficient L is called the *coefficient of self-induction* of the circuit. It may be defined as the magnetic flux which, in consequence of a current of unit strength in the circuit, traverses the area of the circuit. It is a quantity of the same kind as the coefficient of mutual induction (§§ 184, 261) of two circuits.

295. Electromotive Force due to Self-Induction.—If we connect what has been said in the last paragraph with the result arrived at in § 292 as to induced electromotive force, we see that the self-induction of a circuit must give rise, in the case of any variation of the current, to an electromotive force tending to oppose the variation. In fact, the equations

$$e = -\frac{dQ}{dt} \text{ and } Q = LC$$

give, when L is constant,

$$e = -\frac{d(LC)}{dt} = -L\frac{dC}{dt}.$$

Hence when $\frac{dC}{dt}$ is positive, that is, when the current is increasing, there is a negative electromotive force due to self-induction; and when $\frac{dC}{dt}$ is negative, or the current is decreasing, the electromotive force of self-induction is positive. Whenever the strength of the current is constant, $\frac{dC}{dt} = 0$, the electromotive force of self-induction is nothing.

These results contain the proper interpretation of the phenomenon of the "extra current" (§ 291).

296. Work Spent in Establishing a Current—Intrinsic Energy of a Circuit.—If there were no such thing as self-induction, an electromotive force, E, applied to a circuit of resistance, R, would instantly give rise to a current of strength, $C = E/R$. The electromotive force of self-induction, however, causes the establishment of a current to be a more or less gradual process. It is true that in many cases it is so rapid as to need delicate observation to detect that it is not instantaneous, but in cases where the coefficient of self-induction has a high value, the growth of the current occupies a very appreciable time. This point is discussed more particularly in § 310.

During the variable period at the commencement of a current, more energy is imparted to the circuit by the battery than is given out by it in the form of heat or otherwise. The excess is consequently stored up, and goes to increase the energy of the circuit. Take again the equation

$$ECdt = C^2Rdt + CdQ,$$

already employed in § 292. In the paragraph referred to, dQ was an increment of magnetic flux through the circuit due to a deformation of the circuit in an independent magnetic field; in the present case, we take it to stand for an increment of the magnetic flux due to an increase in the strength of the current in a circuit of constant size and shape. Expressing the total flux by means of the coefficient of self-induction, we have $Q = LC$, and therefore $dQ = LdC$. Hence we may write

$$ECdt - C^2Rdt = LCdC.$$

In this expression, $LCdC$ clearly appears as the excess of the energy imparted to the circuit over that given out as heat.

If we put W for the total energy stored up while the strength of the current changes from an initial value, C_o, to a final value, C, we have

$$W = \int_{C_o}^{C} LCdC = \tfrac{1}{2}L(C^2 - C_o^2).$$

If the initial value $C_o = 0$, this gives

$$W = \tfrac{1}{2}LC^2$$

as the value of the work expended in establishing the current, or that of the intrinsic energy of the circuit.

297. Analogy between Electro-Magnetic Energy and Electro-Static Energy.—Using, as before, Q for the magnetic flux, we have the three equivalent expressions—

$$W = \tfrac{1}{2}LC^2 = \tfrac{1}{2}QC = \tfrac{1}{2}\frac{Q^2}{L},$$

which may be compared with the three forms in which the energy of an electro-static field (§ 55) may be expressed, namely,

$$W = \tfrac{1}{2}C(V - V')^2 = \tfrac{1}{2}Q(V - V') = \tfrac{1}{2}\frac{Q^2}{C}.$$

It is to be remembered that, in the latter case, C stands for electro-static capacity, and Q for electro-static charge.

The analogy between the energy of a conducting circuit and that of an electro-static field goes beyond the mere parallelism of the formulæ.

The seat of the energy of an electric current, or, as we may call it more shortly, electro-magnetic energy, is not the conductor along which the current is said to flow, but rather the surrounding medium, the seat of the magnetic field, which, as we have pointed out, is implied in the existence of the current. So long as the strength of current remains constant, the energy depends only on the geometrical conditions of the conductor, and not at all upon its physical properties; it does, however, depend on the magnetic permeability of the medium, for this is a factor of the coefficient of self-induction. In the electro-static case, the conducting boundaries of the field affect the energy only by virtue of their geometrical characters; but the dielectric medium affects it by virtue of its specific inductive capacity.

In either case, the distribution of energy in the field might be mapped or represented graphically, if we suppose drawn the tubes of force and equipotential surfaces. These, by their mutual intersections, divide the field into a number of compartments or cells, each of which is the seat of the same quantity of energy (§ 56). As the electro-static charge, in the one case, or the strength of the current in the other, increases, we must imagine more energy flowing out from the conducting surfaces into the medium, and the number of cells increasing. They become smaller and more closely crowded together; that is to say, more energy is accumulated in a given space. As the charge, or the strength of the current, diminishes, energy returns from the medium into the conductors, the cells expanding and decreasing in number till all are gone.

If f is the intensity of electro-static force at any part of a field, and K the dielectric coefficient, the energy in a portion of the field, whose volume, dv, is so small that the force may be taken as constant throughout, is

$$\frac{f^2 K}{8\pi}\, dv.$$

The corresponding expression in the electro-magnetic case is

$$\frac{F^2 \mu}{8\pi}\, dv,$$

where F is the intensity of magnetic force, and μ the magnetic permeability of the medium (comp. § 337).

298. Transfer of Electric Energy.—Besides the existence of a magnetic field, another necessary condition of the production of a current by a galvanic battery, or any equivalent arrangement, is a difference of potentials between different parts of the circuit, and this difference of potentials implies the existence of electro-static energy, which might be represented by tubes of force and equipotential surfaces. If the current is of constant strength, the potential remains constant at each point of the circuit and of the field, and consequently the equipotential surfaces are stationary, but each of them intersects the surface of the conductor at the part where its potential has the corresponding value. The tubes of force, which are necessarily orthogonal to the equipotential surfaces, consequently meet the surface of the conductor obliquely; they cannot, therefore (§ 33), be in equilibrium. Their ends run along the conductor towards the side on which they meet the surface at an acute angle, and at the same time they, as it were, sweep the electro-static energy into the conductor, where it appears as heat. The result would be the almost instantaneous disappearance of all energy, were it not for the battery, which pours electro-static energy into the field as fast as it is absorbed into the conductor. Part of the process going on in a closed circuit is thus a flow of electro-static energy from the battery across the field into the conducting wire: this corresponds to the generation of heat at the rate C^2R.

This flow or transfer of electro-static energy across the field is accompanied by the existence of magnetic force, and therefore of magnetic energy in the field. The lines of magnetic force coincide with the lines in which the surfaces of equal electro-static potential and of electro-static force intersect each other. So long as the flow of electric energy remains constant, the distribution of magnetic force and energy remains stationary. A constant electro-magnetic field thus accompanies and requires a constant rate of transfer and dissipation of electric energy. For a given form and size of circuit, the difference between what is called a good and a bad conductor may, from our present point of view, be stated thus: with a good conductor an electro-magnetic field of given intensity can be maintained with a smaller rate of dissipation of electric energy than with a bad conductor.

299. Energy of Two Circuits.—If two circuits, each carrying a current, are in presence of each other, the total energy is the sum of the intrinsic energies of the two circuits and of the energy due to their co-existence. If C and C' are the strengths of the two currents, L and N the two coefficients of self-induction, and M the coefficient of mutual induction, the whole energy is

$$W = \tfrac{1}{2} LC^2 + MCC' + \tfrac{1}{2} NC'^2.$$

300. Quantity of Electricity.—The quantity, dm, of electricity put in motion in the circuit during the time, dt, by an induced current is

$$dm = Cdt = -\frac{dQ}{R}.$$

The total quantity, m, corresponding to a finite variation of flux, is the sum of all such expressions. If we consider a variation in which both the initial and final values of the strength of the current are zero, the variations of the flux due to the current itself give a sum zero, and any resultant change of flux is due to the external field. If this be called $Q_2 - Q_1$, we have

$$m = \frac{Q_1 - Q_2}{R},$$

consequently *the total quantity of electricity put in motion by an induced current, the limiting values of which are zero, is equal to the quotient of the total variation of magnetic flux through the circuit by the resistance of the circuit.*

It depends neither on the time the variation has occupied nor on the manner in which it has taken place. The induced flow, m, is to be reckoned positive or negative in accordance with what has been already said in § 293.

CHAPTER XXVI.

SPECIAL CASES OF INDUCTION.

301. Applications.—We proceed now to discuss the application of the general principles pointed out in the last chapter to some important special cases.

The simplest is that in which, passing from a stationary condition where the flux through a circuit is Q_1 to another stationary condition in which the flux is Q_2, we wish to determine the quantity of electricity put in motion. The result of the last paragraph applies at once. If the variation takes place very quickly, the quantity, m, of electricity traverses the circuit in a momentary current or a sudden rush, and may be easily measured (§ 354).

A second equally simple case is that in which the variation of flux takes place uniformly. The circuit is the seat of a constant electromotive force, the value of which is given directly by the formula arrived at in § 293, namely, $e = -\frac{dQ}{dt}$, where $\frac{dQ}{dt}$ is the rate of increase of flux.

A more complicated problem is that of determining the strength of the current at each instant during the variable period. This is always given by the equation $ECdt = C^2Rdt + CdQ$, but the solution of problems of this kind often involves mathematical difficulties that we cannot attempt to deal with. We must, therefore, in some cases, only give the results.

302. Momentary Current due to the Displacement of a Circuit — Movable Circuit in a Uniform Field.—Consider a closed circuit of any given shape; let S be its surface, which we will suppose is plane. If the circuit is in a uniform field of intensity, F, and its plane, which at first is perpendicular to the field, is turned half round, the total variation of flux is $-2SF$, and if R is the total resistance of the circuit, the quantity of electricity put in motion is

$$m = \frac{2SF}{R}.$$

Suppose the case of the earth's field. If the rotation is about a *vertical* axis, the horizontal component, H, alone comes into play. The circuit, at first perpendicular to the meridian, being turned half round, we have

$$m_1 = \frac{2SH}{R}.$$

If the rotation is about a *horizontal* axis, the plane at first being horizontal, we need only consider the vertical component, Z, and we have

$$m_2 = \frac{2SZ}{R}.$$

Combining these two equations we get (§ 228)

$$\frac{m_2}{m_1} = \frac{Z}{H} = \tan I.$$

This method was used by Weber to determine the magnetic dip. The apparatus (Fig. 271) employed for this purpose is known as *Weber's Inclinometer.*

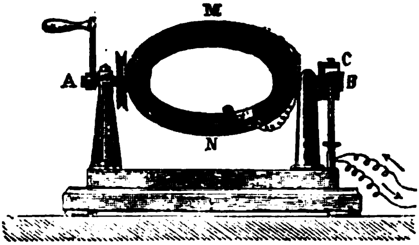

FIG. 271.

303. Measurement of a Magnetic Field.—By means of the same method, and using a very small coil movable about an axis in its plane, the intensity of a variable field may be measured. The plane of the coil, at first perpendicular to the direction of the field, is suddenly turned through 180°, and the resulting current is measured by the angle of throw of the galvanometer (§ 354). Thus, if F is the intensity of the field, and R the total resistance,

$$F = \frac{Rm}{2S}.$$

A measurement of the surface of the coil may be dispensed with by observing the effect of suddenly reversing the coil in a field of known intensity, that of the earth, for example.

304. Measurement of the Normal Component at the Surface of a Magnet.—Suppose that a small coil is applied to the surface of a magnet, as a proof plane might be, and that it is then turned half round, that is, through an angle of 180°. The current will give twice the flux which traverses the coil, and dividing by the surface, we should have the mean value of the normal component for the portion of the surface covered by the coil.

Another method may be adopted. Suppose that the bar is cylindrical, and is surrounded by a ring connected with a galvanometer. If the ring is moved from M to M′, the deflection of the galvanometer measures the flux cut during the displacement. This flux divided by the area of the cylindrical surface comprised between M and M′ gives the mean value of the component of magnetic force normal to the surface of the magnet.

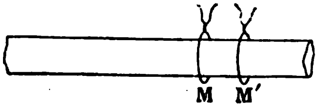

FIG. 272.

If the ring were pulled off the bar and were carried to an infinite distance, the current would measure the total flux from the part of the bar beyond the point M. If the ring fits closely round the cylinder, this is, by Gauss's theorem (§ 41), the flux which traverses the section M of the magnet.

305. Induced Current produced by a Variation of the External Field—Case of Two Coils.—Suppose the circuit stationary and the variation of magnetic flux to be produced in the external field. Let Q be the flux through the circuit; if this flux is suddenly withdrawn, we have an induced flow, $m = \frac{Q}{R}$; if the flux is restored, there is an equal flow in the opposite direction; if the direction of the flux is reversed without changing its value, we get twice the flow,

$$m = 2\frac{Q}{R}.$$

This is the case of a current induced in a closed circuit, A, by breaking, making, or reversing the current in a neighbouring circuit, B. Let C be the strength of the steady current in B, and M the value of the flux due to B which would pass through A if B were traversed by unit current, or, in other words, let M be the coefficient of mutual induction of the two circuits (§ 261), then $Q = MC$, and the value of the induced flow, corresponding either to breaking or making the inducing circuit B, is

$$m = \pm\frac{MC}{R}.$$

In both cases the same quantity of electricity is put in motion; but the law according to which the intensity varies is not the same. The time occupied by the cessation of the inducing current, even when it is prolonged by the spark (§ 312), is usually less than that which corresponds to making. Hence the electromotive force of the direct current is usually much greater than that of the inverse current, but its duration is less.

306. Coefficient of Mutual Induction. — Suppose a long uniformly wound coil, B, containing n_1 turns per unit length, and of section S, to be surrounded about the middle (Fig. 273) by another coil, A, composed of n turns of wire.

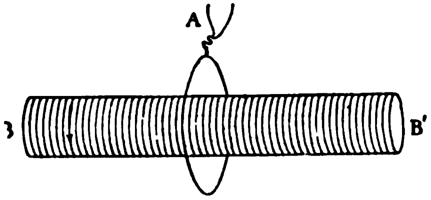

FIG. 273.

The flux in the interior of the coil B is $4\pi n_1 S$ (§ 256), and as this is enclosed n times by the coil A, the coefficient of mutual induction for this special case is

$$M = 4\pi n_1 n S.$$

307. Determination of Coefficients of Magnetisation.—Suppose that the coil B contains a long cylinder of soft iron, NS,

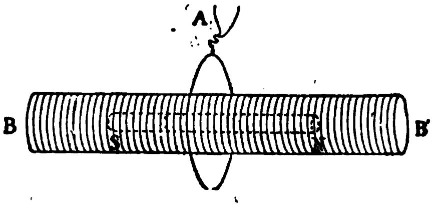

FIG. 274.

placed parallel to the axis (Fig. 274). Let S' be the section of the cylinder, and C the strength of the current in the coil B.

The intensity of the field inside the coil is $F = 4\pi n_1 C$, and the induction in the interior of the cylinder $B = \mu F$. The total flux Q, which traverses the n turns of the coil A is

$$Q = nF(S - S' + \mu S').$$

If the current is reversed, the induced current will be proportional to twice Q, and from this μ can be deduced.

If the coil B is wound directly on the iron rod, we have $S = S'$, and therefore

$$Q = nBS = n\mu FS = 4\pi n_1 n\mu SC.$$

The result is the same whatever be the form and size of the windings of the coil A, provided that neither the inner coil nor the core produce any external flux. This condition is strictly fulfilled with an annular coil (§ 273), and very approximately with a coil and core which both project a long way on each side of the outer coil. In all cases this coil, A, if wound directly on the core, will measure the flux of induction which traverses the corresponding section.

308. Constant Current.—Let us return to the case considered in § 292. If the sliding bar move with uniform velocity, v, and if l is the distance between the rails, and H the component of the field perpendicular to the plane of the rails, we have already seen that the electromotive force is

$$e = lvH,$$

and, if R is the resistance of the circuit, the strength of the induced current is

$$C = \frac{e}{R} = \frac{lH}{R} v.$$

The direction of the current is that shown by the arrows in Fig. 275, it being assumed that the direction of the field is from front to back, and that the bar moves away from AA′.

For unit field and a velocity of 1 centimetre per second, the electromotive force per centimetre in length of the bar is one C.G.S. unit or 10^{-8} volts. A bar a metre long, moving at right angles to its own length and to the vertical component of the earth's field with a uniform velocity of 20 metres per second, would give an electromotive force for $Z = 0.438$ (§ 232), of

$$100 \times 0.438 \times 2000 = 87600 \text{ C.G.S.}$$
$$= 8.76 \times 10^{-4} \text{ volts.}$$

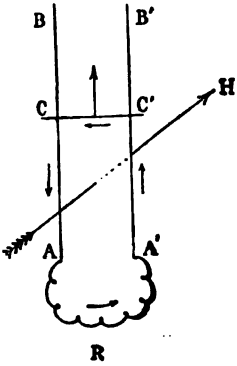

FIG. 275.

309. Faraday's Disc.—A second example of a uniform electromotive force is furnished by Faraday's disc (§ 267). Suppose that the two brushes, one of which presses against the circumference of the disc, and the other against the axis, are connected by a wire of resistance, R, and suppose the disc is made to rotate uniformly (Fig. 276).

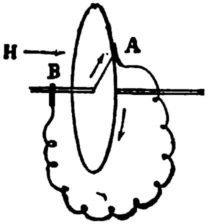

FIG. 276.

Let a be the radius of the disc, ω the angular velocity, and H the component of the magnetic field parallel to the axis: the radius OA, fixed as to its position in space, but movable relatively to the disc, cuts in each unit of time a flux equal to $\frac{1}{2}a^2\omega H$; hence the resulting current has a constant strength,

$$C = \frac{1}{2}\frac{a^2\omega H}{R}.$$

If T is the time occupied by a single revolution, and S is the surface of the disc, we have $\omega T = 2\pi$ and $S = \pi a^2$: the above expression therefore becomes

$$C = \frac{SH}{RT}.$$

Suppose that the disc has a surface of one square metre, and let it revolve at the rate of 10 turns per second about a horizontal axis in the magnetic meridian; then if $H = 0.2$, the electromotive force equals

$$E = 10^4 \times 0.2 \times 10 = 2 \times 10^4 \text{ C.G.S.} = 2 \times 10^{-4} \text{ volts.}$$

If the circuit had a resistance of only 10^{-4} ohms, the strength of the current would be 2 amperes.

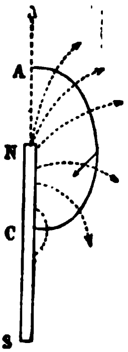

FIG. 277.

In like manner, if an arc of any form (Fig. 277) be made to turn uniformly about the north pole of a magnet, the strength of the pole being m, the arc will, in each unit of time, cut a flux equal to $2m\omega$, and in a circuit of resistance, R, will give a constant current of strength

$$C = \frac{2\omega m}{R}.$$

Let us suppose that at A and at C the arc is in contact by means of two brushes with the surface of the magnet, or with a conducting surface rigidly connected therewith; the same current will be produced going from C to A by the arc, whether the arc is moved about the magnet in the direction of the hands of a watch, or, the arc being stationary, the magnet is moved about its axis in the opposite direction. This latter effect is often referred to as *unipolar induction.*

310. Variable Period—Establishment of a Current.—Let E be the total electromotive force of the battery, R the total resistance, L the coefficient of self-induction of the circuit, and $C_0 = E/R$ the ultimate strength of the current when it has become uniform. While the current is increasing in strength from 0 to C_0, the strength at each instant satisfies the equation (§ 296)—

$$E = CR + L\frac{dC}{dt}.$$

Dividing throughout by R, we may write this

$$\frac{E}{R} - C = \frac{L}{R} \cdot \frac{dC}{dt},$$

$$= -\frac{L}{R} \cdot \frac{d\left(\frac{E}{R} - C\right)}{dt},$$

since E and R are both constant. Rearranging this expression, we get

$$\frac{d\left(\frac{E}{R} - C\right)}{\frac{E}{R} - C} = -\frac{R}{L}dt,$$

where the left-hand side is the differential of $\log_e\left(\frac{E}{R} - C\right)$. If we count time from the instant of closing the circuit, the strength of the current = 0 when $t = 0$. Putting C for the current after any assigned interval of time, t, and integrating on the left between the limits 0 and C, and on the right between 0 and t, we get

$$\log_e\left(\frac{E}{R} - C\right) - \log_e\frac{E}{R} = -\frac{R}{L}t;$$

or, passing from logarithms to exponentials and rearranging the expression—

$$C = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) = C_0\left(1 - e^{-\frac{t}{\lambda}}\right).$$

Where e is the base of natural logarithms, the never-ending decimal 2.71828. . . The self-induction and resistance of the circuit are permanent characteristics of it, and their quotient L/R determines the behaviour of the circuit in all cases of varying currents. It is commonly called the *time-constant* of the circuit, and we shall denote it by the symbol λ, as in the last formula.

This formula shows that when t is great, that is, when the circuit has been closed for a long time, the strength of the current is equal to $C_0 = E/R$, for e raised to a very high negative power is sensibly equal to nothing. In order to trace the growth of the current, we may consider what fraction of its ultimate strength, C_0, it has acquired at the end of various intervals from making contact: for instance, suppose t to have in succession the values λ, 2λ, 3λ . . . . and let C_1, C_2, C_3 . . . . denote the corresponding strengths of the current. From the above equation we obtain

$$\frac{C_1}{C_0} = 1 - \frac{1}{e} = 0.6321. \qquad \frac{C_3}{C_0} = 1 - \frac{1}{e^3} = .9502.$$

$$\frac{C_2}{C_0} = 1 - \frac{1}{e^2} = .8647. \qquad \frac{C_4}{C_0} = 1 - \frac{1}{e^4} = .9817.$$

According to the formula, the ratio C/C_0 never becomes $= 1$ exactly in any finite time, but in most cases it attains so nearly this value in a very short time, usually less than a second, unless the circuit includes the coils of electromagnets, that the subsequent change is imperceptible.

311. Cessation of a Current.—Suppose that, when a current has attained a steady strength, the electromotive force is withdrawn from the circuit, the resistance, however, remaining unchanged. The equation from which we started in § 310 will still apply if we give to E the value corresponding to the conditions of the case, namely, 0. We then get

$$CR + L\frac{dC}{dt} = 0.$$

Dividing by C and L and rearranging the terms

$$\frac{dC}{C} = -\frac{R}{L}dt.$$

We again count time from the instant when the current begins to change; consequently, for $t = 0$ we have the current $= C_0 = E/R$. Integrating from C_0 to C, and from 0 to t, we get

$$\log_e \frac{C}{C_0} = -\frac{R}{L}t = -\frac{t}{\lambda},$$

which is equivalent to

$$C = C_0 e^{-\frac{t}{\lambda}}.$$

If, as before, we put C_1, C_2 . . . for the strength of the current at intervals of time equal to *once*, *twice*, ... the time-constant, λ, after the electromotive force has ceased to act, we get

$$\frac{C_1}{C_0} = \frac{1}{e} = 0.3679. \qquad \frac{C_3}{C_0} = \frac{1}{e^3} = 0.0498.$$

$$\frac{C_2}{C_0} = \frac{1}{e^2} = .1353. \qquad \frac{C_4}{C_0} = \frac{1}{e^4} = .0183.$$

The gradual growth and cessation of a current in a circuit of constant self-induction are represented graphically in Fig. 278 (§ 313).

The case here supposed might be furnished by a circuit in which the current was due to the motion of a Faraday's disc (§ 309) or other equivalent arrangement: if the motion of the disc were suddenly stopped, the electromotive force would cease to act, but the resistance would be unaltered, and the current would die out according to the law stated. The more usual case of stopping a current is by increasing the resistance to a practically infinite amount while the electromotive force continues to act. The change of resistance may occur very quickly, but it is never absolutely sudden, and often, even after the metallic circuit has been interrupted, conductivity is maintained for a short but perceptible time by a spark caused in a manner that we shall explain immediately.

If the resistance were suddenly increased from R to R_1, the current would change from the initial value $C_0 = E/R$ to $C_1 = E/R_1$, its value at any instant during the period of change being given by the formula

$$C = C_1 + (C_0 - C_1)e^{-\frac{t}{\lambda_1}},$$

where $\lambda_1 = L/R_1$ is the time-constant corresponding to the new resistance R_1. If the change of resistance takes place gradually, the resulting variation of the current is much more complicated; but, other things equal, it is obvious that the more rapidly the resistance of a circuit is increased, the more rapidly the strength of the current will diminish.

312. Spark on Breaking Circuit.—A rapid fall of current strength, however, has an effect that we have now to consider. We have seen (§ 293) that the electromotive force of induction is always numerically equal to the rate of change of the magnetic flux through the circuit; therefore, in the case we are now considering, the electromotive force of induction is

$$\epsilon = -L\frac{dC}{dt}.$$

When the strength of the current is falling, dC/dt is negative, and therefore the resulting electromotive force is positive,—that is, it acts in the direction of the existing current, which it tends to maintain.

By Ohm's law, the strength of the current at any instant multi-

and BND the way in which it dies away when the electromotive force ceases. Time is represented by the abscissæ in terms of the time-constant λ, taken as unity. The point O denotes the instant of applying the electromotive force, and C the instant at which it is withdrawn. The ultimate strength which the current would attain is represented by the ordinate OA; after the lapse of seven times the time-constant, the strength of the current differs from this by less than one part in a thousand, and it would be impossible to represent any further increase on a diagram of this size. The extra-current on making contact is represented by the area between OMB and the horizontal line through A; similarly, the extra-current corresponding to the cessation of the electromotive force is represented by the area between the curve BND, which represents the dying out of the current, and the axis of time. This curve is the same as that for the increasing current, but its position is reversed, as represented in the figure. After the withdrawal of the primary electromotive force, the electromotive force of self-induction is the only one acting in the circuit, so that, in this case, the whole quantity that passes is given by the formula

$$m' = -\lambda C_0 \left(e^{-\frac{t_2}{\lambda}} - e^{-\frac{t_1}{\lambda}}\right),$$

where t_1 and t_2 are both counted from the instant at which the primary electromotive force ceases to act. In the figure the value of m' is represented by the area below the curve BN which stands on the part of the base corresponding to the interval of time $t_2 - t_1$.

If we put $t_1 = 0$, and make t_2 very great, the above formula becomes

$$m' = \lambda C_0, \text{ or } m' = -m.$$

Hence, when the resistance of the circuit remains constant, the total quantity of the extra-current due to the cessation of the primary is equal and opposite to that of the extra-current due to the commencement of the primary, or the sum of the two is zero. This is also true in a more general form which we may state as follows: If a given electromotive force is applied to a circuit of invariable resistance for any given period and then withdrawn, the total quantity of the resulting current is the same as if the circuit had no self-induction. In such a case, self-induction affects the rate of rise or fall of the current, but not the total quantity. This result is of much importance in many practical cases and methods of electrical measurement. An obvious consequence is that the quantities conveyed by two or more conductors in parallel are connecting the same two points, when a momentary current is sent through them, is independent of the self-induction of the several branches.

314. Energy of the Extra-Current.—The work done against the electromotive force of self-induction, while an elementary quantity of electricity dm traverses the circuit, is equal to the product of dm into the electromotive force $L\frac{dC}{dt}$; but $dm = Cdt$, so that the work done in the element of time dt is

$$dW = LCdC,$$

and the work done while the strength of the current varies from nothing to any given value C is the integral of this expression, or

$$W = \tfrac{1}{2}LC^2,$$

as already shown in § 296.

As was there pointed out, this work is spent by the battery in imparting magnetic energy to the medium surrounding the conducting circuit. Properly speaking, it has not passed out from the conductor into the medium: it rather represents energy which, though it has left the battery, has not yet reached the external conductor. When the primary electromotive force ceases to act, the energy of the field flows in upon the conductor, causing the temporary continuation of the current which we recognise as the extra-current, and being converted into heat.

From this point of view we see that a circuit of great self-induction is a circuit in which a relatively large quantity of energy is poured into the field from the battery while the strength of the current is rising to a given value.

315. Self-Induction in Parallel Circuits.—If two conductors whose coefficients of self-induction are L_1 and L_2, and whose resistances are R_1 and R_2, are employed to connect the same two points of a circuit, we know (§ 117) that the total current ultimately divides between them in the inverse ratio of their respective resistances; but this proportion does not hold good while the currents are varying in strength. Seeing that the conductors are in contact at each end, the total electromotive acting in each of them is the same: this is expressed by the equation

$$c_1R_1 + L_1\frac{dc_1}{dt} = c_2R_2 + L_2\frac{dc_2}{dt}.$$

When the currents have become steady, the second term on each side vanishes, and we get $c_1R_1 = c_2R_2$, the relation stated above. But at the first instant, neither c_1 nor c_2 has as yet reached a finite value, and the equation becomes

$$L_1\frac{dc_1}{dt} = L_2\frac{dc_2}{dt};$$

that is to say, the currents in the two branches *begin* to rise at rates which are inversely proportional to the coefficients of self-induction and independent of the resistances. As the currents increase, their relative strengths are influenced more and more by the resistance, and less and less by the self-induction of the two conductors.

In the case of a current which rises in strength from any given value (which may be zero) to some higher value, and then falls to the same strength again, like that resulting from the discharge of a condenser, or from the closing or opening of a neighbouring circuit, the effect of self-induction during the increase of strength is, in each branch of a divided circuit, equal and opposite to that during decrease. Consequently the ultimate quantity of electricity traversing each branch in such a case is the same as if there were no self-induction, and is determined by the resistances only.

316. Induction in Conductors which are not Linear.—Any variation of magnetic flux produces induction currents not only in linear conductors, like wires, but also in conductors which are not linear. Gambey, as long ago as 1824, observed that the oscillations of a bar magnet are rapidly damped when the magnet is placed over a plate of copper. This phenomenon was studied by Arago, and was at first attributed to a particular form of magnetism developed by motion, and called *magnetism of rotation.* It was only properly explained after Faraday's discovery of induction.

The relative displacement of a magnetic pole and of a metal plate determines induction currents. Let us suppose that lines are drawn on a surface representing at each point the direction of the current; these lines are evidently closed curves, some of them enclosing others, but without ever intersecting. On the other hand, the space between two infinitely near lines may be regarded as a closed linear current, which is equivalent to a magnetic shell of the same contour (§ 254).

Let the surface of the conductor be thus subdivided into infinitely narrow belts by lines of flow, and let each of these belts be replaced by the corresponding magnetic shell; all the currents which rotate about a point are equivalent to a complex shell, the density of which at each point is evidently the sum of the densities of the superposed shells. Fig. 279 represents the lines of flow in the case of an unlimited plate, moving from left to right below a south pole, whose projection would be at the centre of the figure.

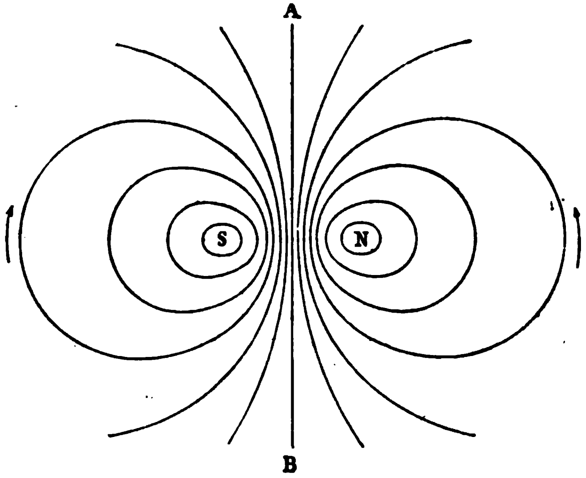

FIG. 279.

There is a flux of magnetic force directed upwards through the plate, and this, though constant as regards space, is increasing relatively to the plate in the part which is on the left of the central line, and decreasing in the part to the right of the central line. Consequently (§§ 292, 293), the part of the plate on the left becomes the seat of closed currents circulating like the hands of a watch, and the part on the right the seat of closed currents circulating in the opposite direction, as indicated in the figure. The former are equivalent to a number of magnetic shells superposed with their negative (south) faces upward, and the latter to similar shells with their positive faces upward. The mutual forces exerted between the circulating currents (or the equivalent magnetic shells) and the fixed pole are such as to oppose the existing motion of the plate.

The experiment may be easily made by means of a disc which is made to rotate between the poles of an electromagnet (Fig. 280). The disc, which before the passage of the current turns easily, offers a considerable resistance as soon as the electromagnet is excited. The effect is greater the greater the conductivity of the disc, but it is almost wholly destroyed if the continuity of the material is broken by a number of radial saw-cuts (Fig. 281).

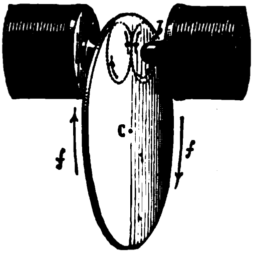

FIG. 280.

In like manner, if a cube of copper, C, is suspended between the two poles, B and A, of an electromagnet by a twisted thread, the cube, if left to itself, spins round rapidly, but it stops the moment the current is turned on (Fig. 282).

It is clear that a pole which can rotate about an axis concentric with the disc must follow the motion of the disc. This was Arago's first experiment: a magnetic needle is suspended horizontally above a copper disc mounted on the vertical axis of a whirling table; when the disc is made to rotate, the needle follows its motion, though more slowly.

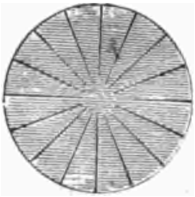

FIG. 281.

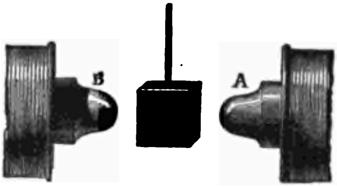

FIG. 282.

317. Damping the Vibrations of Magnets.—The phenomenon in the form originally observed by Gambey is used for deadening or damping the oscillations of compass-needles and galvanometer-needles.

The forces called into play act as though there were a frictional resistance to the relative motion of the needle and the conductor, the effect at each instant being proportional to the strength of the current, and therefore to the relative velocity.

318. Foucault Currents.—The energy absorbed is found again in the form of heat developed by the current in accordance with Joule's law. Foucault, who first observed this, made the experiment in a very striking form. By means of a system of toothed wheels, D, worked by a handle, a stout disc of pure copper is made to rotate between the branches, AB, of a powerful electromagnet (Fig. 283).

As soon as the current passes, a considerable effort is required to keep up the motion of the disc, and this soon rises to a very high temperature. This experiment furnishes thus a beautiful illustration of the transformation of mechanical work into heat.

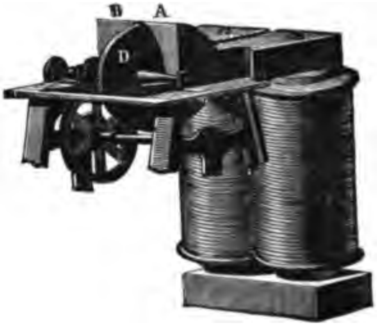

Fig. 283.

The term *Foucault currents* is applied to the induced currents produced in the core of an electro-magnet whenever the strength of the current in the surrounding wire varies.

The heat which results from this cause is considerable when the wire is traversed by a rapid succession of currents either in the same or in opposite directions.

The currents which cause this heating, and the loss of work which is the consequence, are greatly diminished by constructing the core of a bundle of separate wires, or of a number of thin plates which are insulated from each other by varnish or by paper, and are arranged parallel to the axis, and therefore perpendicular to the windings, and therefore to the direction in which induced currents tend to be set up.

As these currents are thus suppressed, any heating effects which still remain are due to the work of magnetisation or hysteresis (§ 201).

319. Electromagnetic Screens.—Let us suppose that a pole is rapidly brought near an infinite conducting surface. The currents induced in the plate produce a magnetic flux which, for all points behind the plate in respect to the pole, is in the opposite direction to that from the pole. Theory shows that if the plate had no resistance, the magnetic forces at any point would be always equal and in opposite directions, that there would therefore be no inductive action behind the plate, which would thus play the part of an *absolute screen.* Experiment shows, in fact, that the magnetic force is lessened, and the more so, the better the plate conducts, and the more rapidly the changes of magnetic force occur.

CHAPTER XXVII.

SPECIAL CASES OF INDUCTION.

ALTERNATING CURRENTS AND ELECTRICAL OSCILLATIONS.

320. Currents varying Harmonically.—Consider the case of a coil wound in a narrow groove, and rotating uniformly about a vertical diameter in a uniform magnetic field like that of the earth (Fig. 284). Let H be the horizontal component of the magnetic field, and S the area enclosed by the coil: then, when the plane of the coil is perpendicular to the meridian, the magnetic flux through the coil is HS; and, when the coil has been turned through an angle a from this position, or when its axis makes an angle a with the meridian, the magnetic flux is $Q = HS \cos a$.

If ω be the velocity of rotation, and T the period, or the duration of one complete revolution, we have at time t, measured from an instant at which the axis of the coil is in the meridian

$$a = \omega t = 2\pi \frac{t}{T}.$$

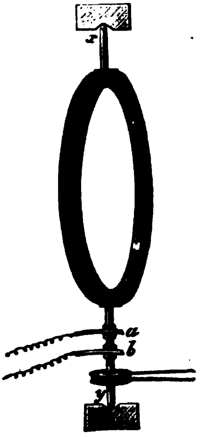

FIG. 284.

The electromotive force due to the variation of the flux through the coil is given at each instant by the corresponding value of $-\frac{dQ}{dt}$. Hence, seeing that H and S are constant, we may write

$$E = -HS\frac{d\cos a}{dt} = HS\sin a\frac{da}{dt} = \omega HS\sin\omega t = \frac{2\pi}{T}HS\sin\frac{2\pi t}{T}.$$

The magnetic flux through the coil varies most rapidly when its instantaneous value is zero; this is when the plane of the coil is parallel to the meridian, and therefore a is a right angle. At this instant, therefore, the induced electromotive force is a maximum, and if we denote this by E_0, we have for the electromotive force at any time t

$$E = E_0 \sin a = E_0 \sin \omega t = E_0 \sin \frac{2\pi t}{T}.$$

In such a case the electromotive force is said to vary ***harmonically***. A variation of this kind is expressed graphically by the ordinates of the sine-curve OAB (Fig. 285). In this figure the distance OB

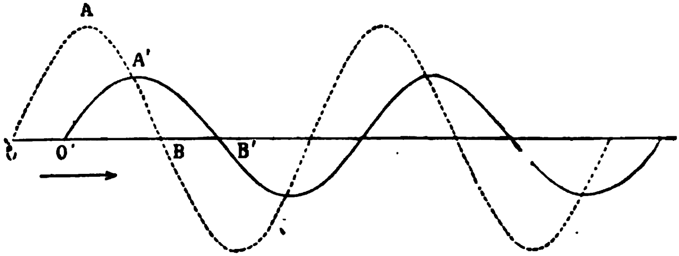

FIG. 285.

represents half the period T, and the maximum ordinate represents E_0, the ***amplitude***.

If self-induction produced no effect, the current would be proportional at every instant to the existing electromotive force, or $C = E/R$. The maximum strength of current would be E_0/R, and the current at any instant would be

$$C = \frac{E_0}{R} \sin \frac{2\pi t}{T},$$

that is, the current would also vary harmonically in the same period as the electromotive force, and might be represented by a similar sine-curve, having the same period and the same nodes as the curve OAB.

321. Effect of Self-Induction.—From what has been said already (§§ 310, 311) as to the way in which self-induction retards the rise or fall of the current when a steady electromotive force is suddenly applied or removed, it is easy to infer that, in the case we are now considering, the changes of current-strength will lag behind those of electromotive force, and that the maximum strength of current due to a rapidly alternating electromotive force will be smaller than that which an equal electromotive force would produce if it alternated more slowly. We will accordingly assume that the strength of the current, when the effects of self-induction are appreciable, still varies harmonically in the same period as the electromotive force, but that the maximum value is less than E/R, and that it is retarded in *phase* relatively to the electromotive force. This assumption is expressed by the equation

$$C = a\frac{E_o}{R}\sin 2\pi\left(\frac{t}{T} - \phi\right) \quad . \quad . \quad . \quad . \quad (1),$$

where a is a proper fraction and ϕ represents the retardation of phase. We have now to investigate the conditions on which these two quantities depend.

To simplify notation we will write

$$C_o = aE_o/R, \quad \omega = 2\pi/T, \quad \text{and } p = 2\pi\phi.$$

The last equation then becomes

$$C = C_o \sin(\omega t - p),$$

and the electromotive force of self-induction

$$L\frac{dC}{dt} = LC_o\omega\cos(\omega t - p).$$

Now the effective electromotive force at any instant is the excess of the applied electromotive E over the electromotive force of self-induction, and it is equal to the strength of the current at the same instant multiplied by the resistance of the circuit; or, in symbols

$$E - L\frac{dC}{dt} = CR.$$

Re-writing this equation with the values already adopted, we get

$$E_o \sin\omega t = C_oR\sin(\omega t - p) + LC_o\omega\cos(\omega t - p) \quad . \quad . \quad . \quad (2).$$

If we put $t = 0$, the applied electromotive force vanishes, while for $t = \frac{1}{4}T$ it has a maximum value $= E_o$. In these two cases, equation (2) takes the respective forms

$$0 = -R\sin p + L\omega\cos p \quad . \quad . \quad . \quad . \quad (3),$$

and

$$\frac{E_o}{C_o} = R\cos p + L\omega\sin p \quad . \quad . \quad . \quad . \quad (4).$$

From (3) we get at once

$$\tan p = \frac{L}{R}\omega = \lambda\omega \; ; \quad . \quad . \quad . \quad . \quad (5),$$

while, by squaring (3) and (4) and adding, we get

$$E_0^2 = C_0^2 (R^2 + L^2\omega^2),$$

or

$$C_0 = \frac{E_0}{R} \cdot \frac{1}{\sqrt{1 + \lambda^2\omega^2}} = \frac{E_0}{R}\cos p \; ; \quad . \quad . \quad . \quad . \quad (6) ;$$

whence

$$a = \frac{1}{\sqrt{1 + \lambda^2\omega^2}} = \cos p.$$

Using these results in the original expression for the current-strength (equation 1), we have

$$C = \frac{E_0}{R\sqrt{1 + \lambda^2\omega^2}} \sin(\omega t - p) \quad . \quad . \quad . \quad . \quad (7a).$$

$$= \frac{E}{\sqrt{R^2 + L^2\omega^2}} \sin(\omega t - p) \quad . \quad . \quad . \quad . \quad (7b).$$

$$= \frac{E_0 \cos p}{R} \sin(\omega t - p) \quad . \quad . \quad . \quad . \quad (7c).$$

It thus appears that, besides causing the phase of the current to lag behind that of the electromotive force, the effect of self-induction is equivalent to dividing the electromotive force by $\sqrt{1 + \lambda^2\omega^2}$, or to multiplying it by $\cos p$, or again to increasing the resistance in the ratio $R : \sqrt{R^2 + L^2\omega^2}$. The quantity last written, which forms the denominator of the expression (7b), is sometimes called the apparent resistance, or the *impedance* of the circuit. If a right-angled triangle be drawn with the sides about the right angle proportional to R and $L\omega$ respectively, the hypothenuse will represent the impedance. At slow speeds of rotation, or when the self-induction is small, this reduces to the simple resistance R; as the speed and self-induction increase it becomes more and more nearly independent of the resistance, and approaches to $L\omega$.

322. Geometrical Representation.—The problem we are considering admits of a very simple geometrical treatment. Let O (Fig. 286) be the projection of the axis of rotation on the horizontal plane, which we take as the plane of the figure, OH the direction of the field, and OB the direction at a given instant of the plane of the coil. We suppose the direction of rotation to be that of the hands of a clock.

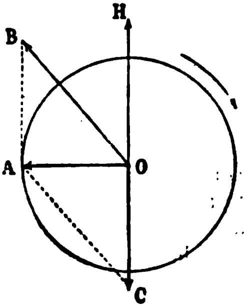

FIG. 286.

Let the length OB represent the maximum value of the electromotive force due to the field, that is, $E_0 = \omega HS$; draw OA so as to make an angle with OB equal to the retardation p or $2\pi\phi$ measured in the opposite direction to the rotation; then if BA be drawn perpendicular to OA, the length intercepted, or OA = OB $\cos p$, represents the maximum effective electromotive force. Lastly, draw OC so as to complete the parallelogram OBAC, the angle AOC being a right angle again measured in the opposite direction to the rotation.

Now let the whole figure rotate in its own plane about the point O, and consider the projection at any instant of the three straight lines OA, OB, OC, upon the direction OH of the field. The projection of OB represents the instantaneous value of the impressed electromotive force due to the field; the projection of OC represents the electromotive force of self-induction; and that of OA, which is equal to the difference of the other two, gives the effective electromotive force, that is to say, the product of the strength of the current at the instant considered, into the true resistance R of the circuit.

It is evident that the phase of the electromotive force of self-induction must be one-quarter of a vibration behind that of the effective electromotive force, or what comes to the same thing, behind that of the current, since the maxima of one concur with the zero-values of the other, and reciprocally. This is expressed in Fig. 286 by the angle AOC being an angle of 90°.

323. Average and Effective Strength of a Current.—The average strength of a variable current, for an interval of time t, is the quotient by t of the total quantity of electricity which has traversed any given section of the circuit in this time.

If the current is represented by a curve as a function of the time, the quantity of electricity, Cdt, which corresponds to the interval of time MM′ (Fig. 287) is represented by the area of the infinitely narrow quadrilateral M P P′ M′, and that which corresponds to a finite time AB by the area below the curve comprised between the corresponding two ordinates AA′ and BB′. If this area is divided by the time AB, we have the *average strength of current*, or, more shortly, the *average current.*

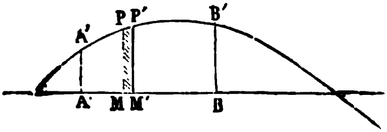

FIG. 287.

It is manifest that if the current varies harmonically, the average current corresponding to an entire period is zero; and the same applies to any interval of time divided into two equal parts by an instant at which the sign of the current is reversed. The mean strength for a half period from zero to zero is

$$C_m = \frac{C_o}{\frac{1}{2}T}\int_o^{\frac{1}{2}T} \sin \omega t dt = \frac{2C_o}{\pi},$$

C_o being the maximum current.

If the ordinates of the curve (Fig. 287), instead of being equal to the currents, were taken equal to the squares of the currents at each instant, the area AA′ BB′ multiplied by the resistance R of the circuit, would represent the energy transformed into heat in the time t. The quotient of this area by t would be the mean square of the current, and the square root of the quotient would give the strength of the constant current, which would develop the same quantity of heat in the circuit.

We shall call this the *effective* strength of the current $= C_e$. It is given directly by the electrodynamometer, or, still better, by the electrometer. With a current varying harmonically, the effective strength for any whole number of half-periods is

$$C_e = \frac{C_o}{\sqrt{\frac{1}{2}T}}\sqrt{\int_o^{\frac{1}{2}T} \sin^2 \omega t dt} = \frac{C_o}{\sqrt{2}},$$

it is equal to the maximum divided by $\sqrt{2}$.

The term *effective electromotive force* is applied to the square root of the mean square of the electromotive force. It is in like manner

$$E_e = \frac{E_o}{\sqrt{2}}.$$

Dividing E_e by C_e we have the apparent resistance R_a. Accordingly the *apparent resistance* may be defined as the factor by which the effective current must be multiplied to give the effective electromotive force: it is the same as impedance (§ 321).

324. Work done by an Alternating Current.—We have seen (§ 126) that the *power* of an electric circuit, or the rate of doing work by the current, is equal to the product of the strength of the current into the electromotive force. In the case of alternating currents, both electromotive force and current-strength vary from one moment to the next, but at a particular instant we may write for the power

$$P = EC = E_0C_0 \sin \omega t \sin (\omega t - p),$$
$$= \tfrac{1}{2}E_0C_0 [\cos p - \cos (2\omega t - p)].$$

Now, during half a period, that is while t changes to $t + \frac{1}{2}T$, the angle $2\omega t - p = \frac{4\pi t}{T} - p$ changes by 2π, and its cosine goes through all possible values, positive and negative, the mean value being nothing. Hence, the average rate of doing work, during any space of time which is a multiple of the half-period, is

$$P = \tfrac{1}{2}E_0C_0 \cos p = E_eC_e \cos p \quad . \quad . \quad . \quad . \quad (8).$$

Hence *the power of an alternating current is equal to the product of the effective electromotive force and effective current into the cosine of the difference of phase.*

Referring to § 321, we see that $C_0 \cos p = E_0 \cos^2 p/R$, and that $\cos^2 p = \frac{R^2}{R^2 + L^2\omega^2}$; hence we get

$$P = \tfrac{1}{2}E_0^2 \frac{R}{R^2 + L^2\omega^2}.$$

The multiplier of $\frac{1}{2}E_0^2$ in this expression is a maximum for a given speed of rotation (ω constant), when

$$R = L\omega = \frac{2\pi L}{T}.$$

In this case the rate of working is

$$P = \frac{E_0^2}{4R},$$

and

$$\tan p = 1, \text{ or } p = 45^\circ;$$

that is, the difference of phase between electromotive force and current is $\frac{1}{8}$ of a complete period.

325. Case of Soft Iron.—Suppose a soft iron core surrounded by a coil of wire between the ends of which a difference of potential is maintained which varies as the ordinate of a sine curve.

In order to simplify the problem, we will assume that the magnetic induction is at each instant proportional to the existing strength of the current. This simplification, which is justifiable as a first approximation, amounts to disregarding hysteresis and the variations of permeability; in other words, it amounts to replacing the closed curve, which represents a complete cycle of magnetisation, by a simple straight line passing through the origin (Fig. 174).

If n is the number of turns, and R the magnetic reluctance (§ 276), we have at each instant for the relation (§ 273) between the strength, C, of the current and the total flux of induction, Q,

$$\mathrm{R}Q = 4\pi nC.$$

Consequently, supposing the strength of current to be unity, and remembering that the flux is encircled n times by the wire, we get for L, the coefficient of self-induction,

$$L = \frac{4\pi n^2}{\mathrm{R}} = \frac{4\pi n^2 S\mu}{l};$$

and by putting this value into the formulas (5), (6), and (7) of § 321, we get the retardation of phase and the strength of current.

The high value of μ causes the self-induction to be great, and consequently, for high frequencies, the retardation is nearly 90°, and therefore $\cos p$ differs but little from zero, and formulas (6) and (8) show that the current and rate of doing work are very small.

A coil with a soft iron core placed in a circuit carrying a rapidly alternating current has thus the curious property of *choking* the current, but without the expenditure of energy which would take place with a simple resistance.

326. Action on an Adjacent Circuit.—A harmonically varying current causes a harmonically varying electromotive force to act on an adjacent circuit, and therefore gives rise to a current which is also harmonic, and of the same period as itself, but which differs in phase. The difference of phase can never be less than $\frac{\pi}{4}$, nor greater than $\frac{\pi}{2}$. For the secondary electromotive force has a retardation of $\frac{\pi}{4}$ in respect of the primary current, and self-induction produces a further lag comprised between 0 and $\frac{\pi}{4}$.

Professor Elihu Thomson has shown that, under these conditions, there is always repulsion between the two circuits. Fig. 288 represents one of his experiments. An electromagnet is traversed by a powerful alternating current; when a ring is brought near one end and left to itself, it is strongly repelled.

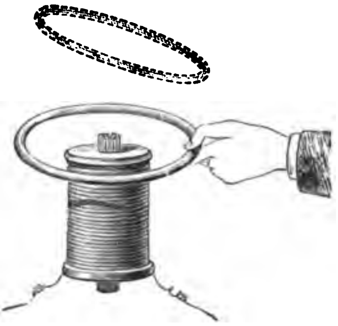

FIG. 288.

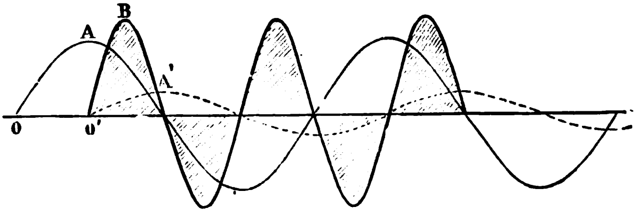

FIG. 289.

The electrodynamic force at each instant is proportional to the product of the strength of the current equivalent to the electromagnet and that induced by it in the ring. From Ampère's law, the two currents attract if they are in the same direction, and repel if they are opposite. In Figs. 289 and 290, two sine curves, A and A′, have been traced with the same period, but with amplitudes which are as 4 : 1. The ordinates of the curve B are taken equal to the products of the corresponding ordinates of A and A′, and are drawn above the line if the product is positive, and below if it is negative. When the difference of phase is equal to $\frac{\pi}{4}$ (Fig. 289), the attraction and repulsion are equal both in intensity and in duration, and the resulting effect is therefore nothing. But if the difference of phase exceeds $\frac{\pi}{4}$, and this is what always takes place (Fig. 290), repulsion preponderates over attraction, and the more so the greater the difference of phase.

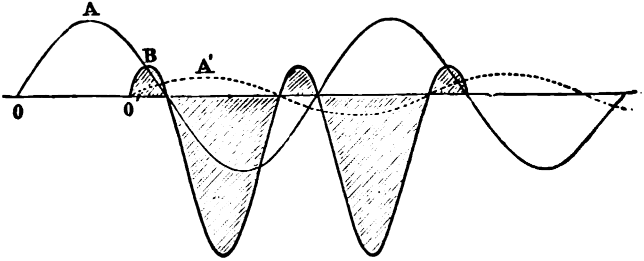

FIG. 290.

Professor E. Thomson has planned several arrangements in which the effect of repulsion may give rise to a continuous rotatory motion. In that shown in Fig. 291, a copper disc, B, mounted on a spindle, rotates rapidly when it is placed excentrically in reference to the alternating pole, and when another disc, A, partly screens the first. If the disc A is itself movable, it will turn in a contrary direction to the disc B.

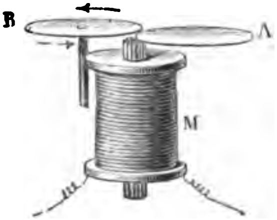

FIG. 291.

327. Distribution of Alternating Currents in Conductors.—In the case of alternating currents the apparent resistance greatly exceeds the real resistance; this latter is itself increased by the fact that the current is not distributed uniformly throughout the whole section of the conductor, but is concentrated near the surface.

Experiment shows, in fact, that a current at starting is first formed on the surface, and that it only gradually, though very quickly, reaches the deeper layers. If its direction is reversed at very short intervals, the current never fully reaches the centre. The case may be compared with that of an infinitely long tube filled with liquid, which is rapidly moved to and fro lengthwise. If the liquid is a perfect fluid. it will remain at rest; if it is viscous, the layers in contact with the sides of the tube will share the motion of the tube, but the inner layers will oscillate with an amplitude which decreases, and with a retardation of phase which increases, from the periphery to the axis.

The results in the electrical case are in full agreement with what has been already stated (see especially §§ 296–298, and 314) as to the seat of electric and magnetic energy. It has been pointed out that the energy of an electric current enters the conductor from the surrounding non-conducting medium. It is a necessary consequence of this that it arrives first at the outer layers, and that its penetration to the inside, though a rapid, is not an instantaneous process.

What we recognise as an electric current is, in one aspect, the process by which energy, whether electric or magnetic, passes from the field and generates heat in the conductors. Whenever the result of a current in a conductor would be a diminution of the energy of the field, such a current takes place. If energy is poured into the field in any way as fast as it is removed by the conductors, the condition known as that of a steady current is set up, but in all cases the ultimate distribution of the current among the available conductors is such that the total energy is a minimum. The electric energy corresponding to a given strength of current is proportional to the difference of potentials between the ends of the conductor, or, what comes to the same thing under the conditions specified, to the resistance—and this is least if the current makes use of the whole cross-section, distributing itself so that the current-density is uniform. On the other hand, the magnetic energy is least when the current is confined to the outer skin of the conductor; for it is easily proved, by an adaptation of the reasoning employed in § 19 in relation to spheres, that a current distributed uniformly on the surface of a circular cylinder exerts no magnetic force at an internal point, and that at external points it acts as though it were concentrated in the axis. It follows that the external magnetic field is the same whether the current is confined to the surface of a conductor, or uniformly distributed through the cross-section; whereas, in the former case the magnetic field inside the conductor itself is nothing, and in the latter case the internal magnetic force is proportional to the distance from the axis. The magnetic energy of the conductor itself is greater the greater the magnetic permeability, and hence the effect in question is more marked with iron than with copper.

For a frequency of 80 periods per second the virtual increase of resistance due to concentration of the current near the surface is 1 per cent. in a copper wire 1 centimetre in diameter, and 8 per cent. in a wire of 2 centimetres diameter. For large diameters the conducting power of a cylindrical conductor increases almost as the perimeter instead of as the section. Hence we have the practical conclusion, that for very strong alternating currents tubes should be used, and not solid conductors.

328. Influence of Capacity.—A difference of potential cannot exist between two conductors without correlated electrostatic charges. Each element of a wire in which a current is established acts as a capacity, and the assumption we have hitherto made that at each instant, every section of the conductor is traversed by the same quantity of electricity, is strictly true only when the current has reached the steady state.

If a condenser is connected in circuit with a coil revolving in a magnetic field, as described in § 320, or with any other equivalent source of alternating electromotive force, it acquires alternate positive and negative charges, the difference of potentials of the coatings varying harmonically, and the period being the period of revolution of the coil. The difference of potentials of the coatings is equivalent to an electromotive force opposing that due to the rotation of the coil. Calling the potential-difference at any instant V, the strength of current at the same instant C, and the capacity of the condenser S, the charge is SV, and its rate of change, $S\frac{dV}{dt}$, is equal to the strength of the current. Writing, as before, $E_0 \sin \omega t$ for the instantaneous electromotive force, we have

$$E_0 \sin \omega t = CR + L\frac{dC}{dt} + V.$$

To express that V varies periodically in the same period as the electromotive force, we may write

$$V = V_0 \sin (\omega t - r),$$

where V_0 is the maximum value and r is a constant determining the difference of phase between the electromotive force and the charge of the condenser. From this expression for V we get for the strength of current

$$C = S\frac{dV}{dt} = S\omega V_0 \cos (\omega t - r),$$

and for the rate of variation of the current

$$\frac{dC}{dt} = -S\omega^2 V_0 \sin(\omega t - r).$$

Using these values in the original equation, we may write it as follows :—

$$E_0 \sin \omega t = [RS\omega \cos(\omega t - r) + (1 - LS\omega^2)\sin(\omega t - r)]V_0.$$

If we now make $\omega t - r$ first $= 0$, and then $= \frac{\pi}{2}$, this gives

$$E_0 \sin r = RS\omega V_0 \quad \text{and} \quad E_0 \cos r = (1 - SL\omega^2)V_0$$

respectively. The potential-difference at time, t, therefore becomes

$$V = \frac{E_0 \sin r}{RS\omega} . \sin(\omega t - r),$$

or

$$= \frac{E_0 \cos r}{1 - SL\omega^2} . \sin(\omega t - r),$$

and the retardation of phase r is determined by the equation

$$\tan r = \frac{RS\omega}{1 - SL\omega^2}.$$

The simultaneous strength of current is given by the equation

$$C = S\frac{dV}{dt} = \frac{E_0}{R} \sin r . \cos(\omega t - r).$$

The product RS in the case of a condenser has a significance similar to that of the ratio $\frac{L}{R}$ in the case of a coil : if the surfaces of a charged condenser of capacity S, are connected through a conductor of resistance, R, the difference of potentials falls to the fraction $\frac{1}{e} = \frac{1}{2.718...}$ of its initial value in the time RS. This product is, consequently, called the "time-constant" of the condenser. If we denote it by ν, using, as before, λ for the corresponding value L/R, we may write

$$\tan r = \frac{\nu\omega}{1 - \lambda\nu\omega^2},$$

and remembering that ω stands for $2\pi/T$, where T is the period of rotation of the coil, we see that if

$$T = 2\pi\sqrt{\lambda\nu},$$

the angle denoted by r becomes a right angle, and the expression for the current-strength becomes

$$C = \frac{E_0}{R} \sin \omega t.$$

That is to say, under the specified conditions, the self-induction and capacity of the circuit exactly neutralise each other's effects, both the strength of the current and its phase being the same as they would be in a simple circuit of resistance, R, without self-induction.

329. Electrical Oscillations.—When the surfaces of a charged condenser are suddenly connected by a conductor of small resistance, the electric energy is converted into heat, but the process is, comparatively speaking, a gradual one, and involves the charging of the condenser a large number of times in succession, and alternately in opposite directions; the energy represented by each charge being a constant fraction of that of the preceding one. In fact, a series of electric oscillations of gradually decreasing amplitude is set up.

In order to follow out the theory of the process, we will, in the first instance, suppose the resistance of the conductor to be inappreciable. Denoting the charge of a condenser by Q, and its capacity by S, the electrostatic energy is $\frac{1}{2}Q^2/S$ (§ 55). When the coatings are connected by a conductor of self-induction, L, the electrostatic energy disappears, giving rise to a current and an amount of electromagnetic energy represented (§ 296) by $\frac{1}{2}LC^2$. This in its turn dies away, reproducing the original amount of electrostatic energy in the form of a reversed charge of the condenser. This, as soon as it is formed, begins to diminish again, giving rise to an inverse current and reproducing electromagnetic energy; and so the process goes on, the original energy of the condenser being transformed into the electromagnetic and electrostatic forms alternately.

From the principle of conservation of energy we conclude that the total energy, electrostatic and electromagnetic together, is constant, or

$$\frac{1}{2}\frac{Q^2}{S} + \frac{1}{2}LC^2 = \text{constant.}$$

Differentiating with respect to time, we get

$$\frac{Q}{S}\frac{dQ}{dt} + LC\frac{dC}{dt} = 0,$$

which simply means that an increase of one kind of energy is accompanied by an equal decrease of the other kind. But $C = dQ/dt$, and therefore $dC/dt = d^2Q/dt^2$: consequently, the last equation may be written

$$\frac{d^2Q}{d^2t} = -\frac{1}{LS}\cdot Q.$$

Let us compare this with the formula that expresses the acceleration of a particle of mass, m, which, when displaced from a position of stable equilibrium, is urged back by a force proportional to the displacement, x, namely, the formula

$$\frac{d^2x}{dt^2} = -\frac{f}{m}x,$$

where f stands for the restoring force corresponding to unit displacement. This equation states that the acceleration is proportional to displacement and in the opposite direction. We know that, under the conditions here specified, the motion of the particle is simply harmonic, and may be represented by

$$x = x_0 \cos \omega t,$$

if x_0 is the amplitude and t denotes time reckoned from an instant when the particle is at the positive extremity of its path. The velocity is

$$v = \frac{dx}{dt} = -\omega x_0 \sin \omega t,$$

and the acceleration

$$\frac{d^2x}{dt^2} = -\omega^2 x_0 \cos \omega t = -\omega^2 x.$$

Equating this with the expression given above, we get

$$\omega^2 = \frac{f}{m},$$

or, for the periodic time, we have

$$T = 2\pi\sqrt{\frac{m}{f}}.$$

All these mechanical results have strict analogues in the electrical case. The charge, Q, of a condenser may be taken as a measure of the displacement from the condition of electrical equilibrium; $\frac{dQ}{dt}$ then becomes the rate of change of electric displacement, and might be called electric velocity; and $\frac{d^2Q}{dt^2}$ becomes acceleration of electric displacement. The expression we obtained above for this quantity shows it to be directly proportional to the electric displacement, Q. Hence, following the mechanical analogy, and putting Q_0 for the initial charge of the condenser, we may write

$$Q = Q_0 \cos \omega t;$$

$$C = \frac{dQ}{dt} = -\omega Q_0 \sin \omega t;$$

$$\frac{d^2Q}{dt^2} = -\omega^2 Q_0 \cos \omega t = -\omega^2 Q.$$

From this, by the original expression for $\frac{d^2Q}{dt^2}$, we get

$$\omega^2 = \frac{1}{LS}, \text{ or } T = 2\pi\sqrt{LS} = 2\pi\sqrt{\lambda\nu},$$

if, as before, we use λ and ν for the two time-constants.

330. Comparison with Experiment.—These equations represent the discharge of a condenser, under the conditions stated, as giving rise to an endless series of oscillations of the same amplitude taking place in the periodic time T[1] $= 2\pi\sqrt{\lambda\nu}$, the geometric mean of the electromagnetic and electrostatic time-constants of the system × 2π.

For two reasons, however, these results do not accurately agree with what takes place in real cases. On the one hand, the true seat of the changes which we have spoken of as electrostatic and electromagnetic oscillations is not the conductor connecting the surfaces of the condenser, but the dielectric medium of the condenser and that by which the conductor is surrounded. Any change of condition, such as those accompanying electric charge and discharge, produced at one part of this medium, is propagated with a definite velocity to more and more distant parts, and consequently a series of electrical alternations, such as we have been discussing, gives rise to a series of waves which travel outwards in all directions into the surrounding space. Electric energy is thus radiated away from the oscillating system, and the energy of the latter is consequently diminished in proportion. The process is closely analogous to the gradual loss of energy by a vibrating tuning-fork consequent upon its radiation of sound-waves into the air.

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[1] This may be called the natural oscillation-period of the circuit, and it may be noted that the condition pointed out at the end of § 328, under which a condenser can neutralise the effects of self-induction in the case of an alternating electromotive force, is the condition of equality between the period of the electromotive force and the oscillation-period of the circuit.

We shall return presently (§ 333) to the subject of electric radiation.

Another respect in which the conditions assumed in § 329 differ from those of actual experiment is that they take no account of the resistance of the conductor. In certain cases this resistance may be so small that the formulæ we have arrived at represent the results with fair accuracy, but it can never be got rid of altogether. The existence of resistance causes a certain amount of electric energy to be converted into heat at every oscillation, just as friction does in the case of ordinary mechanical vibrations. The electric energy which is thus expended in an element of time *dt* is, in agreement with Joule's law,

$$RC^2dt = R\left(\frac{dQ}{dt}\right)^2 dt.$$

Adding this term to the equation from which we started in § 329, we get after simplification,

$$\frac{d^2Q}{dt^2} + \frac{R}{L} \cdot \frac{dQ}{dt} + \frac{Q}{LS} = 0.$$

This equation may be taken as a statement in mathematical language that the rate of generation of electromagnetic energy, together with the rate at which electric energy is expended in generating heat, is equal to the rate of decrease of electrostatic energy; or, again, as a statement that the electromotive force due to self-induction, together with that required to maintain the existing current, is equal and opposite to the difference of potentials between the surfaces of the condenser. The discussion of the equation would involve more elaborate mathematical processes than we can enter upon here: we must, therefore, be content with merely stating the most important conclusions to which it leads.

The character of the discharge depends essentially on the relative magnitudes of the electromagnetic time-constant $\frac{L}{R} = \lambda$, and the electrostatic time-constant $RS = \nu$. If λ is less than $\frac{1}{4}\nu$, the charge of the condenser decreases continuously by a constant fraction of its existing amount at any instant in a given interval of time. On the other hand, if λ is greater than $\frac{1}{4}\nu$, the discharge is oscillatory, the amplitude of the oscillations decreasing in geometrical progression, and the period depending on the two time-constants as expressed by the formula

$$T = \frac{2\pi\sqrt{\lambda\nu}}{\sqrt{1 - \frac{\nu}{4\lambda}}} = \frac{2\pi}{\sqrt{\frac{1}{LS} - \frac{R^2}{4L^2}}}.$$

As R diminishes, this expression approaches more and more nearly to the value $T = 2\pi\sqrt{LS}$ previously obtained. Further, as R occurs in ν as a multiplier, and in λ as a divisor, it is the determining factor in the relative magnitude of these two quantities, and therefore in the nature of the discharge. One and the same condenser will give an oscillatory discharge when the surfaces are connected by a conductor of small resistance, and a continuous discharge when the resistance is increased.

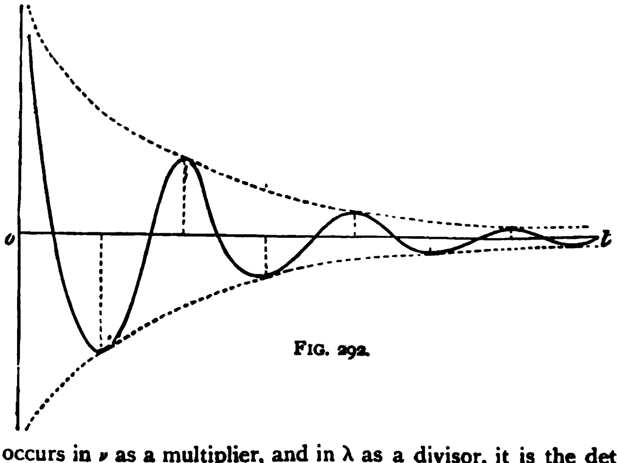

FIG. 292.

The general character of an oscillatory discharge is indicated in Fig. 292. The ordinates of the sinuous curve represent successive values of the electric charge. The periodic time is denoted by the distance along the axis *ot* between two points where the curve cuts it in the same direction. In the discharge of a Leyden jar, the oscillations may occupy from 10^{-4} to 10^{-8} of a second, according to the size of the jar and the nature of the discharging conductor. The intermittent character of the discharge can be directly observed by examining it by reflection in a rapidly revolving mirror, or by photographing it by means of a moving camera.

A continuous discharge would be represented graphically by a curve such as BNC (Fig. 278, § 313), the time-constant, $\nu = RS$, being indicated by one of the horizontal divisions.

As an example, take the case of a Leyden jar of diameter 10 centimetres, with tinfoil coatings extending to a height of 12.5 centimetres, and glass of thickness 0.2 centimetre. If the inductive capacity of the glass were six times that of air, the capacity of the jar would be 1125 K, if K denotes the inductive capacity of air. Suppose the jar discharged by connecting the coatings by a copper wire 1 metre long and of 0.2 centimetre radius, bent into an approximate circle. If the specific resistance of the copper be taken as 1600, the resistance, R, of the wire would be $1.273 \times 10^6 \mu$. The self-induction, L, would be about 1430μ. In both these formulæ μ denotes the magnetic permeability of air. The time-constant of the discharging circuit would therefore be $\lambda = L/R = 1.123 \times 10^{-3}$, and the period of oscillation, derived from the simple formula,

$$T = 2\pi \sqrt{LS},$$

becomes $7968 \sqrt{K\mu}$. Now, although there is no means of ascertaining with certainty the separate values of K and μ, we know that the product denotes the reciprocal of the square of a velocity, which for air is 3×10^{10} centimetres per second (see §§ 337 and 372). We have thus finally the value

$$T = \frac{7968}{3 \times 10^{10}} = 2.66 \times 10^{-7}.$$

The period of oscillation given by the more complete formula is not appreciably different. The amplitude decreases in the ratio $e : 1 = 2.72 : 1$, in the time $2\lambda = 2.25 \times 10^{-3}$ seconds; or the amplitude decreases in the ratio named in the time occupied by $\frac{2.25 \times 10^{-3}}{2.66 \times 10^{-7}} = 8460$ oscillations. It is as though the amplitude of vibration of a tuning-fork sounding "middle C" (256 vibrations per second) decreased in the ratio 2.72 : 1 in just over half a minute.

331. Effects of an Instantaneous Discharge—Lodge's Experiments.—Very sudden discharges give rise to phenomena which, at first sight, seem inconsistent with familiar electrical experience, but which are explained when we consider that, in the case of rapidly varying currents, the properties of a conductor depend essentially upon its self-induction, and only to a small extent upon its resistance. These effects may be compared with those produced by the sudden application of mechanical forces. A block of gun-cotton placed on a steel plate burns quietly if it is lighted with a match; it smashes the plate if it is exploded by fulminating powder. In both these cases the same quantity of gas is developed, and the same energy put into play; but in the second case the explosion is so sudden that the resistance of the air becomes comparable with that of steel.

Many of these effects have long been known, but they have only been set in their proper light by the experiments of Professor Lodge.

1. The conductor by which a charged Leyden jar or battery is discharged is capable of giving a strong spark to any conductor brought near it; this is what is called a *lateral discharge.* Hence arises the necessity, which has long been known, of having glass handles to discharging rods (§ 60).

If the two branches of the discharger are connected by a wire, covered with silk, bent backwards and forwards several times, sparks pass not only at the points where the wire is connected, but, notwithstanding the insulation, wherever the loops of the wire cross each other.

2. If a long copper wire 7 to 8 millimetres in diameter is used for discharging the jar, and if this wire is led round a large room, not only does it everywhere give lateral discharges, but gas or water pipes connected with it give sparks to any conductor brought near them, whether insulated or not. If taken at the end of a gas jet, these sparks ignite the gas as it issues.

The induction on all adjacent bodies is often so strong that they become capable of giving sparks without being connected with the discharging wire.

3. Two Leyden jars, M and N (Fig. 293), are joined in cascade with the two poles, A, of a Holtz machine, for instance; the two

outer coatings are connected by a continuous wire, L, and with two spark knobs, B. Whenever a spark passes at A, a spark also passes at B, if the distance of the two knobs is not too great. The limiting distance depends on the wire L, but it may be greater than that of the knobs A. The greater part of the discharge of the external armatures, a discharge which takes place suddenly and *without preparation*, as in the case of the discharge at A, passes through the air in preference to the wire L, although the resistance of the air-break may be many million times as great.

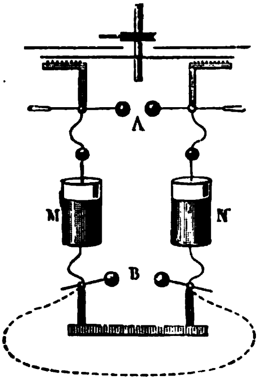

Fig. 293.

4. The internal coating of a Leyden jar is connected with one of the poles of the machine, the other coating being connected by a long conducting wire L, with the second pole (Fig. 294). Every time a spark passes at A, sparks are seen to strike between the two coatings along the surface of the glass, and the jar is *self-discharged*. When the difference of potential between the two coatings has reached the value which corresponds to the striking distance, a powerful spark passes between the two knobs; this sets up for a short time *a path* of feeble resistance, across which electricity oscillates from one armature to another. When the spark ceases, the path closes suddenly, and the moving electricity strikes across the gap between the two coatings by an action like that of the hydraulic ram.

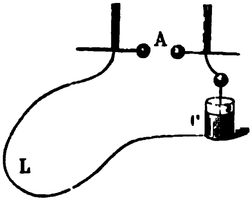

Fig 294.

332. Hertz's Experiments.—Hertz obtained still more rapid oscillations by means of a discharger or *vibrator* formed of two spheres or of two plates, AA′, connected by a nearly continuous brass rod, C (Fig. 295).

If the two spheres are suddenly raised to the potentials + V and − V, and the system is left to itself, equilibrium is attained after a series of oscillations the duration of which is determined by the formula already given (§ 329),

$$T = 2\pi\sqrt{SL},$$

which, according to the dimensions of the apparatus, may vary between 1 and 30 thousand millionths of a second.

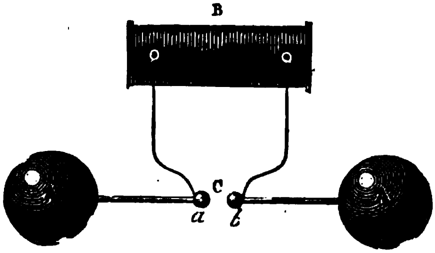

FIG. 295.

To get an instantaneous charge of the vibrator, a break is left at C between the two parts of the connecting rod, and these are provided with two small knobs which are connected with the poles of a powerful induction coil, B (§ 419).

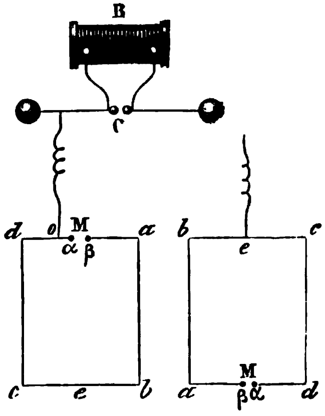

FIG. 296.

When induction takes place on the secondary wire of the coil, the two arms of the vibrator which form the ends of the wire are raised to different potentials, and at the same instant a spark strikes between the knobs *a* and *b*. This, as was explained above, *makes a path* for the oscillations, and the vibrator now discharges itself independently, as if it were insulated from the coil. The same succession of phenomena takes place at each to and fro motion of the secondary current of the coil.

The duration of these oscillations is so small that the effects of their propagation can be detected. If a conductor, such as *abcd* (Fig. 296), with a break at M, is connected by a wire of any length with one of the branches of a vibrator, sparks are obtained at M if the point of attachment, *o*, is unsymmetrical with respect to the interval; they disappear if the point of attachment is exactly at *e*. In this case the two parts into which the wave divides arrive at the same time at the knobs α and β, and cannot, therefore, produce the difference of potential which is necessary for a spark. On the other hand, the spark is stronger the nearer the point of attachment is to one of the knobs. Experiment shows, moreover, that there is a certain size of the circuit for which the spark is a maximum; it is that for which the time which the waves take to traverse the wire is equal to half the natural period of oscillation of the vibrator; the impulse increases at each oscillation, and the circuit acts like a *resonator*.

333. Propagation of Vibratory Motion.—When the vibrator works well, sparks can be taken from every piece of metal, large or small, insulated or not, contained in the room where the vibrator is, or in the adjacent ones. They strike between two coins or two keys which are brought near together; they can be taken from water or gas pipes when the point of a penknife is presented. Dr. Hertz used a wire bent in the form of a circle, one end provided with a small knob, and the other with a point whose distance from the knob could be varied at will (Fig. 297). The intensity of the action at any point is indicated by the length and brightness of the spark. The effect is greatest for a particular size of the circuit, namely, that which causes it to act as a resonator.

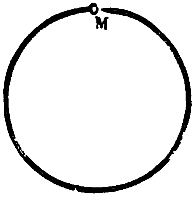

FIG. 297.

Near the vibrator, the resonator gives sparks of 7 to 8 millimetres; and sparks of only a few hundredths of a millimetre, but still visible, at distances of 15 to 20 metres. A stone or brick wall or a door does not stop the action; a metal surface, according to its thickness and conducting power, reflects the vibrations and acts like a more or less perfect screen.

The question arises, whether this action is instantaneously transmitted in the medium, or whether it has a finite velocity. In the first case, all points should be at the some moment in the same phase as the source; in the second, the propagation must take place in the form of waves. If V is the uniform velocity of propagation, and T the period, the wave-length,

$$\lambda = VT,$$

is the distance through which the effect is propagated during a complete vibration. At points distant by λ, 2λ, 3λ . . . the electrical disturbance is the same at each instant as that of the source, except as to amplitude; at points distant by $\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, $\frac{5}{2}\lambda$, the motion is in the contrary direction.[1]

In order to render these waves manifest Hertz changed them by reflection into stationary vibrations. It is known that if waves are reflected from a plane surface, the reflected waves interfering with the direct waves give stationary vibrations separated by fixed nodes. In order to see this, we need only draw a sine curve and fold it back on itself about any perpendicular to its axis (Fig. 298).

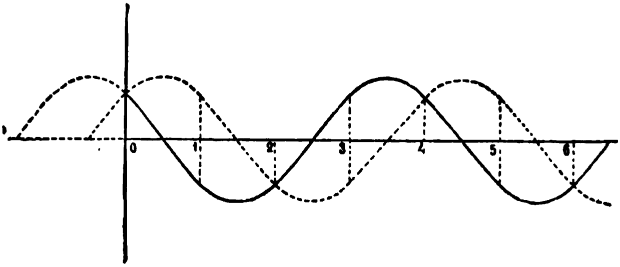

FIG. 298.

The points where the ordinates of the two curves are equal are always, if we do not take account of the sign, at distances 0, $\frac{1}{4}\lambda$, $\frac{2}{4}\lambda$ . . . from the reflecting surface. If the reflection takes place with change of sign, the distances 0, $\frac{1}{2}\lambda$, $\frac{2}{2}\lambda$ correspond to the nodes, the distances $\frac{1}{4}\lambda$, $\frac{3}{4}\lambda$ . . . to the loops.

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[1] Let us imagine two wires stretched parallel to each other, one belonging to a primary and the other to a secondary circuit. If the former is traversed by an alternating current, and the latter is at a distance from the inductor equal to one wave-length, it will be acted upon in the same way as if it were in contact, except as regards intensity. But if it is at a distance of half a wave-length, the induced electromotive force will at each instant be in the opposite direction to that which would be produced close to the primary conductor. In such a case, the usual laws as to the direction of the induced currents would be reversed. The experimental proof of this fact would be the most direct proof that the propagation occupies time; but if the velocity were that of light, the wave-length for an alternating current of 100 periods in a second would be 3000 kilometres, and the two wires would have to be placed 1500 kilometres apart.

This is exactly what takes place when waves from an electrical vibrator strike against a metal wall parallel with the vibrator. There is a node at the wall, and others follow at equal distances. For a vibrator whose period is thirty thousand-millionths of a second, the interval between the nodes is about 5 metres. The wave-length in this case is 10 metres, which makes the velocity of propagation 300,000 kilometres; that is to say, exactly the velocity of light.

334. Rays of Electrical Force.—True rays of electrical force may be obtained by placing a small vibrator along the focal line of a parabolic cylinder (Fig. 299).

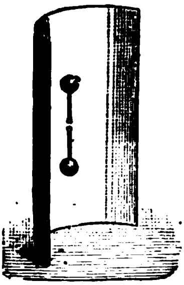

FIG. 299.

The field in which the phenomenon can be perceived, and in which sparks can be obtained with the resonator, is bounded by two vertical planes which pass through the edges of the mirror, and are parallel to the axis of the parabola. This forms a *parallel electric beam*, identical with the luminous beam which a source of light would give if put in the place of the vibrator.

If this beam is received upon a mirror identical with the first, the well-known experiment of the two conjugate mirrors may be repeated, and it may be shown that the vibratory motion is concentrated on the focal line of the second mirror.

The ray may also be reflected at a vertical plane, and the angle of reflection proved to be equal to the angle of incidence.

The radiation can be transmitted through a prism with vertical edges; in this case it is deflected towards the base of the prism, and the refractive index of the substance for the wave-length used can be deduced from the amount of deviation.

Lastly, a flat grating formed of parallel copper wires intercepts the beam if the wires are parallel to the vibrations, and allows them to pass if they are perpendicular. It thus acts like a tourmaline on a ray of polarised light.

The analogy between the motion produced in the ether by electrical oscillations and by luminous phenomena appears complete, and we are led to the conclusion that they only differ as to their respective periods of vibration, and therefore in their wave-lengths. The wave-length of visible rays is about 0.00005 centimetre; that of thermal rays, as observed by Langley, is 0.003 centimetre; the smallest wave-length which has hitherto been observed for electrical rays is about 50 centimetres, that is to say, a million times as great as for the visible rays.

335. Propagation along Wires.—A straight insulated wire fixed perpendicularly to the centre of a metal plate placed close to one of the knobs of the discharger is traversed by waves which, reflected from the end of the wire, give rise to stationary vibrations. The distance from one node to another is constant whatever be the nature of the wire, and this value is the same as for air. It follows from this that the propagation takes place through the air, and not through the wire.

We might infer from the extreme rapidity of the oscillations that the phenomenon does not penetrate beyond the surface of the wire (§ 327). Hertz demonstrated this directly by the following arrangement (Fig. 300). The wire is cut at A, and the gap is enclosed in a kind of cage made of metal wires stretched between two discs, α and β. The disc α is in contact with the wire; the disc β is supported by a tube $\gamma\delta$, which surrounds the wire, but does not touch it. As the waves arrive in the direction of the arrow, there is no spark at A if the tube is connected at δ with the wire; the electrical action stops at the outer surface. The sparks reappear, however, if the tube is insulated at δ; in this case the oscillations travel through the dielectric between the wire and the inner surface of the tube.

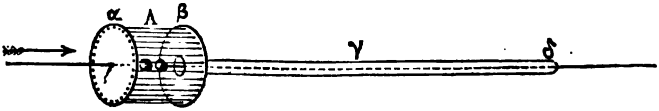

FIG. 300.

336. The Electromagnetic Field.—The mutual relations between electrical and magnetic phenomena, and especially phenomena such as those discussed in the latter part of the present chapter, afford strong confirmation of the view, already referred to several times in the course of this work, that electric and magnetic forces do not emanate directly from electrified conductors or from wires conveying currents, but that they are the result and manifestation of a special state, or rather states, of the dielectric medium surrounding the bodies on which the forces are exerted. The fact that, if the charges and currents remain of the same strength, the resulting effects depend upon the physical nature of the dielectric medium in which they are produced, and not at all on the physical nature of the conductors, may itself be taken as proving the essential truth of the view in question. The intervention of conductors in bringing about the required condition of the medium seems to be essential; but, when once this condition exists, the function of conductors is purely geometrical in localising and bounding the electric and magnetic fields.

At any part of a dielectric medium, electric and magnetic force may exist either separately or simultaneously, and in the latter case the effects of one are not affected by the existence of the other. Otherwise expressed, an electric or magnetic field may exist independently of each other, or they may be superposed without the direction or intensity of either being modified by the coexistence of the other.

But this mutual independence of the two fields applies only to the steady state of each. The movement of lines of electric force is accompanied by magnetic force, which at any point is perpendicular to the plane containing the tangent to the line of electric force through that point and the direction of motion; and, reciprocally, the movement of lines of magnetic force is accompanied by electric force at right angles to the direction of motion and to that of the magnetic force.

A complete theory of electricity and magnetism would require as its basis such a conception of the structure of dielectric media as would afford a mechanical explanation of the properties of electric and magnetic fields and of their mutual relations. Various more or less satisfactory attempts to form a conception of this kind have been made, but, without discussing them, we may say that the evidence seems conclusive that the phenomena we are studying are due to special conditions of the medium occupying the field, although we cannot say definitely what these conditions are.

Electric and magnetic force can act, not only through ordinary matter in any of its forms, solid, liquid, or gaseous, but also through what we are accustomed to speak of as a perfect vacuum. We must suppose, therefore, that the electromagnetic medium is something different from ordinary matter, existing where matter is not, and, indeed, extending throughout space to the farthest limits of which we have any cognisance.

Even where ordinary matter exists, say atmospheric air, its mechanical properties are not such as can explain the electric and magnetic forces which act through it. Hence, the required medium must be something to which the property of impenetrability, a property inseparable from our common idea of matter, does not apply. We must think of the medium as existing throughout space, and retaining everywhere essentially the same properties, whether the space is otherwise vacuous or occupied by any kind of matter.[1] When an electric field is filled with air, glass, or any other dielectric, we must suppose that the properties of the field depend essentially on the electromagnetic medium coexisting with the ordinary matter, though the properties of the medium are modified quantitatively by the nature of the ordinary matter with which it is associated.

337. Electricity and Optics.—In order to explain the propagation of light across stellar space, and to account for the properties of light as ascertained by experiment, men of science have long recognised the necessity of supposing some medium, the so-called "luminiferous ether," possessing mechanical properties different from those met with in ordinary matter, to exist throughout all transparent substances and to extend through space.

After imagining one intangible, all-pervading ether, in order to explain optical phenomena, it seems highly artificial and complicated to imagine a second intangible medium to account for the phenomena of electricity and magnetism. It turns out, however, that these two hypotheses mutually support each other; that, in fact, the mechanical properties which must be attributed to the ether, in order to explain the transmission and properties of light, are just those which the electromagnetic medium must possess in order to produce the effects of electricity and magnetism. We may say, therefore, that the unknown electromagnetic medium has been found in the long-admitted luminiferous ether.

It appears, indeed, that in studying optics and electricity we are really investigating the properties of the ether from two different sides. In optics, we are dealing with effects due to vibrations of enormous rapidity;[1] whereas in electricity we observe the effects of very much slower vibrations, or of steady, non-vibratory conditions of the ether. Such at least seems to be the fair inference from what is known so far.

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[1] If we think of matter as made up of discrete molecules, the electromagnetic medium may be imagined to occupy the space between the molecules, so that it may not be necessary to suppose penetration in the ultimate sense.

Without attempting to give a complete account of the evidence in favour of the statements we have just made, we will endeavour to explain the nature of some of the chief steps in the argument in favour of regarding electrical and optical phenomena as different manifestations of the same agency.

The fundamental property that must be ascribed to the luminiferous ether is that of receiving and transmitting energy in the form of vibrations; hence it must have inertia and elasticity, and thus be capable of possessing energy either in the kinetic or potential form. The kinetic energy of a portion of the ether possessing inertia m, and moving with velocity c, is of course $=\frac{1}{2}mc^2$. Comparing the ether with an ordinary elastic material possessing the same properties in all directions, we may express the potential energy of a given portion of it as follows:—Consider a rectangular block of an elastic material having one face, call it M, held fast, while a force f is applied uniformly all over the parallel face N. For definiteness, suppose f to be parallel to M and N, and to another pair of the faces of the block. If a is the area of one of the faces M, the intensity of the applied force, or the *stress*, tending to deform the block is $p = f/a$. The result will be that every point of the face M will move relatively to N in the direction of the force through a distance λ, say, producing a *strain* (in this case a shear), $s = \lambda/l$, if we put l for the distance between the surfaces M and N. The applied force f will be proportional to the displacement λ, and during its application work will be done equal to the product of its average value into the final value of the displacement; this is easily seen to be

$$\tfrac{1}{2}f\lambda = \tfrac{1}{2}pa\,.\,ls = \tfrac{1}{2}ps\,.\,v,$$

where $v = a\,l$ represents the volume of the elastic block.

The work done in producing the strain is stored up as potential energy in the strained material, and from the above it appears that the energy per unit volume of a uniformly strained elastic substance is equal to half the product of the stress into the strain.

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[1] About 600 million million vibrations per second for the mean rays of the spectrum.

The criterion of perfect elasticity is that the ratio of stress to strain is constant, and this constant ratio is taken as the measure of the elasticity. Denoting it by N, we may write the potential energy of unit volume of an elastic substance in the three alternative forms

$$\tfrac{1}{2}ps = \tfrac{1}{2}\frac{p^2}{N} = \tfrac{1}{2}Ns^2.$$

In the case of electrical energy, it seems natural to regard the energy of an electrostatic field as potential, and the energy corresponding to the movement of lines of force, that is, the magnetic energy of a current, as kinetic. We will consider first electrostatic energy. If the charge of a field of capacity S is Q, and the difference of potential between the boundaries is $V - V'$, the total energy may be written

$$\tfrac{1}{2}(V - V')Q = \tfrac{1}{2}(V - V')^2 S = \tfrac{1}{2}\frac{Q^2}{S}.$$

To obtain expressions for the energy of unit volume, we will consider the energy of a portion of a field bounded by area a of two parallel plane surfaces at a distance l from each other. The energy in this case is

$$\tfrac{1}{2}(V - V')^2 \frac{aK}{4\pi l} = \tfrac{1}{2}\left(\frac{V - V'}{l}\right)^2 \cdot \frac{K}{4\pi}v,$$

where v is the volume of the field $= al$. If we put f for the intensity of electric force in the field, or $(V - V')/l$, and express the charge as $Q = \sigma a$, where σ is surface-density, we get the three following expressions for the energy of unit volume of the field—

$$\tfrac{1}{2}f\sigma = \tfrac{1}{2} \cdot \frac{f^2K}{4\pi} = \tfrac{1}{2} \cdot \frac{4\pi\sigma^2}{K}.$$

The electric force f per unit area of the field, and σ the charge per unit area, may be looked upon as representing respectively electric stress and electric strain, and their ratio, $\frac{4\pi}{K}$, then appears as the electric elasticity of the field. From this point of view, the three shapes in which we have written the expression for electric energy per unit volume correspond exactly with the three forms of the expression for elastic energy per unit volume.

As we have said, if the energy of an electric field is assimilated to the potential energy of elastic strain, the energy of a current must be of the nature of kinetic energy, and must, therefore, be expressible in a form comparable with $\frac{1}{2}mu^2$, where u is a velocity. An electric current implies the motion of lines of force (§ 298), and the strength of a current is an expression for the number of lines which pass a given section of the circuit in unit of time. We may, therefore, take u as equal to or proportional to C. It remains to consider what we are to take as representing the inertia m. To investigate this point we will again choose a case that admits of easy calculation. Consider the energy of a length l of an electromagnetic cylinder (§ 256) of cross-section a, having n turns of wire per unit length, through which a current of strength C is flowing. The energy is (§ 296) $\frac{1}{2}LC^2$, and L, the coefficient of self-induction, is the magnetic flux through the circuit multiplied into the electric flux round the circuit, both for the case of unit current. This gives (§ 273)

$$L = a \,.\, 4\pi n\mu \,.\, nl,$$

μ being the magnetic permeability of the medium inside the coil. The energy may, therefore, be written

$$\tfrac{1}{2}LC^2 = \tfrac{1}{2}al \,.\, 4\pi\mu \,.\, (nC)^2.$$

Here al is the volume of the electromagnetic field considered. The energy of unit volume is therefore

$$\tfrac{1}{2} \,.\, 4\pi\mu \,.\, (nC)^2,$$

where nC is the effective electric flux round unit length, and may be taken as the measure of electric velocity. In this case $4\pi\mu$ appears as the electric mass or inertia of unit-volume, or more simply as the electric density of the medium.

We may sum up the results of this discussion by saying that electrostatic energy and electromagnetic energy may be expressed as respectively the potential and the kinetic energy of the medium occupying the field, if $\frac{4\pi}{K}$ be taken as a measure of the electric elasticity of the medium, and $4\pi\mu$ as a measure of its electric density.

We began by recalling that the luminiferous ether, in order to fulfil its function of transmitting light, must possess elasticity and density. The velocity with which vibrations travel through an elastic medium is equal to the square root of the ratio of the elasticity to the density of the medium. If, then, the values of the elasticity and density that we have arrived at for the medium concerned in electric and magnetic phenomena are the values of the elasticity and density of the luminiferous ether, we should expect the velocity of light to be equal to

$$\sqrt{\frac{4\pi}{K} \div 4\pi\mu} = \frac{1}{\sqrt{K\mu}}.$$

Now, although, as has been said, no means is known of measuring the separate values of K and μ, the product $K\mu$ is measurable in various ways, and many experiments agree in indicating for its value for air $1 \div 9 \times 10^{20} \left(\frac{\text{centim.}}{\text{second}}\right)^2$ (§§ 386-390). The square root of the reciprocal of this, or 3×10^{10} centimetres per second, is almost exactly the mean of the best determinations of the velocity of light in air.

For other transparent media, the value of the magnetic permeability, μ, does not differ to any important extent from its value for air, therefore the difference of the velocity of light in different media must, if our theory is right, depend upon differences of specific inductive capacity, K. That is, if u is the velocity of light in air and u' its velocity in another medium, and K and K' the corresponding inductive capacities, we ought to have

$$\frac{u}{u'} = \sqrt{\frac{K'}{K}}.$$

The numbers that are recorded as the values of the specific inductive capacity for various substances are really comparative values referred to air, and therefore give at once the ratio K'/K. On the other hand, the undulatory theory of light indicates that the ratio u/u' of the velocities in air and any other medium is the *index of refraction*, n, of light from air into this medium. We thus come to the further conclusion that the *dielectric coefficient* of a transparent substance (that is, its relative specific inductive capacity compared with air) should be equal to the *square of the index of refraction* of light from air into that substance.

In many cases this conclusion is closely verified by direct experiment; in other cases there is considerable discrepancy, but, considering the enormous rapidity of the vibrations of light, it is quite conceivable that the virtual elasticity of the ether in reference to them may not be the same as that which comes into play in the case of the statical processes, or of the relatively slow vibrations that have been employed in determinations of dielectric coefficients. On the whole, it may be said that comparisons of indices of refraction and dielectric coefficients have afforded a remarkable confirmation of the general truth of our theory.

CHAPTER XXVIII.

GALVANOMETERS.

338. Galvanometer.—A galvanometer, in the strict sense of the word, is an instrument which serves to measure the intensity of a current by the action which it exerts on a magnetic needle.

It consists essentially of a frame on which wire is coiled in a vertical plane, and of a magnetic needle suspended horizontally at the centre of the coil (Fig. 301). The plane of the wire is placed in the magnetic meridian, and therefore the needle is parallel to it in its position of equilibrium. The field due to the earth and that due to the current near the middle of the coil are thus at right angles.

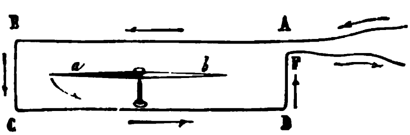

FIG. 301.

Let us suppose that the field due to the current is uniform in the region which is occupied by the needle, and let G be its strength for unit current; it will be GC for a current of strength C, and under the action of the two fields the needle will come to rest in a position which makes an angle α with its original direction. If H is the horizontal component of the magnetic force of the earth and M the moment of the needle, the angle α is given by the equation of equilibrium

$$MH \sin \alpha = MGC \cos \alpha,$$

from which

$$C = \frac{H}{G} \tan \alpha.$$

Thus, in the case in which the two fields are uniform and at right angles, the deflection is independent of the magnetic moment M of the needle, and therefore of its shape and magnetisation, and *the tangent of the deflection is proportional to the strength of the current.*

This condition that the field be uniform is of prime importance for the galvanometer. It dispenses with any empirical graduation. It is easily realised if small needles and small angles of deflection are used. This is the case with all galvanometers of modern construction. The inconveniences of small deflections are amply compensated by the accuracy with which they can be measured by the method of reflection (§ 80).

The scale reading gives directly tan 2α. If the deflection does not exceed 3 to 4 degrees, the error may usually be neglected which arises from taking $2 \tan \alpha = \tan 2\alpha$, and therefore in assuming that the current-strength for a fixed distance of the scale is proportional to the displacement $x - p$ of the image. For a deflection of 2° the error due to this assumption is less than $\frac{1}{800}$.

339. Absolute Measurement of Currents—Tangent Galvanometer.—The absolute measurement of current-strength depends on the measurement of G and H. We know how H is obtained (§ 231). The value of the quantity G, which we will call the *galvanometer-constant*, is deduced from the dimensions of the coil of wire. This requires that the windings should be accurately circular, and wound with such regularity that the radius can be exactly measured. The radius should, moreover, be very large in comparison with the dimensions of the needle, if we are to assume that the field in which the needle hangs is uniform.

A much more uniform field is obtained by using two equal circular coils, placed parallel to each other at a distance equal to their common radius (§ 251).

A current of unit strength going once round a circle of radius a, produces a magnetic force which, at any point on the axis of the circle, is directed along the axis, and at distance x from the centre is equal to

$$\frac{2\pi a^2}{(a^2+x^2)^{3/2}} \text{ (§ 249; compare Fig. 217).}$$

The values which this expression assumes for various values of x are shown graphically in Fig. 302, by the ordinates of the curve A_1CB_1. The value $x=0$ corresponds with the centre of the circle; the force here is greatest, and is represented by the maximum ordinate O_1A_1. As x increases, the magnetic force, as shown by the curve, decreases, at first slowly, then more and more rapidly until x becomes $= \frac{1}{2}a$, after which the rate of decrease of force again gets slower and the representative curve becomes less steep.

At the point of greatest steepness, C, the curve changes from being concave towards the axis to being convex, and at this point there is no curvature either way. In other words, at a distance along the axis equal to half the radius of the circle, the magnetic force decreases for a little way at a uniform rate. Hence, if two equal circles or circular coils are placed so that their axes coincide, and at a distance apart equal to their common radius, the magnetic field on the axis at a point half-way between the circles is uniform, since the decrease of force corresponding to a small increase of distance from one circle is compensated by the equal increase of force due to decrease of distance from the other. The curve

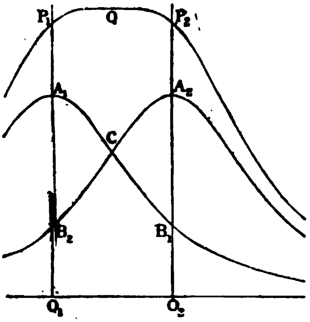

FIG. 302.

P_1QP_2, which represents the magnetic force due to two equal circular currents placed as described above, exhibits this result. The ordinates of this curve are the sums of the corresponding ordinates of the curves A_1CB_1 and A_2CB_2 representing the forces due to each of the two circles acting separately. For a distance on either side of the middle point equal to one-tenth of the common radius, the force decreases only by about one part in 3000, so that, with two circles, each of 25 centimetres radius, the magnetic force is almost exactly uniform for a distance of 2.5 centimetres on each side of the middle point.

Fig. 303 represents a galvanometer constructed upon this principle, which is due to Helmholtz. With an instrument of this kind, having in all n turns of wire arranged in two equal coils of mean radius a, with a distance $2x = a$ between the mean planes of the coils, the magnetic field at the centre due to a current of unit strength in the wire is

$$G = \frac{2\pi n a^2}{(a^2+x^2)^{3/2}} = 4.496\,\frac{n}{a}.$$

If the circles are accurately set with their common axis perpendicular to the magnetic meridian, the field $= CG$ due to the current is at right angles to the earth's horizontal field, H, and the resultant force makes with the meridian an angle, α, given by the relation

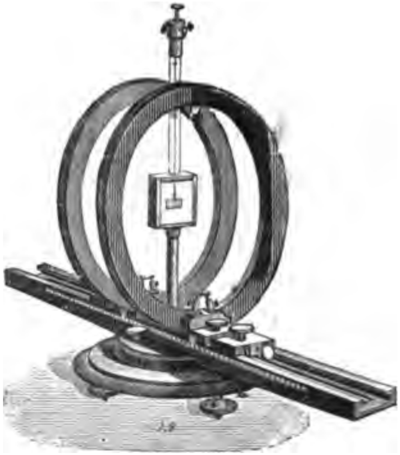

FIG. 303.

$$\tan \alpha = \frac{CG}{H},$$

and the axis of a small delicately suspended magnet at the centre of the instrument sets itself in this direction. The strength of a current which causes a deflection, α, of the magnet is therefore

$$C = \frac{H}{G}\tan \alpha = \frac{Ha}{4.496n}\tan \alpha = 0.2224\,\frac{a}{n}\,H\tan \alpha.$$

For example, suppose there are 60 turns of wire, 30 on each circle, that the mean radius is 20 centimetres, and $H = 0.1825$: then the current which deflects the magnet through 45° is

$$C = 0.01353 \text{ (C.G.S.)} = 0.1353 \text{ ampere.}$$

If the deflections are observed by the method of reflection, with a scale at a distance of 1 metre, a movement of the reflected ray through 1 centimetre indicates an angle a, such that $\tan 2a = \frac{1}{100}$. In such a case we may take $\tan a = \frac{1}{200}$, and the strength of the current is

$$\frac{.1353}{200} = .0006765 \text{ ampere.}$$

In the case of a single circle with the magnet hung at the centre, the value of x in the above formula $= 0$, and the field at the centre due to a unit current is given by the simpler formula

$$G = \frac{2\pi n}{a},$$

and the strength of a current which produces a deflection, a, becomes, in C.G.S. measure,

$$C = \frac{Ha}{2\pi n} \tan a.$$

340. Sine Galvanometer.—Fig. 304 represents a sine galvanometer. The circle is set at first in the plane of the magnetic meridian, and, when a current passes, it is turned about a vertical diameter in the direction of the deflection, until it has again the same position in regard to the needle. This condition is indicated when the pointer *cd* fixed to the needle has the same position as at first on the graduated circle A. Let a be the angle through which the instrument has been turned, read off on the horizontal limb, C, then the force due to the current, MGC, balancing the action of the earth, $MH \sin a$, we have

$$MGC = MH \sin a,$$

whence

$$C = \frac{H}{G} \sin a.$$

The current-strength is proportional to the sine of the angle through which the instrument is turned, whence the name of the apparatus. It is not necessary that the field due to the current should be uniform.

By this apparatus current-strengths greater than H/G cannot be measured.

341. Shape of the Coil.—In ordinary galvanometers, no attempt is made to calculate the constant G directly; sensitiveness is the only character looked to. This is greater according as the value of tan α is greater for the same current. The sensitiveness is then measured by the ratio $\frac{G}{H}$, and therefore is greater as G increases and H is less.

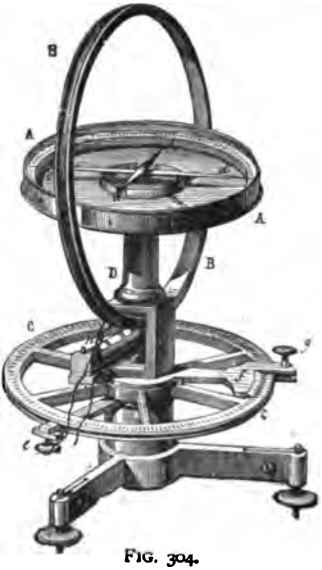

FIG. 304.

The magnetic field at the centre due to unit current in a circle of radius a is $\frac{2\pi}{a}$; the length of wire being $2\pi a$, the field due to unit-length of the wire, or the specific effect, is $\frac{1}{a^2}$, and therefore varies inversely as the square of the radius. We are thus led to diminish as much as possible the radius of the windings and therefore the length of the needle.

The shape to be given to the coil, so as to get the greatest advantage from the wire used, is determined by the condition that the specific effect is to be the same at all parts of the outer surface; for, if it were otherwise, there would be an advantage in transferring some of the wire from the position which it occupies to another where the specific effect is greater. This leads to the adoption for the meridian section of the coil of a figure approximately like that obtained by squeezing a circle along its vertical diameter (Fig. 305).

For a circle of radius r the magnetic force along the axis at a point O, distant ρ = OP from the circle (Fig. 305) is $\frac{2\pi r^2}{\rho^3}$ (§ 249), and the force due to unit-length of wire is $\frac{r}{\rho^3}$. The condition just stated, that this quantity is to be constant $\left(= \frac{1}{N^2} \text{ say}\right)$ at all points on the surface of the coil, may be written

$$\rho^2 = N^2 \sin\theta.$$

For $\theta = 90°$, this evidently gives $\rho = N$.

The figure gives successive curves for values of N, which increase in arithmetical progression; the dotted parts of the curve correspond to the central space left for the magnet, which is hung at O, with its axis perpendicular to the plane of the figure. In practice, the cross section of the coil has not the exact shape represented in the figure, but is a rectangle agreeing as nearly as may be with the theoretical shape. In any case, the magnet is at the centre of a long coil, so that the small part of the field in which it moves may be regarded as uniform.

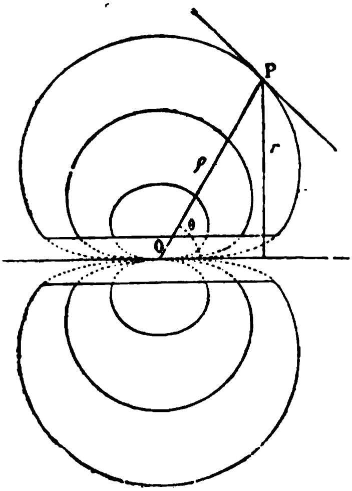

FIG. 305.

Considerations similar to those of § 271 show that the action exerted by a coil of given shape is independent of the size of the wire if the density of the current is constant. But in the case of a given electromotive force, the current depends on the resistance of the wire, and the external action is a maximum when the resistance of the galvanometer-coil is equal to that of the rest of the circuit.

342. Compensating Magnet.—Two methods may be employed either separately or simultaneously to diminish the value of H.

The effect of the earth may be compensated by that of a magnet which produces in the place occupied by the needle a sensibly uniform field in the opposite direction to that of the earth. This magnet, which is called the *compensating* or *adjusting magnet*, is usually supported on a rod (Fig. 306), to which it is fixed by a clamping screw, whereby it can be set at any desired height, and by means of a tangent screw, the rod itself, along with the magnet, may be turned about its axis.

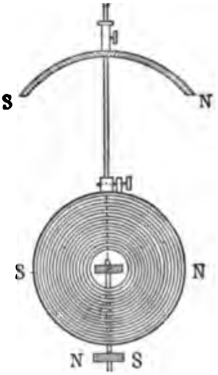

FIG. 306.

The direction of equilibrium of the needle is that of the resultant of the two fields. Thus, if OA (Fig. 307) is the direction of the terrestrial field, OB that of the field of the magnet, OC will be the direction of the needle. It will be seen that when the two fields are nearly equal, the resultant makes an angle of nearly 90° with the common direction. In this case, the direction of the resultant, and therefore the position of equilibrium of the needle, is powerfully influenced by variations of the earth's field.

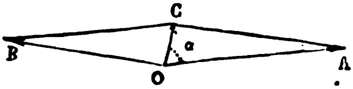

FIG. 307.

343. Astatic Needles.—The second method consists in using a system of astatic needles (§ 173). The force of the earth on the system may be reduced at will. On the other hand, if only one of the needles is placed inside the coil and the other outside, the force due to the coil is slightly increased, as the force which it exerts on both needles is in the same direction (Fig. 306).

But a more symmetrical apparatus and greater directive force is obtained by using two coils, one over the other (Fig. 308), and placing one of the needles in the middle of each.

If the current passes in opposite directions on the two coils, the effects are obviously concordant.

344. Damping.—When a current corresponding to a permanent deflection a is started in a galvanometer, the needle receives an impulse which, neglecting passive resistances, would cause it to pass beyond its position of equilibrium by an angle a, so that the first throw would be equal to $2a$. The needle then continues to oscillate until it has expended its energy against passive resistances.

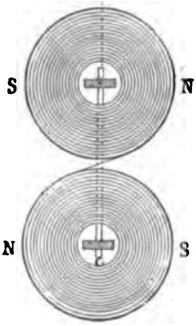

FIG. 308.

The oscillations may at once be reduced to a very small amplitude if the current is broken after a time equal to one-third of the period of a single swing of the needle. From the laws of simple harmonic motion, the needle will in this time have traversed the arc $\frac{a}{2}$, and acquired just sufficient velocity to bring it to a. The circuit is then closed again and the needle remains practically at rest.

An analogous method is used to bring it back to zero with no velocity; the circuit is broken during the first third of the oscillation, it is made again during the second, and is then permanently broken.

Even with these precautions the needle would oscillate for a long time before coming perfectly to rest if the oscillations were not damped. This may be effected either by increasing the resistance of the air, by light vanes attached to the magnet and oscillating with it, or by causing the motion of the magnet to generate induction currents in the conductors which surround it; these currents, according to Lenz's law (§ 292), tend to oppose the motion. When the resistance of the circuit is small, the currents produced in the wire itself are sufficient to damp the motion; in galvanometers of high resistance the needle is sometimes surrounded by a mass of copper.

Whether the damping is due to the resistance of the air or to induced currents, experiment shows that the amplitude of oscillation decreases as the terms of a geometrical progression. We may infer from this that the retarding causes are at each instant proportional to the velocity of the needle. If a_0, a_1, a_2, . . . are the successive amplitudes, we have

$$\frac{a_0}{a_1} = \frac{a_1}{a_2} = \frac{a_2}{a_3} = \text{constant.}$$

Let the value of this constant be called e^{λ}, e being the base of the Napierian logarithms. The quantity λ, or the Napierian logarithm of the ratio of two consecutive amplitudes, is called the *logarithmic decrement* of the oscillations, and is taken as a measure of the damping. The duration, t, of the damped oscillation is a little greater than the duration, T, of the oscillation of the same system without damping.

By gradually increasing the damping, the oscillations may ultimately be suppressed, and an *aperiodic* or *dead-beat* motion obtained; the needle, when removed from its position of equilibrium, returns thither with a velocity which increases at first, passes through a maximum, and finally becomes zero, when the needle attains its position of equilibrium.

345. Shunts.—The limits within which a galvanometer may be used may be greatly extended by means of *shunts*. This term is applied to a branch circuit connecting the two binding screws of a galvanometer, whereby a known fraction only of the current is allowed to pass through the galvanometer.

Let g be the resistance of the galvanometer, s that of the shunt, C the total current, and c the current in the galvanometer which produces the observed deflection; then, from the law of compound circuits (§ 117),

$$C = c\frac{g + s}{s}.$$

The factor $\frac{g + s}{s} = m$, by which the galvanometer current must be multiplied to get the total current, is called the *multiplying power* of the shunt. Galvanometers are usually provided with three shunts, which have respectively the multiplying powers 10, 100, 1000, and the resistances of which are therefore $\frac{1}{9}, \frac{1}{99}, \frac{1}{999}$ that of the galvanometer.

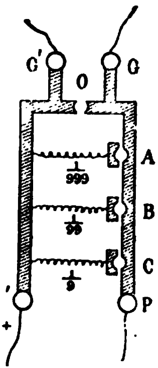

FIG. 309.

The wire of the galvanometer being attached to the terminals

G and G′, and those of the battery either to the same terminals or to others, P and P′ (Fig. 309), if one of the holes, A, B, C, is stopped with a plug, the corresponding shunt is introduced. When the plug is placed at O, the galvanometer is "short-circuited," and is thus protected against the passage of an accidental current.

346. Differential Galvanometer.—If two identical wires are coiled close together, their effects are added together, if currents in the same direction are passed through both; but their combined effect is the difference of their separate effects, if the currents are in opposite directions, and therefore vanishes when the currents in opposite directions are equal.

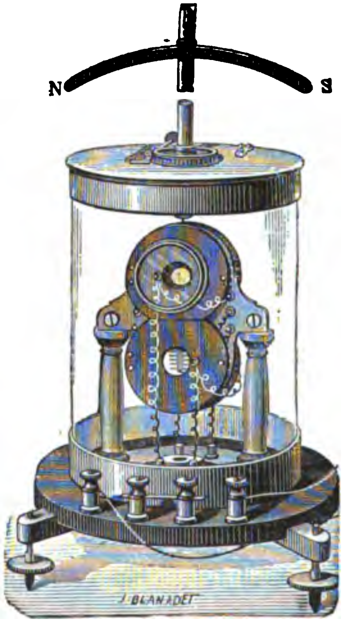

FIG. 310.

A differential galvanometer should satisfy two conditions: the two circuits should exert equal effects on the needle, and should have the same resistance. The first condition can be tested by connecting the two coils in series, but so that the current traverses them in opposite directions ; the second by connecting the coils in parallel, so that the currents again circulate in opposite directions ; the needle should be stationary in both cases.

347. Thomson's Galvanometer.—The general plan of this is represented in Figs. 305 to 308. It has either one or two coils. The one-coil galvanometer is not, in general, astatic. The galvanometer with two coils is represented in Fig. 310. It has usually a considerable resistance—from 8000 to 10,000 ohms, or even more. The needles are about 8 millimetres in length, and usually consist of a series of small parallel bars (Fig. 311), thus giving a greater magnetic moment for the same weight.

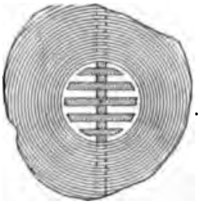

FIG. 311.

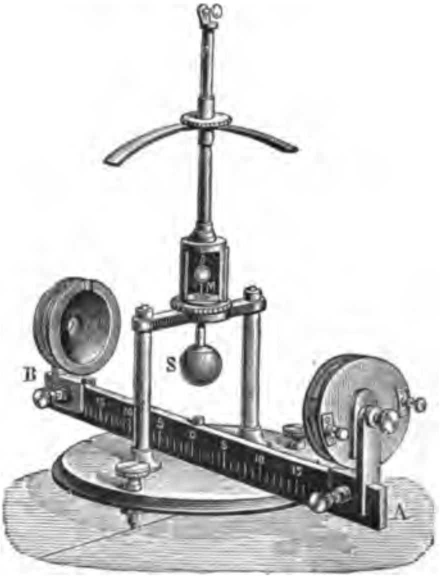

FIG. 312.

The small mirror is fixed to one of the sets of needles, or, better still, between the two coils, between which a sufficient space is left. The damping is effected by a thin plate of mica, that moves with the needle. The two coils can be connected in series or in parallel circuit, or the instrument can be used as a differential galvanometer.

Thomson's galvanometers attain the highest degree of sensitiveness. With a current of a thousandth of a microampere, deflections of 50 millimetres are obtained on a scale at a distance of a metre.

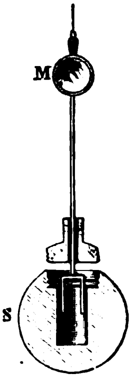

FIG. 313.

348. Wiedemann's Galvanometer. — Fig. 312 represents a galvanometer of the form introduced by Professor G. Wiedemann. The needle is a horse-shoe magnet which oscillates in a cavity in a solid sphere of copper (Fig. 313). By this means a damping effect is obtained sufficient to make the instrument aperiodic. The coil is divided into two parts, which may be moved along a divided scale, and thus the sensitiveness may be varied at pleasure. There may be several sets of coils with wires of various thicknesses. The mirror is placed in a small chamber with a glass front, and can be turned in any direction without its being necessary to twist the rest of the instrument.

By means of an external magnet, the strength and direction of the field can be modified at pleasure, as in Thomson's galvanometer.

If the instrument is to be used as a ballistic galvanometer, the copper sphere is removed, to allow oscillation to take place freely.

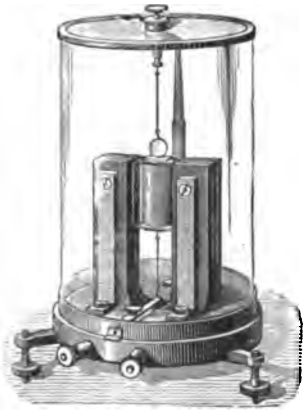

FIG. 314.

349. Deprez and d'Arsonval's Galvanometer. — A rectangular coil (Fig. 314), movable about a vertical axis formed by two metal wires by which the current enters and leaves, is placed between the poles of a horse-shoe magnet, and a cylinder of soft iron, which becomes magnetised by induction, is supported inside the coil. When the current passes, the coil tends to set itself with its plane perpendicular to the field (§ 261), and takes such a position that the deflecting couple due to the electromagnetic forces balances the torsion of the suspending wire. The damping is produced by induction currents due to the motion of the coil in the field, and as the field is very strong, the apparatus is nearly dead-beat when the circuit is closed by a low resistance. The damping effect is often increased by coiling the wire on a light silver frame.

The deflections are read by means of a small mirror attached to the coil. If S is the total effective surface enclosed by the coil, and H the average strength of the field, and T the coefficient of torsion of the wire, the equation of equilibrium is

$$SHC \cos a = Ta$$

from which is deduced

$$C = \frac{Ta}{SH \cos a}.$$

For small deflections a is equal to sin a, and the current is sensibly proportional to the tangent of the deflection.

350. Electrodynamometer.—In electrodynamometers the current is measured by the force which two conductors exert on each other. Other things being equal, the action is proportional to the product CC' of the two currents, or to the square of the current if the two conductors are each traversed by the same current. In the latter case the direction of the force is not changed when the direction of the current is reversed.

The apparatus usually consists of two coils, one fixed, the other movable, the movable coil being placed inside the fixed coil so that the centres coincide and the axes are at right angles to each other.

Under the influence of the current the axes tend to set parallel, and the moment of the couple which acts on the movable coil is equal to the product of its magnetic moment into the intensity of the field due to the fixed coil. For a current C, the moment of the movable coil formed of n turns of wire enclosing a total surface $S = n\pi r^2$ is equal to SC; the strength of the field is GC, G being the constant of the fixed coil. The moment of the couple is therefore SGC^2. In Weber's electrodynamometer the movable coil is supported by a bifilar suspension, which at the same time serves to convey the current. The angle of deflection a represents the equilibrium between the electrodynamic couple and that of the suspension, so that if T is the coefficient of torsion of the bifilar,

$$SGC^2 \cos a = T \sin a$$

and therefore

$$C^2 = \frac{T}{SG} \tan a:$$

the square of the current is proportional to the tangent of the deflection.

351. **Pellat's Absolute Electrodynamometer.**—In Pellat's absolute electrodynamometer (Fig. 315) the movable coil is fixed at the end of a balance-beam, so that its axis is vertical, and it is placed in the uniform field due to a long regularly wound coil. If n_1 is the number of turns per centimetre of the fixed coil, we have

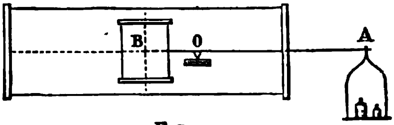

FIG. 315.

$$G = 4\pi n_1,$$

and the moment of the couple which acts on the beam is $4\pi n_1 SC^2$. This is counterpoised by the weight of a mass, p, placed at a distance, a, from the knife-edge; we have thus

$$pga = 4\pi n_1 SC^2,$$

from which we can deduce

$$C = A\sqrt{p},$$

where

$$A = \frac{1}{2\pi r}\sqrt{\frac{ag}{nn_1}},$$

a constant which is determined once for all by direct measurements

352. **Balance Electrodynamometers.**—The strength of a current may also be deduced from the square of the attraction or repulsion exerted between two parallel coils kept at a fixed distance. In an apparatus of this kind, one of the coils is suspended to the beam of a balance, and the force between it and the fixed coil is weighed.

Electrodynamometers are not capable of such a high degree of sensitiveness as is readily obtainable with a galvanometer. But they have the advantage, as compared with galvanometers, that their indications are independent of the external magnetic field, and when two coils are traversed by the same current, they are independent also of changes in the direction of the current.

CHAPTER XXIX.

ELECTROMAGNETIC MEASUREMENTS.

353. Ordinary Measurements—Measurement of the Strength of a Current.—The measurements most frequently met with in practice are those of *current*, of *quantity*, of *electromotive force*, of *resistance*, and of *capacity*.

The absolute measurement of the strength of a current is made directly by a tangent galvanometer, or by the absolute electrodynamometer. The tangent galvanometer gives (§ 339)

$$C = \frac{H}{G} \tan \alpha.$$

According to this formula the strength of the current depends upon the constant G, which has to be calculated from the dimensions of the instrument, and H, which has to be determined by a separate experiment.

The electrodynamometer has the advantage of not requiring this latter determination, which is always a matter of some difficulty; on the contrary, when associated with a tangent galvanometer which is traversed by the same current, it furnishes one of the best methods of obtaining the value of the horizontal component.

Currents are most frequently measured by a calibrated galvanometer. For this purpose the deflection produced by a known current must be ascertained. The same current is passed simultaneously through a tangent galvanometer, and through the galvanometer to be calibrated, which may be shunted if necessary. Let α and α' be the simultaneous deflections observed on the two instruments; G' the field due to unit-current in the galvanometer under examination, and H' the horizontal component at the place where the needle hangs, we have

$$C = \frac{H}{G} \tan \alpha = \frac{H'}{G'} \tan \alpha';$$

an equation which gives the constant by which the indication of the latter galvanometer must be multiplied to obtain the value of the current either in C.G.S. units or in amperes.

With galvanometers of high sensitiveness, the calibration may be effected more directly. It is only needful to observe the deflection produced when the galvanometer is joined up in a circuit of very high resistance, with a cell of known electromotive force.

Suppose we are using a Daniell's cell, that the total resistance of the circuit is 30,000 ohms, and that the galvanometer shunted to one-thousandth gives a deflection of 100 scale divisions.

The total current is $\frac{1.07}{30{,}000}$ amperes, but only one-thousandth of this passes through the galvanometer. Hence one division corresponds to a current of

$$\frac{1.07}{30{,}000 \times 1000 \times 100} = 3.6\ 10^{-10} \text{ amperes} = 3.6\ 10^{-11} \text{ C.G.S.}$$

The resistance of the coil is not needed, as it is negligible in comparison with the total resistance of the circuit. If the resistance of the galvanometer is g, its effective resistance when shunted to the mth is g/m, and is therefore usually a small part of the resistance of the circuit.

354. Measurement of a Quantity of Electricity—Ballistic Galvanometer.—When a discharge traverses a galvanometer, the needle is thrown from its position of equilibrium through a certain arc, and comes to rest after a series of oscillations. If the discharge is completed before the needle has had time to be appreciably displaced, the angle of throw measures the quantity of electricity put in motion.

The mechanical problem is the same as that of an instantaneous force acting on a pendulum. Let m be the magnetic moment of the needle, K its moment of inertia in reference to the axis of rotation, and G the galvanometer constant; when the strength of the discharge current is C, the deflecting moment exerted on the needle is GmC, and in the infinitely short time dt this gives the needle an angular velocity, $\frac{GmCdt}{K}$. If the needle has not appreciably moved during the time occupied by the whole discharge, the final velocity is the sum of all the terms of the same kind, a sum which is evidently equal to $\frac{GmQ}{K}$, Q being the total quantity of electricity put in motion. Hence if ω is the angular velocity imparted to the needle,

$$K\omega = GmQ \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Again, the initial energy of motion of the needle, namely, $\frac{1}{2}K\omega^2$, is expended, during the swing, in doing work against the horizontal component of the earth's magnetic force; if θ is the angle of deflection, this work is (§ 172) $mH(1 - \cos\theta)$, whence we get

$$\tfrac{1}{2}K\omega^2 = mH(1 - \cos\theta) = 2mH\sin^2\tfrac{1}{2}\theta \quad . \quad . \quad (2)$$

If τ be the period of one complete vibration of the needle, we have

$$\tau = 2\pi\sqrt{\frac{K}{Hm}}, \quad \text{or } K = \frac{mH\tau^2}{4\pi^2} \quad . \quad . \quad . \quad (3)$$

Multiplying together equations (2) and (3), and taking the square root, gives

$$K\omega = \frac{mH\tau}{\pi}\sin\tfrac{1}{2}\theta,$$

and by substituting this value in (1), we obtain, to determine the quantity of electricity that has traversed the galvanometer,

$$Q = \frac{H\tau}{G\pi}\sin\tfrac{1}{2}\theta,$$

or, if θ is small enough to be identified with its sine,

$$Q = \tfrac{1}{2}\frac{H\tau}{G\pi}\theta = A\theta.$$

The constant A is found by determining the factor $\frac{H}{G}$ as above (§ 353), and the time of oscillation of the needle,—or, more simply, by determining the angle of throw corresponding to the discharge of a known quantity of electricity, for instance, that obtained by turning a coil of a known surface (§ 302) in the earth's field through an angle of 180°.

In the preceding calculation, the arc θ is that through which the needle would be deflected if there were no damping. The angle really observed is a smaller angle θ_1. To get the necessary correction, the next deflection, θ_2, on the same side should be observed, then

$$\theta = \theta_1 + \frac{\theta_1 - \theta_2}{4}.$$

For between the two deflections, θ_1 and θ_2, the needle has made four quarter vibrations, and it may be assumed that the damping during the first quarter vibration, when the needle swings from rest to θ_1, is equal to the mean damping during each of the four following quarter vibrations.

355. Alternating Currents.—If a series of discharges, alternating in opposite directions, and succeeding each other at very short intervals, are passed through the galvanometer, the deflection measures the algebraic sum of the electricity which passes, and, if this is zero, the needle remains stationary, or only undergoes accidental deflections. This is the case with a sinusoidal current and with the alternate currents obtained in a circuit by rapidly making and breaking an adjacent circuit (§ 305). In this case the electrodynamometer may be employed; the deflection is proportional to the mean square of the current, that is to say, to the number which is the mean of the squares of successive current-strengths (§ 323). It must not be forgotten that the electrodynamometer brings into the circuit not only its own resistance, but also its self-induction. The electrometer gives a method free from this difficulty.

356. Measurement of Resistance.—A resistance is measured by comparing it with a known resistance by means of a current. It is impossible to compare two resistances without passing a current through them any more than we can compare two masses without submitting them to the action of a force,—that of gravity, for example.

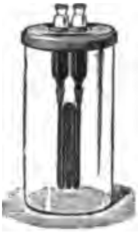

FIG. 316.

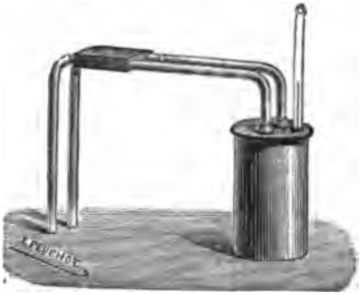

FIG. 317.

The legal unit of resistance is the ohm, defined as being the resistance at 0° of a column of mercury of one square millimetre in cross section and 106.3 centimetres long. Copies are made either in mercury (Fig. 316) or in wire; the wire, frequently german-silver or platinum-silver, is wound on a bobbin and sunk in paraffin (Fig. 317). These copies have their marked value for a definite temperature. The variation of the resistance of the mercury in a glass tube is given by the formula

$$R = R_0(1 + 0.0008726t + 0.000000106t^2),$$

that of german-silver by the formula

$$R = R_0(1 + 0.00044t).$$

A thermometer in the box containing the wire indicates the temperature.

357. Resistance Boxes.—It is necessary to have at our disposal a series of resistances, the values of which increase regularly. They consist ordinarily of coils placed in a box, and provided with plugs, by means of which any one or more of them may be inserted into the circuit at pleasure.

The wire is usually coiled after having been folded back on itself. In this way the effects of self-induction are diminished, but the capacity of the wire is increased by bringing close to each other parts of the wire which are at different potentials. It is best to form the coils of successive layers wound in opposite directions. The wire should be completely insulated.

The two ends of the same coil terminate in two contact pieces of copper, A and B (Fig. 318), with an interval between them which may be closed by a plug. When the plug is in its place the resistance is zero, that of the bar being negligible. When the plug is removed the corresponding coil is brought into the circuit.

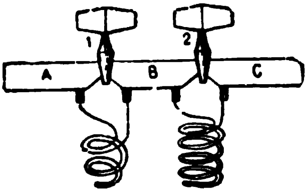

FIG. 318.

The most convenient arrangement is that of "dial" boxes (Fig. 319). Each dial consists of nine equal coils connected with each other by ten copper blocks numbered from 0 to 9. There is no coil between the blocks 9 and 0. In the centre is a circular block connected to the zero of the following dial by one of the copper strips, L, L′, L″. The plugs are placed between the central blocks and the blocks of the dial.

The first dial gives units, the others give tens, hundreds, &c. If the plugs are placed as shown in the figure, they bring in a resistance of 2305 ohms between the terminals B and C. K is a safety plug, which, when it is removed, cuts off all connection between the two plugs B and C. By means of special plugs provided with binding screws which can be inserted in holes in the blocks of the dial, conductors can be connected with any intermediate point of the dials. The rest of the figure is explained at the end of the next section.

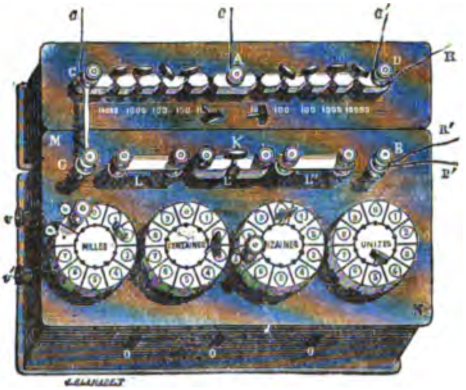

FIG. 319.

358. Wheatstone's Bridge.—The method of comparison most usually adopted is that of *Wheatstone's Bridge.* It is an application of the following problem:—

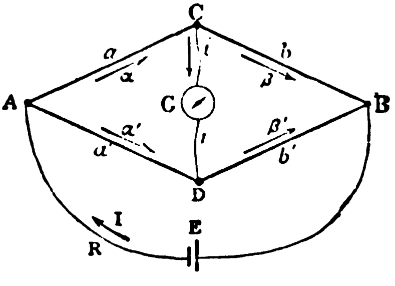

FIG. 320.

Two points, A and B, of a circuit (Fig. 320) are connected by two conductors. It is required to make a connection or *bridge*, CD, between the two branches, such that no current passes by this bridge.

For this it is necessary and sufficient that the two points C and D be at the same potential.

Now, if a a', b and b' are the resistances of the four branches AC, AD, CB, DB, V the potential at the point A, V' that of the point B, the fall of potential from A to C is

$$(V - V')\frac{a}{a+b},$$

and from A to D

$$(V - V')\frac{a'}{a'+b'}.$$

Consequently, for the potentials at C and D to be the same, we must have

$$\frac{a}{a+b} = \frac{a'}{a'+b'},$$

and therefore

$$\frac{a}{a'} = \frac{b}{b'}, \text{ or } ab' = a'b;$$

in other words, the resistance of any two adjacent sides of the quadrilateral, ABCD, must be to each other in the same ratio as those of the other two sides.

It will be seen that the state of equilibrium is independent of the resistance of the branches, which contain the battery and the galvanometer. These two branches are said to be *conjugate:* an electromotive acting in one of them produces no current in the other.

This arrangement furnishes a very simple means of comparing two resistances, b and b'; for when there is no current in the bridge CD, their ratio is that of the two resistances a and a', which can be adjusted at pleasure.

Suppose that the four sides of the quadrilateral are made up of metal bars having no appreciable resistance, but that any desired resistance can be inserted in any of them (Fig. 321). Let b be the resistance to be measured, b' the standard resistance with which it is to be compared, and which can be adjusted so as to produce equilibrium. If we take $a = a'$, then $b = b'$; but the terms a and a' may be in any ratio, and the condition of equilibrium is that the resistances of the branches b and b' shall be in the same ratio.

This arrangement of Wheatstone's is frequently compared to a balance, the resistances a and a' corresponding to the two arms of the beam; but, contrary to what occurs in the case of weights, equilibrium requires that the resistances to be compared shall be in the same ratio as the arms of the balance.

Each of the arms a and a' contains resistances of 10, 100, 1000 ohms. In the figure the ratio of the two arms is shown as that of

10 to 1, and, since the resistance b' is equal to 15 ohms, the resistance measured is 150 ohms.

In order to avoid the effects of induction on the galvanometer, the circuit containing the galvanometer is only closed by the key K_2, after the circuit of the battery has been closed by depressing the key K_1.

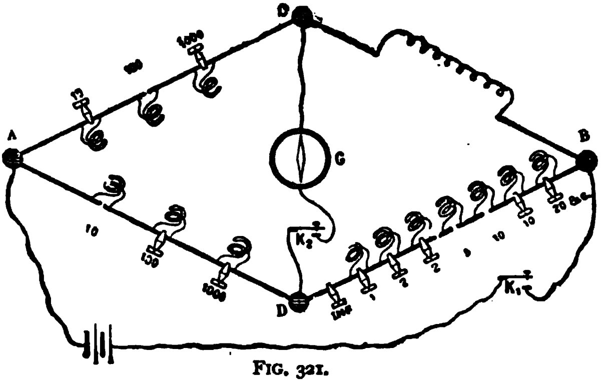

FIG. 321.

The portion CAD of Fig. 319 represents the balancing coils of a dial resistance box. The coils in the arms a and a' are 10, 100, 1000, and 10,000 ohms. Hence we can get any decimal ratio between 10^{-3} and 10^3.

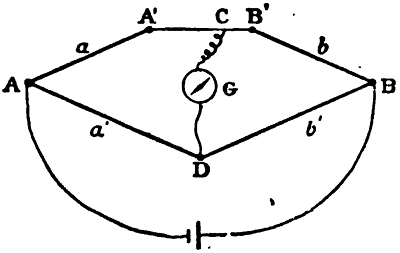

FIG. 322.

358A. Wire Bridge.—Suppose that in the ordinary quadrilateral (Fig. 322), one of the summits, C, is replaced by a straight wire A′B′, along which the end, C, of the galvanometer branch may be moved. If l is the length of the wire A′B′, which should be perfectly cylindrical and homogeneous, x the distance A′C, and if we suppose resistances expressed in units of length of the wire A′B′, we have for the condition of equilibrium

$$\frac{a'}{b'} = \frac{a + x}{b + l - x}.$$

The wire, which is usually of german-silver, forms one side of an elongated rectangle (Fig. 323), the other three sides of which are made up of broad copper bands of negligible resistance, at gaps in which the two resistances to be compared, a' and b', and two known resistances, a and b, are inserted. If these two latter are replaced by bars without resistance, we have simply

$$\frac{a'}{b'} = \frac{A'C}{B'C}.$$

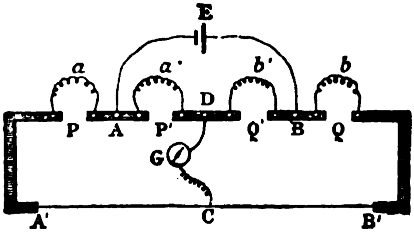

FIG. 323.

The apparatus is particularly convenient for comparing resistances which are almost identical with each other, such as the comparison of a copy with a standard.

A good plan for this purpose is to insert the resistances to be compared, which may be represented by a and b, in the two end gaps P and Q, the two middle gaps P′ and Q′ being filled by any two equal or nearly equal resistances a' and b'; then, after finding the balancing position of the sliding contact C, we interchange the resistances a and b, and again find the balancing position. Suppose that in the first case, with a at P and b at Q, the distance A′C is found to be m centimetres, and that, after interchanging a and b, a balance is obtained with A′C $= n$ centimetres, then if r is the resistance of 1 centimetre of the wire A′B′, we have

$$a - b = (m - n)\,r;$$

that is to say, the difference between the two resistances to be compared is equal to the resistance of the part of the graduated wire A′B′ over which the movable contact C is shifted. To prove this, we have only to remember that the total resistance AA′B′B is not altered by interchanging a and b, and that, since a' and b' remain unaltered, the second position of the point C must divide AA′B′B in the same ratio as the first.

359. Resistance of a Battery.—The poles of the battery are connected by a conductor of very high resistance, g, containing a galvanometer, and a current, c, is observed. The poles are then connected by a second conductor of resistance, s, not very different from that of the battery, arranged in multiple arc with g, and the current c' is read off. If ρ is the resistance of the battery,

$$\frac{\rho}{s} = \frac{c - c'}{c'}.$$

For if E is the electromotive force of the battery, we have (§ 117),

$$c = \frac{E}{\rho + g} \text{ and } c' = \frac{E}{\rho + \dfrac{gs}{g + s}} = \frac{Es}{g\rho + gs + s\rho}.$$

Hence

$$\frac{c}{c'} = \frac{g\rho + gs + s\rho}{gs + s\rho},$$

and

$$\frac{c - c'}{c'} = \frac{g\rho}{gs + s\rho},$$

which reduces to the value given above when g is very much greater than ρ.

360. Measurement of Electromotive Forces.—(1.) **Electrometer Method.**—The electrometer at once gives the electromotive force of a couple by comparison with a standard. A galvanometer may also be used provided we join it up with a high resistance. If the resistance of the element can be neglected in comparison with that of the rest of the circuit, the electromotive forces are, as when using the electrometer, proportional to the deflections.

The difference of potential between two points, A and B, of a wire traversed by a current may also be measured in the same way. If C is the strength of the current, and R the resistance of the wire between the two points, we have $e = RC$; a measurement of e thus enables us to determine C when R is known. This method is frequently used for measuring strong currents.

It is specially applicable to the measurement of alternating currents. In using the electrometer, the needle being connected with one pair of quadrants (§ 79), the two pairs of quadrants are put respectively in connection with the two ends A and B of the resistance R. The deflection is proportional to the square of the difference of potential, and therefore does not change with the sign of this difference. If the alternate currents succeed each other at intervals which are very short in comparison with the time of oscillation, the needle takes up a fixed deflection proportional to the mean square of the current, that is, to the square of the effective current (§ 323). If θ is the deflection, C_e the effective current-strength, and A the constant of the instrument, we have

$$\theta = AV_e^2 = AR^2C_e^2,$$

and therefore

$$C_e = \frac{1}{R}\sqrt{\frac{\theta}{A}}.$$

361. (2.) **Method of Compensation.**—A current is passed (Fig. 324) through a homogeneous wire so that the fall of potential from A to B is greater than the electromotive forces which are to be compared. The positive pole of the couple is connected through a galvanometer with the point A: the negative pole is connected with a sliding key, C, which is moved along the wire. The distance x = AC is determined at which the key must be placed so that the galvanometer needle may be at zero. If C is the strength of the current in the wire AB, and k the resistance of one centimetre of this wire, then, in order that there may be no current from A to C through the galvanometer, the difference of potentials between A and B must be equal to the electromotive force to be measured; or

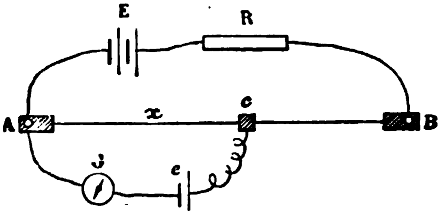

FIG. 324.

$$e = Ckx.$$

With the standard cell, we shall have, similarly,

$$e' = Ckx',$$

whence

$$\frac{e}{e'} = \frac{x}{x'}.$$

362. Closed Circuits.—We have thus obtained the electromotive force of a couple on open circuit. It is also a matter of interest to learn the electromotive force of the couple when at work, that is to say, the electromotive force as modified by the effects of polarisation. The two couples to be compared, E and E′, are put in the same circuit (Fig. 325), so that the electromotive forces act in the same direction round the circuit; a bridge containing a galvanometer is interposed between any point A, and a point B, which is found by trial, so that the needle remains at zero. Let R and R' be the resistances of the two segments AEB and AE′B of the circuit; when there is no current in AB, the current is of the same strength in the two segments. Kirchhoff's second law (§ 116) gives

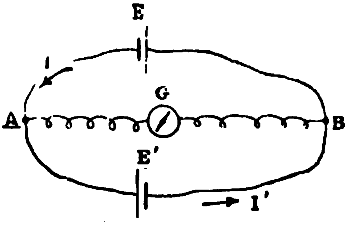

FIG. 325.

$$E = RC,$$

$$E' = R'C,$$

from which

$$\frac{E}{E'} = \frac{R}{R'}.$$

In order to avoid a measurement of the two resistances R and R', a known resistance is added to each of them, x for the first, x' for the second, so that there is still no current in AB; we have then

$$\frac{E}{E'} = \frac{R}{R'} = \frac{R+x}{R'+x'} = \frac{x}{x'}.$$

363. Standards of Electromotive Force.—The standard elements most frequently used are those of Daniell and Latimer Clark.

A *Daniell's* cell is not well adapted for a standard, as its electromotive force varies with the concentration of the liquids from 1.06 to 1.14 volt. Even with the same solutions there are variations of as much as 3 per cent. due to the condition of the metals. The variation with temperature depends also on the state of concentration of the liquids.

Latimer Clark's element consists of pure mercury as a negative metal covered with a paste obtained by mixing mercurous sulphate with a saturated solution of zinc sulphate. The positive metal is a zinc rod dipping into this paste of sulphate. Insulated wires, leading to the mercury and the zinc respectively, form the connections. This battery is not well adapted for continuous work, as it polarises easily, but it furnishes a very constant and trustworthy standard of electromotive force. Its electromotive force is 1.434 volts at 15° C., and it diminishes by about 0.0011 volt for an increase of temperature of one degree.

Thermo-electric couples furnish stable standards, provided the temperatures of the junctions are accurately known. If the junctions are at 0° and 100° respectively, the electromotive force of an iron-copper couple is 0.001307 volt (§ 143).

364. Measurement of Capacity.—In order to compare the capacities of two condensers, the condensers are charged by connecting the surfaces of each in succession with the poles of the same battery, and are then discharged through a ballistic galvanometer: the amounts of charge, and therefore the capacities, are as the angles of throw of the needle.

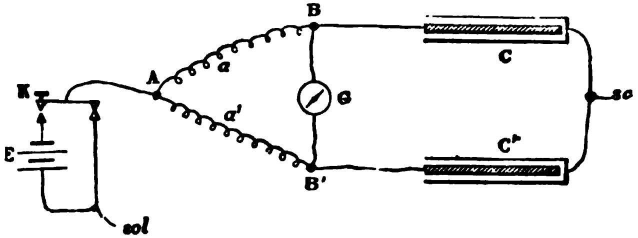

Fig. 326.

By charging the same condenser successively by means of two different batteries, we may compare in the same way the electromotive forces.

Two capacities may also be compared by an arrangement analogous to Wheatstone's bridge (Fig. 326). The two condensers, *C* and *C'*, take the place of the branches *b* and *b'* of the usual quadrilateral, one surface of each being connected to earth, or to the back contact of the key K. The resistances *a* and *a'* are then adjusted until there is no deflection of the galvanometer on raising or lowering the key K.

The condition of equilibrium is that the two points B and B' shall be at the same potential, that is to say, that at each instant the charges *Q* and *Q'* shall be proportional to the capacities *C* and

C' : as these charges are proportional to the currents which produce them in the same time, and as these latter are inversely as the resistances, we have the proportion

$$\frac{C}{C'} = \frac{a'}{a}.$$

365. Coefficient of Mutual Induction.—The two coils whose coefficient of mutual induction, M, is to be measured, are connected up in two separate circuits. One coil, with a battery, a key for making or breaking contact, and a tangent galvanometer, forms a primary circuit, and the other is connected with a ballistic galvanometer to form a secondary circuit. The primary circuit is closed, and the strength of the primary current, C, is read off on the tangent galvanometer as soon as the deflection has become steady. The primary circuit is then broken, and the throw, θ, of the ballistic galvanometer is observed, the reading being corrected for damping if needful. If r is the total resistance of the secondary circuit (including the galvanometer), the quantity of the secondary current is (§ 305)

$$m = MC/r;$$

but it is also given by the ballistic galvanometer (§ 354) as

$$m = \frac{H\tau}{G\pi} \sin \tfrac{1}{2}\theta.$$

Equating these two values, we get the coefficient of mutual induction—

$$M = \frac{rH\tau}{CG\pi} \sin \tfrac{1}{2}\theta.$$

The ratio H/G may be found as described in § 353 ; or, if C' is the strength of the steady current which gives the small permanent deflection θ' when passed through the ballistic galvanometer, we have

$$H/G = C'/\tan\theta',$$

and therefore

$$M = r\frac{C'\tau}{C\pi} \cdot \frac{\sin \frac{1}{2}\theta}{\tan\theta'}.$$

Lastly, if the same battery is used for giving the primary current C and the current C', and if R and R' are the total resistances respectively in circuit in the two cases, we may dispense with a measurement of the primary current and write

$$M = r\frac{R\tau}{R'\pi} \cdot \frac{\sin \frac{1}{2}\theta}{\tan\theta'}.$$

CHAPTER XXX.

ELECTRICAL UNITS.

366. Physical Magnitudes.—Every statement as to the numerical value of a physical quantity consists of *two* factors, one of which is a *number*, and the other the *unit* to which the number refers. Thus, in the statement that the strength of a current is 20 amperes, the factors are 20 and *one ampere:* the former answers to the question *how many?* the latter, to the question how many *of what?* Often the answer to the second question may be taken as understood from the nature of the matter in hand, in which case it does not need to be expressed. Thus, if it is understood that, in speaking of the strength of a current, we always refer to the ampere as the unit, the number 20 would be a sufficient statement by itself; but, in the absence of such an understanding, even if we knew that the statement before us related to the strength of a current, the mere number would leave it uncertain whether the current referred to was 20 amperes, or 20 C.G.S. units, or 20 units of any other kind.

The measurement of a physical quantity consists in ascertaining, by an appropriate experimental process, the relation which it bears to some other quantity of the same kind chosen as a standard of comparison or unit of value for the kind of magnitude in question. This unit constitutes the physical factor, expressed or understood, in the statement of the result of the measurement.

So long as we have to deal merely with magnitudes of one kind, the only points of fundamental importance to be considered in the choice of a unit are that it should be permanent, well-defined, and capable of being accurately compared with the quantities to be expressed by means of it. Thus, the yard, the metre, the mile, or the wave-length in a vacuum of the yellow light of incandescent sodium-vapour might, in principle, serve equally well as the unit of length: the choice of one rather than another is a matter of convenience.

But the main problem of physics does not consist in the comparison of quantities of the same kind, but rather in the discovery and accurate investigation of relations between quantities of different kinds.

Hence it is important that, in choosing our units, we should take account not only of the conditions just mentioned, but should consider also the greater or less degree of clearness and simplicity with which they enable the mutual relations between different quantities to be expressed. The smaller the number of independent units any statement of the mutual relations of quantities contains, the simpler and clearer it is. Hence the last-mentioned condition is equivalent to saying that the best system of units is that which makes it possible to express the results of physical investigations by means of the smallest number of independent units.

An example will make these matters clearer. The relation between length and volume is such that the product of the three linear dimensions at right angles to each other of any rectangular space is equal to its volume. But whether the numerical value of a volume is equal to the product of the numerical values of the three linear dimensions depends on the units of volume and of length respectively. Let V denote any concrete volume, and v the unit of volume, then the numerical value is $\frac{V}{v}$. Similarly, if a, b, and c are the corresponding linear dimensions, and l is the unit of length, $\frac{a}{l}$, $\frac{b}{l}$, $\frac{c}{l}$ are their numerical values, and $\frac{abc}{l^3}$ is the numerical value of their product.

We cannot, in general, equate this with $\frac{V}{v}$, but require to employ a factor of proportionality, say ρ, depending on the values of v and l. We may then write

$$\frac{V}{v} = \rho \cdot \frac{abc}{l^3}.$$

Since V and abc are different expressions for the same concrete volume, they are necessarily equal, and therefore

$$\rho = \frac{l^3}{v}.$$

For example, if v is *one gallon* and l is *one foot*, $\rho = 6.232$, and the statement contained in the first equation may be expressed in words as follows: *the number of gallons in a given volume is equal to* 6.232 *times the number obtained by multiplying together its length, breadth, and depth, each expressed in feet.* Here we adopt two independent units, for volume and length respectively. If, however, while keeping the gallon as unit of volume, we take as unit of length the edge of a cube measuring one gallon, we get $l = v^{\frac{1}{3}}$; or, again, keeping the foot as unit of length, if we take the cubic foot as unit of volume, we have $v = l^3$. In either case, $\rho = 1$, and our first equation takes the much simpler form

$$V = abc,$$

which is now numerically true.

It will be seen that the essential condition of this simplification is the avoidance of an unnecessary number of independent units.

367. Fundamental and Derived Units.—We have seen that a single independent or *fundamental* unit suffices for the measurement of both length and volume. We may take a certain length, l, as a fundamental unit, and derive from this l^3 as unit of volume. Or we may adopt a certain volume, v, as a fundamental unit, and derive from it $v^{\frac{1}{3}}$ as unit of length. Again, we may take a definite superficial area, s, as a fundamental unit, and derive from it $s^{\frac{1}{2}}$ as unit of length and $s^{\frac{3}{2}}$ as unit of volume.

Other units may be derived by referring simultaneously to two or more fundamental units. Thus if t denote the unit of time, the unit of velocity derived from this and the unit of length is the ratio l/t of the unit length to the unit time. Arithmetically, we can give no meaning to the ratio of two heterogeneous quantities like length and time, but this example shows that such a ratio may be capable of being interpreted physically as a quantity of a third kind. Reciprocally, we might adopt a definite velocity, c, as unit of velocity and derive from this the unit of length, as $l = ct$. Here the product of two heterogeneous quantities is interpretable as a quantity of a third kind.

We may extend similar considerations to the case of electrical quantities. For instance, it results from the discussion of electrical capacity in Chapter VI. that the capacity of an electric field may always be represented as the product of two factors, one of which is purely geometrical, while the other, the dielectric coefficient of the medium occupying the field, is purely physical. The geometrical part of the expression, in its turn, consists of the first power of the unit of length multiplied by a numerical factor.

Consequently it is not needful to adopt an independent unit of electrostatic capacity; the unit may be derived by defining it as the capacity of a field occupied by the standard dielectric substance, air or a vacuum, and such that the geometrical factor applying to it is the unit of length.

368. Choice of Fundamental Units.—It will be seen from what has been said, that if we adopt as a principle that quantities are to be represented by means of as small a number of independent units as possible, the question still remains,—what are these units to be? Are we, for example, to take the unit of velocity as fundamental and derive from it the unit of length? or are we to treat the unit of length as fundamental and found on it the unit of velocity?

In order to answer such questions, we have to consider not only the qualities that have already been pointed out as essential to a satisfactory physical unit of any kind, but also the natural or logical sequence of ideas. This no doubt is, in part at least, relative to the historic order in which physical knowledge has been developed, and to the point of view from which we habitually approach the consideration of the various scientific conceptions. But taking, as we inevitably must do, these conceptions as we find them, the notion of length, or of the distance between two points, is simpler than that of velocity, and therefore logically precedes it.

The examination of the other quantitative notions of physics, and the discussion of them in the same sort of way as we have here discussed velocity and electrostatic capacity, has led physicists to the adoption of *three* fundamental units, namely, a unit of *length*, a unit of *mass*, or of quantity of matter, and a unit of *time*, in terms of which and of three other physical ideas—change of *temperature*, *dielectric coefficient*, and *magnetic permeability*—it is found that all the physical magnitudes that have been hitherto recognised can be expressed.

369. Absolute Units.—Units derived from the above-mentioned fundamental units are spoken of as *absolute* units, in contradistinction to the special units that might be adopted for each different kind of physical quantity; and magnitudes stated in terms of them are said to be expressed in absolute measure.

For example, we might adopt as unit of velocity the velocity acquired by a heavy body when it falls in a vacuum for a specified length of time at a specified place, and we might state the velocity of a railway train or of a rifle bullet in terms of this standard.

But such a statement would not tell us how far the train or the bullet would move in a given time ; it would only tell us the ratio between this distance and that corresponding to the velocity of the falling body. If, however, a velocity is stated in absolute units, that is, not merely by comparison with some other velocity, but in terms of the fundamental units of length and time, we can estimate completely its results.

370. Dimensions of Derived Units.—The formulæ which express the various derived units in terms of the fundamental units express at the same time the essential relations between the quantities represented, and show, in many cases, that magnitudes, between which no connection is obvious superficially, are essentially homogeneous. The exponent of the symbol of a fundamental unit in the formula of a derived unit is said to give the *dimension* of the latter with respect to the former. Thus the unit of volume, $v = l^3$, is of the *third* dimension with respect to the unit of length.

We proceed to give the dimensions of the most important derived units, that we have to deal with in studying electricity and magnetism ; but as the experimental measurement of electrical and magnetic magnitudes depends almost exclusively on the observation of mechanical effects, we give in the first place the dimensions of the chief quantities occurring in mechanics.

371. Mechanical Units.—*Velocity.*—We have already seen that the derived unit of velocity is the ratio of the unit of length to the unit of time. This is expressed by the equation

$$v = LT^{-1}.$$

Acceleration (a).—An acceleration is the ratio of an increment of velocity to the time in which it occurs, or the time-rate of increase of velocity. Hence

$$a = vT^{-1} = LT^{-2}.$$

Force (f).—A force is measured by the product of a mass acted on by the force into the resulting acceleration of this mass, or

$$f = Ma = LMT^{-2}.$$

The consideration of force as time-rate of change of momentum leads to the same expression.

Work or *Energy* (w).—The work done by a force is the product of the force into the displacement of its point of application projected on the line of action of the force, or fL. The energy of a moving body is the product of a mass into the square of a velocity, or Mv^2. Writing for f and v their values in terms of the fundamental units, both these expressions give

$$w = L^2MT^{-2}.$$

Turning moment or *Torque* (γ).—This is given by the product of a force into a length measured at right angles to the direction of the force. If we do not distinguish between lengths measured in different directions, we have

$$\gamma = L^2MT^{-2},$$

which represents a moment as apparently homogeneous with work. We might write more appropriately

$$\gamma = L_xL_yMT^{-2},$$

the suffixes denoting that the two L's are to be measured at right angles to each other.

Power (p).—Power is used in mechanics for the *time-rate of doing work;* hence

$$p = fLT^{-1} = L^2MT^{-3}.$$

372. Electrical and Magnetic Units.—The measurement of electric and magnetic magnitudes depends on the observation of mechanical effects produced under accurately defined conditions. These effects are—(*a.*) *electrostatic*, involving, among other conditions, the *dielectric coefficient* of the medium in which they occur; (*b.*) *magnetic*, whether produced by, or exerted upon, ordinary magnets or conductors carrying currents. These involve the *magnetic permeability* of the medium.

We proceed to give dimensional formulæ for the most important magnitudes of each kind, that is, omitting all numerical factors, to give formulæ showing how each magnitude is related to the fundamental physical units. But, in the first place, it will be convenient to point out the relation between the dielectric coefficient and the magnetic permeability of a medium. This will enable us to pass, in the case of a given magnitude, from its value deduced from electrostatic effects to the value deducible from electromagnetic phenomena, or *vice versâ*.

With this object, we will examine the expressions obtained from the electrostatic and electromagnetic points of view respectively, for an electric charge or quantity of electricity.

According to Coulomb's law, two equal electric quantities, Q, concentrated at points whose distance apart is L in a medium of dielectric coefficient, K, exert upon each other a mechanical force, f, given by the equation

$$f = \frac{Q^2}{L^2K}.$$

Putting in the value of f from § 371 and reducing,

$$Q = L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}K^{\frac{1}{2}}.$$

This formula gives the dimensions of Q on the electrostatic system.

To get a corresponding expression founded on electromagnetic action, we may start from the relation between electric current and electric quantity, namely,

$$Q = CT.$$

We may proceed farther by considering that, if a current of strength, C, flows round each of two plane areas, $A (= L^2)$, placed at right angles to each other at a distance great as compared with their linear dimensions, in a medium of permeability μ, each current exerts a turning moment on the other, the value of which is proportional to μ and inversely proportional to the cube of the distance. Hence the dimensional equation

$$\gamma = \frac{CL^2 . CL^2}{L^3}\mu,$$

or

$$C = L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}.$$

Consequently,

$$Q = L^{\frac{1}{2}}M^{\frac{1}{2}}\mu^{-\frac{1}{2}}.$$

Equating this with the electrostatic expression already found, we get

$$K^{\frac{1}{2}}\mu^{\frac{1}{2}} = L^{-1}T,$$

or

$$K = L^{-2}T^2\mu^{-1}; \quad \mu = L^{-2}T^2K^{-1}.$$

By similarly equating the electrostatic and electromagnetic expressions for any other electric magnitude we should arrive at the same result. This may be stated in words as follows: the product, $K\mu$, of dielectric coefficient into magnetic permeability for any medium is homogeneous with the reciprocal of the square of a velocity characteristic of the medium.

373. Electrostatic Units.—Starting from the expression for a quantity of electricity deduced from electrostatic action, we proceed to deduce comparable expressions for the other most important electrical magnitudes.

Electric charge:—

$$Q = L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}K^{\frac{1}{2}}.$$

Surface density, otherwise *Electric displacement* or *Electric induction* (Maxwell).—This is the ratio Q/L^2; hence

$$\sigma = L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}K^{\frac{1}{2}}.$$

Electric force at a point in an electric field.—The dimensions are obtained by dividing surface-density by K; or, conversely, displacement = electric force × dielectric coefficient:

$$f_e = L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}K^{-\frac{1}{2}}.$$

Electric potential.—The product of electric force into length is difference of potentials:

$$V = L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}K^{-\frac{1}{2}}.$$

Capacity.—The capacity of an electric field is the ratio of its charge to the difference of potentials of its boundaries; or, again, it is the ratio of electrostatic energy to the square of difference of potentials:

$$S = LK.$$

Current is time-rate of electric displacement through an area:

$$\begin{aligned} C &= \sigma L^2 T^{-1} = QT^{-1} \\ &= L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}K^{\frac{1}{2}}. \end{aligned}$$

Resistance is ratio of difference of potentials to current:

$$R = L^{-1}TK^{-1}.$$

Magnetic moment.—A current multiplied by an area round which it flows, and by μ, the permeability of the medium, gives the moment of the equivalent magnetic shell:

$$m = CL^2\mu = L^{\frac{3}{2}}M^{\frac{1}{2}}K^{-\frac{1}{2}}.$$

Intensity of magnetisation—Magnetic induction.—The dimensions are the same, namely, magnetic moment divided by L^3:

$$I = L^{-\frac{3}{2}}M^{\frac{1}{2}}K^{-\frac{1}{2}}.$$

Magnetic force, or *Strength of magnetic field.*—The dimensions are magnetic induction divided by permeability:

$$H = L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-2}K^{\frac{1}{2}}.$$

Inductance (*Mutual induction*, *Self-induction*).—This is the ratio of energy to square of current:

$$\frac{L^2MT^{-2}}{L^3MT^{-4}K} = L^{-1}T^2K^{-1}.$$

374. Electromagnetic Units.—We have already (§ 372) obtained the dimensions of *quantity of electricity* and *electric current*:

$$Q = L^{\frac{1}{2}}M^{\frac{1}{2}}\mu^{-\frac{1}{2}}; \quad C = L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}.$$

From these we may obtain the dimensions of the remaining magnitudes. For convenience of comparison we will take the same order as before.

Surface-density— $\sigma = L^{-\frac{3}{2}}M^{\frac{1}{2}}\mu^{-\frac{1}{2}}$.

Electric force— $\frac{\sigma}{K} = L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-2}\mu^{\frac{1}{2}}$.

Electromotive force— $L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}\mu^{\frac{1}{2}}$.

Capacity— $L^{-1}T^2\mu^{-1}$.

Resistance— $LT^{-1}\mu$.

Magnetic moment— $L^{\frac{5}{2}}M^{\frac{1}{2}}T^{-1}\mu^{\frac{1}{2}}$.

Magnetic induction— $L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}\mu^{\frac{1}{2}}$.

Magnetic force— $L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}$.

Inductance— $L\mu$.

375. Remarks on the Two Systems of Units.—Frequently in statements as to the dimensions of electric and magnetic quantities, the factors K and μ are not expressed, that is, it is tacitly assumed that each of them has the value unity, or is of *no* dimensions with respect to the fundamental physical units. If we put $K = 1$ in the above formulæ, we get the expressions usually given for the dimensions of the various units on the "electrostatic system;" while, by making $\mu = 1$, we get the usual corresponding expressions on the "electromagnetic system." The result is that the same magnitude appears as having different dimensions on the two systems: for example, an electric charge appears as having the dimensions $L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}$ on the electrostatic system, and the dimensions $L^{\frac{1}{2}}M^{\frac{1}{2}}$ on the electromagnetic system; resistance, as having the dimensions $L^{-1}T$ on the electrostatic system, and LT^{-1} on the electromagnetic system.

The assumption underlying the formulæ given above, in which K and μ appear, is that each magnitude is of definite invariable dimensions, but that these dimensions cannot be completely and explicitly assigned, inasmuch as all that we can state as to the dimensions of K and μ is that their product is of the same dimensions as the inverse square of a velocity, or, more concisely, that

$$K\mu v^2 = \text{a pure number.}$$

376. Choice of Fundamental Units—C.G.S. System.—The quantities to be adopted as the fundamental units of *length*, *mass*, and *time*, may evidently be chosen arbitrarily according to considerations of convenience. The units now almost universally adopted for scientific purposes, not only in electricity, but in all departments of physics, are those agreed upon by a Congress of Electricians that met in Paris in 1881, namely—

As unit of length—one *Centimetre* (very nearly one thousand-millionth part of a quadrant of the earth measured from the equator to the pole);

As unit of mass—one *Gramme* (very nearly the mass of one cubic centimetre of water at the temperature of its maximum density);

As unit of time—one *Second* (one eighty-six thousand four hundredth part of a mean solar day).

It was agreed that units founded upon these quantities should be called Centimetre-Gramme-Second absolute units, or, more shortly, C.G.S. units.

Such units are sometimes much smaller, in other cases much larger, than the quantities which have to be expressed by means of them. To avoid the consequent necessity of using very large numerical multipliers or divisors, multiplication by one million ($= 10^6$) is expressed by the prefix *mega-*, and division by one million by the prefix *micro-*.

It has been found convenient also to give special names to some of the most important derived units of the C.G.S. system. Thus the unit of force, namely, the force which, when acting on a gramme, causes its velocity to change at the rate of one centimetre per second in a second, is called a *dyne*. The weight of a gramme is equal to about 981 such units. The weight of one milligramme is therefore almost equal to one dyne, and the weight of one kilogramme almost equal to one *mega-dyne*. The intensity of magnetic force at a distance of one metre from a small magnet of unit magnetic moment would be from one to two *micro-dynes*, according to the direction.

The C.G.S. unit of work or energy is the work done or energy expended when a force of one dyne acts through a distance of one centimetre, and is called an *erg*. One *kilogramme-metre* is about 981 × 10^5 ergs, or, roughly, 100 *megergs*.

The unit of heat, or the quantity of heat needed to raise the temperature of one gramme of water by one degree centigrade at a specified part of the scale, may be called a *water-gramme-degree*, or simply a gramme-degree. It is equivalent to about 4.19 × 10^7 ergs, or nearly 42 megergs.

377. Practical Units.—The C.G.S. units of electrical magnitudes are in many cases so much smaller, or so much larger, than the quantities usually dealt with in practice, that subsidiary units, derived from them by multiplying or dividing by powers of 10, are employed for practical purposes. For example, on the electromagnetic system, the C.G.S. unit of resistance is equal to the resistance of only about one twenty-thousandth of a millimetre of copper wire of one millimetre diameter; the C.G.S. unit of electromotive force is about one hundred-millionth of that of a Daniell's cell; on the other hand, the C.G.S. unit of capacity is more than two thousand times the capacity of a spherical electric field filled with air, with boundaries one centimetre apart, and with a radius equal to that of the earth.

Consequently, units of a more convenient size for practical purposes were agreed upon by the Electrical Congress, already mentioned as having met in Paris in 1881, in the case of five of the magnitudes that are most frequently referred to, namely, *resistance*, *electromotive force*, *strength of current*, *quantity of electricity*, and *capacity*.

The practical unit of resistance is called the *ohm*, and is defined as 10^9 C.G.S. electromagnetic units. It is, almost as nearly as can be measured, equal to the resistance at 0° C. of a column of mercury of one square millimetre section, and 106.3 centimetres long.

The practical unit of electromotive force is called the *volt*, and is defined as 10^8 C.G.S. electromagnetic units. It is about 6 per cent. less than the electromotive force of a Daniell's cell. Almost as nearly as can be measured, the electromotive force of a Latimer Clark's cell at 15° is 1.454 volt.

The practical unit of current-strength is called the *ampere*. It is the strength of the current which an electromotive force of one volt can maintain in a circuit of resistance one ohm: its value in C.G.S. electromagnetic units is therefore $10^8/10^9 = 1/10$. Almost as nearly as can be measured, a current of one ampere, passed through an aqueous solution of nitrate of silver, deposits silver at the rate of 0.001118 gramme per second, or 4.025 grammes per hour.

The practical unit of quantity is called the *coulomb*. It is the quantity which, in one second, passes every complete transverse section of a conductor traversed by a steady current of one ampere: hence it may also be called an *ampere-second*, and is equal to 0.1 of a C.G.S. electromagnetic unit.

The practical unit of capacity is called the *farad*. It is the capacity of an electric field which has a charge of one coulomb when the difference of potentials between its boundaries is one volt. In C.G.S. electromagnetic measure a farad is therefore equal to the ratio of $10^{-1} : 10^8$, or $10^{-9} : 1$. Even the farad, though only one thousand-millionth of a C.G.S. unit of capacity, is too large for ordinary purposes, and consequently capacities are very commonly stated in terms of the millionth part of a farad, called a *micro-farad*, equal to 10^{-15} C.G.S. electromagnetic unit of capacity.

In addition to the above, a practical unit of inductance has been adopted and is called the *henry*. The self-inductance of a circuit is one henry if an opposing electromotive force of one volt results from a variation of the strength of current in the circuit at the rate of one ampere per second. The mutual inductance of two circuits is one henry if an electromotive force of one volt in one of them results from a variation of the strength of current in the other at the rate of one ampere per second. The value of the henry in C.G.S. measure is consequently 10^9.

The electric energy expended when a coulomb of electricity falls in potential by one volt is called a *joule*. Its value in C.G.S. units is $10^{-1} \times 10^8 = 10^7$ ergs.

The rate at which energy is expended or power exerted when a current of one ampere flows from a given point of a circuit to a point where the potential is lower by one volt is called a ***watt;*** it represents 10^7 ergs per second.

A gramme-degree of heat, being the equivalent of 4.2×10^7 ergs nearly, is equal to 4.2 joules; conversely, 1 joule is equal to 0.24 gramme-degree. One horse-power is equal to 746 watts; one kilo-watt is 1.34 horse-power.

378. Recapitulation.—We may recapitulate here in tabular form the results of the foregoing discussion as to the dimensions of electrical and magnetic quantities.

Table of Dimensions.

	Dimensions in Terms of—	
	L, M, T, and K.	L, M, T, and μ.
Dielectric coefficient	K	$L^{-2}T^{2}\mu^{-1}$
Magnetic permeability	$L^{-2}T^{2}K^{-1}$	μ
Quantity of electricity	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}K^{\frac{1}{2}}$	$L^{\frac{1}{2}}M^{\frac{1}{2}}\mu^{-\frac{1}{2}}$
Quantity of magnetism	$L^{\frac{1}{2}}M^{\frac{1}{2}}K^{-\frac{1}{2}}$	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}\mu^{\frac{1}{2}}$
Electric potential Electromotive force	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}K^{-\frac{1}{2}}$	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}\mu^{\frac{1}{2}}$
Magnetic potential	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}K^{\frac{1}{2}}$	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}$
Electric force (strength of electric field	$L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}K^{-\frac{1}{2}}$	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-2}\mu^{\frac{1}{2}}$
Magnetic force (strength of magnetic field	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-2}K^{\frac{1}{2}}$	$L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}$
Electric displacement (surface density	$L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}K^{\frac{1}{2}}$	$L^{-\frac{3}{2}}M^{\frac{1}{2}}\mu^{-\frac{1}{2}}$
Magnetic induction	$L^{-\frac{3}{2}}M^{\frac{1}{2}}K^{-\frac{1}{2}}$	$L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}\mu^{\frac{1}{2}}$
Electric current	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}K^{\frac{1}{2}}$	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}$
Conductivity	$LT^{-1}K$	$L^{-1}T\mu^{-1}$
Resistance	$L^{-1}TK^{-1}$	$LT^{-1}\mu$
Capacity	LK	$L^{-1}T^{2}\mu^{-1}$
Inductance	$L^{-1}T^{2}K^{-1}$	$L\mu$
Magnetic moment	$L^{\frac{3}{2}}M^{\frac{1}{2}}K^{-\frac{1}{2}}$	$L\ M^{\frac{1}{2}}T^{-1}\mu^{\frac{1}{2}}$
Strength of magnetic shell	$L^{-\frac{1}{2}}M^{\frac{1}{2}}K^{-\frac{1}{2}}$	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}\mu^{\frac{1}{2}}$

It may be noted that the pairs of magnitudes that are bracketed together in this table are reciprocally analogous, the dimensions of one in terms of K being the same as those of the other in terms of μ. In every case, the ratio of the dimensions in terms of K to the dimensions in terms of μ is a power of $LT^{-1}K^{\frac{1}{2}}\mu^{\frac{1}{2}}$.

379. Comparison of Practical and C.G.S. Units.—It may be useful also to collect in tabular form the values of the practical units giving the corresponding electrostatic and electromagnetic values side by side.

	Name of Practical Unit.	Value in C.G.S. Measure.	
		Electro-magnetic.	Electro-static.
Resistance	Ohm	10^9	$10^9 \div v^2$
Electromotive force . .	Volt	10^8	$10^8 \div v$
Strength of current . .	Ampere	10^{-1}	$10^{-1} v$
Quantity of electricity . .	Coulomb	10^{-1}	$10^{-1} v$
Capacity	Farad	10^{-9}	$10^{-9} v^2$
Inductance	Henry	10^9	$10^9 \div v^2$
Energy	Joule	10^7	
Power	Watt	10^7	

The symbol v in this table stands for the velocity $K^{-\frac{1}{2}}\mu^{-\frac{1}{2}}$ with which electric or magnetic disturbances are propagated in the medium to which K and μ refer. This velocity, in the case of air, is very nearly $3 \times 10^{10} \frac{\text{cms.}}{\text{sec.}}$.

As already stated (§ 375), the electromagnetic or electrostatic dimensions of a given unit are obtained from the complete dimensional formulæ by giving the value unity to the symbol μ or K, as the case may be. When this is done, the electromagnetic dimensions of resistance are the same as those of velocity; those of strength of current, the same as those of the square root of a force; the dimension of inductance becomes a simple length, &c. If now, instead of adopting units of length mass, and time, as fundamental, and deriving the units of electrical quantities from these, we had assumed, say, the ohm as unit of velocity, one (ampere)^2 as unit of force, and the ratio of the henry to the ohm, that is to say, the time-constant (§ 310) of a circuit of inductance one henry and resistance one ohm, as unit of time, we might have obtained the units of length and mass from these as derived units. The reader will easily convince himself that the units which would satisfy the conditions stated are—as unit of length, 10^9 centimetres; as unit of mass, 10^{-11} grammes; as unit of time, 1 second.

CHAPTER XXXI.

DETERMINATION OF THE OHM.

380. Standards.—When once the practical units are defined, the next step is to embody the definitions in actual material standards. This can be practically carried out for the standards of resistance, of electromotive force, and of capacity. In other cases, as, for example, in the case of electric currents, it is not practicable to produce and preserve an actual unit as a standard of reference; we have to depend on measuring instruments whose indications bear a known relation to the theoretically defined units. Similarly, in mechanics we have actual concrete standards of length and mass, but no concrete unit of time or of velocity. If, however, we possess a standard of one kind, and appropriate measuring instruments, the others may be easily deduced from it. As the standard of resistance is that which is most trustworthy, both as to accuracy and permanence, the efforts of many physicists in various countries have been directed to its determination.

The problem to be solved was this: What is the length of a column of mercury at 0°, one square millimetre in cross section, the resistance of which is equal to 10^9 electromagnetic C.G.S. units, that is to say, to one ohm?

The resistance of a conductor in the electromagnetic system is a quantity of the same nature as a velocity (§ 375), that is to say, the quotient of a length by a time. Hence, it follows that the measurement of the absolute value of a resistance amounts ultimately to the measurement of a length and of a time. If other quantities occur in the calculation of the experiments, they can only enter as ratios of magnitudes of the same kind.

The formulas of Ohm and of Joule each furnish a method of measurement. But Joule's formula contains the coefficient J, representing the mechanical equivalent of heat, and in the present state of our knowledge this is not known with sufficient accuracy to form the basis for the determination of the ohm.

Ohm's formula bases the measurement of resistance on a comparison of the absolute value of a current with that of an electromotive force. The galvanometer directly gives the former. In reference to the second, it is to be observed that the only electromotive forces which can be directly measured are those due to electromagnetic induction. The method may be founded on the observation either of an instantaneous or of a continuous effect. We shall explain the principles of four of the simplest and at the same time most trustworthy methods that have been employed hitherto.

381. Rotating Coil—Weber's Method.—A coil of known area, S, movable about a vertical diameter, is connected in circuit with a ballistic galvanometer and set exactly at right angles to the magnetic meridian; it is then rapidly turned through 180°, and the throw of the galvanometer is observed. If R is the total resistance of the circuit, we have on the one hand, for the quantity of electricity which traverses the circuit (§ 302),

$$m = \frac{2SH}{R}\mu;$$

and on the other (§ 354),

$$m = \frac{1}{4}\frac{H}{G}\frac{\tau}{\pi}\theta.$$

From which

$$R = 4SG\mu\frac{\pi}{\tau}\frac{1}{\theta}.$$

It will be observed that the result is independent of the value of the earth's field. Besides the area of the coil and the deflection of the needle, the only quantities to determine are the time of oscillation of the needle and the constant of the galvanometer coil. Of these quantities, G represents a magnetic field due to a unit current; its dimensions are therefore H/C, or (§ 378) L^{-1}; tke dimensions of the area S are L^2, while π and θ are purely numerical. The whole expression is therefore of the dimensions $LT^{-1}\mu$, or if, in accordance with the electromagnetic system, we take $\mu = 1$, the result of the experiment gives R expressed as a velocity.

382. Mutual Induction of Two Coils—Kirchhoff's Method.—Let M be the coefficient of mutual induction of two fixed coils; one of which is joined up with a ballistic galvanometer forming a circuit of total resistance R, while the other is in circuit with a battery which gives a steady current of strength, C. When the current is reversed, we have (§ 305)

$$m = \frac{2MC}{R},$$

and on the other hand,

$$m = \tfrac{1}{2}\frac{H}{G}\frac{\tau}{\pi}\theta.$$

The strength of the current is given by the deflection a of a tangent galvanometer, or

$$C = \frac{H}{G'}\tan a.$$

From these equations we obtain

$$R = 4M\frac{G}{G'}\frac{\tan a}{\theta}\frac{\pi}{\tau}.$$

The result is again independent of H. The ratio $\frac{G}{G'}$ of the constants of the galvanometers, ballistic and tangent, is obtained by passing the same current simultaneously through both (§ 353). The physical magnitudes in terms of which the fixed value is expressed are M and τ, of which the ratio is of the dimensions $LT^{-1}\mu$.

383. Constant Current—Lorenz's Method.—A metal disc rotates uniformly about its geometrical axis: if a is the radius of the disc, T the time of a single revolution, H the component of the field parallel to the axis, the electromotive force acting between centre and circumference is (§ 309)

$$e = \frac{\pi a^2 H}{T} = \frac{AH}{T},$$

A being put for πa^2, the area of the disc.

Instead of using the terrestrial field, let the axis of rotation be placed at right angles to the magnetic meridian, and let it coincide with the axis of a concentric circular coil traversed by a current C. If M is the flux of magnetic induction which passes through the disc from face to face, in consequence of a current of unit strength in the coil, we have

$$CM = AH,$$

and therefore

$$e = \frac{CM}{T}.$$

Suppose, lastly, that the centre and circumference of the disc are respectively connected with two points separated by resistance R on the circuit of the current C, which traverses the coil; then the difference of potentials between these points is CR, and if R is so chosen that

$$CR = e,$$

we can, by making the connections the right way about, cause this difference of potentials to balance the electromotive force due to the rotation of the disc, a condition which is easily recognised by a galvanometer connected in circuit with the disc remaining undeflected. Equating the two values of e, we get the remarkably simple expression

$$R = \frac{M}{T}.$$

384. Lippmann's Method.—This may be regarded as, to a great extent, a modification of Lorenz's method. A coil C (Fig. 327) rotates uniformly about its vertical diameter within the uniform field (§ 256) of a long, horizontal, hollow coil, MN, carrying a constant current, which at the same time circulates in the conductor AB, the resistance of which is to be determined. The revolving coil is connected through the capillary electrometer, E, with the points A and A₁, but the circuit is only closed at the moment when the plane of the coil is parallel to the field, in other words, when the electromotive force due to the rotation passes through its maximum. The speed of rotation and the resistance

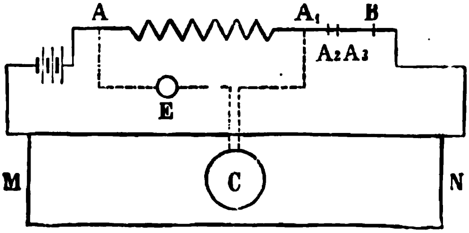

FIG. 327.

R between the points A and A_1 are so adjusted that the induced electromotive force is compensated by the difference of potential between the points A and A_1. As there is no current in the coil, self-induction does not come into account.

If A is the total area of the coil, T the time of a revolution, and n_1 the number of turns per centimetre of the fixed coil, the maximum electromotive force is

$$\frac{2\pi}{T} A\, 4\pi n_1 C = e,$$

and if R is the resistance between the two points A and A_1, we have

$$R = \frac{e}{C} = \frac{8\pi^2 n_1 A}{T}.$$

As in the preceding method, the current cancels out. This method has the advantage, as compared with that of Lorenz, that greater electromotive forces can be used, so that those which may arise at the contact of sliding pieces may be neglected in comparison.

385. Numerical Value of the Ohm.—Many experiments by the above and other methods agree in indicating that the resistance expressed in absolute electromagnetic units, as $10^9 \frac{\text{cm.}}{\text{second}}$, or *one ohm*, is very nearly indeed the same as the resistance at 0° of a column of mercury 1 square millimetre in cross-section and 106.3 centimetres long. Practically, however, the only way of producing and estimating the cross-section of a column of mercury for use as a standard of resistance is to enclose the mercury in a glass tube, and to assume that the cross-section of the column is the same as that of the tube which contains it. Reciprocally, the most accurate method of measuring the cross-section of a glass tube is based upon a determination of the mass of mercury that fills a measured length of it, and upon the determination of the density of mercury. Hence a statement of the value of the ohm, which, like that given above, involves the cross-section of a column of mercury, is not expressed in terms of the quantities which are directly measured. From this point of view it is more direct to specify the length and mass of the mercury column, which has a resistance one ohm, than to specify its length and cross-section. This is the method adopted in the official definition of the ohm for legal and commercial purposes by the Board of Trade, as well as by the Governments of Germany and the United

States. In an Order in Council of 1894, the ohm is stated to be "represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grammes in mass, of a constant cross-sectional area, and of a length of 106.3 centimetres."

From the definition of specific resistance (§ 113) it follows that two conductors of the same material have the same resistance if their masses are proportional to the squares of their lengths: hence the above statement as to the magnitude of the ohm is equivalent to the following—the resistance at 0° of 12.79 grammes of mercury in the form of a column of uniform cross-section 100 centimetres long.

386. Measurement of the Velocity '*v*'.—We have seen (§ 372) that the product $K\mu$ of the absolute inductive capacity into the magnetic permeability of a medium, say air, in which electrostatic and electromagnetic forces can be observed, has the dimensions of the reciprocal of the square of a velocity. Since the properties denoted by K and μ respectively are definite for a given medium under given circumstances, the product $K\mu$ must be expressible in terms of a definite velocity characteristic of the medium. We will denote this velocity by v, and will investigate its numerical value for a vacuum or air, which is from this point of view nearly the same thing.

Suppose a given quantity of electricity, Q, to be expressed, on the one hand, as s electrostatic units, and, on the other hand, as m electromagnetic units. In accordance with § 378, we may write

$$Q = s[L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}K^{\frac{1}{2}}] = m[L^{\frac{1}{2}}M^{\frac{1}{2}}\mu^{-\frac{1}{2}}],$$

or

$$(K\mu)^{-\frac{1}{2}} = \frac{s}{m}[LT^{-1}] = v,$$

where s/m is a purely numerical factor and $[LT^{-1}]$ is the unit of velocity, or, in concrete value, one centimetre per second: v is therefore a velocity of s/m centimetres per second. It is clear that the absolute magnitudes of the quantities expressed by unity on the electrostatic and electromagnetic systems respectively must be in the inverse ratio of the numerical values of the same quantity when expressed in terms of these two units, or

$$\frac{[Q_m]}{[Q_s]} = \frac{s}{m},$$

where $[Q_m]$ denotes the quantity of electricity expressed as unity on the electromagnetic system, and $[Q_s]$ the quantity expressed as unity on the electrostatic system. Consequently, it is often said that the velocity v is equal to the ratio of the electromagnetic to the electrostatic unit of electric quantity.

It will be seen by reference to the table of dimensions of electric magnitudes (§ 378) that the product $K\mu$, and therefore the velocity v, might be expressed in terms of the electrostatic and electromagnetic values of any one magnitude. Thus, if the symbol of any magnitude, with the suffix m or s attached to it, denotes its numerical value in electromagnetic or electrostatic measure respectively, we have

$$\frac{Q_s}{Q_m} = \frac{C_s}{C_m} = \frac{E_m}{E_s} = \sqrt{\frac{R_m}{R_s}} = \sqrt{\frac{S_s}{S_m}} = \sqrt{\frac{L_m}{L_s}}.$$

Practically, the only magnitudes which admit of direct electrostatic measurement are electric charge or quantity, difference of potentials or electromotive force, and capacity. There are consequently three distinct methods that may be employed for the measurement of v.

387. Measurement of a Quantity of Electricity.—The experiment consists in charging a condenser, and measuring by means of Coulomb's balance a known fraction of the total charge, and therefore the charge itself; and, on the other hand, in measuring this charge by the ballistic galvanometer. The quotient of the number furnished by Coulomb's balance, by the number given by the ballistic galvanometer, gives the desired value. This experiment was first made by Weber and Kohlrausch.

388. Measurement of an Electromotive Force.—A steady current is passed through a wire. Let C be the strength of the current and R the resistance between the two ends A and B of the wire, both in C.G.S. electromagnetic units. The electromagnetic difference of potential between the two points is CR. An absolute electrometer (§ 82) connected with the points A and B gives the same difference in electrostatic units. The quotient of the former number by the latter is the value of v.

389. Measurement of a Capacity.—A condenser is formed of two parallel plates separated by a layer of air; one of them is so large that it may be considered as infinite, and the other is surrounded by a guard ring (§ 82). If A is the area of the movable plate, and e the distance between the two plates, the electrostatic capacity is given by the formula (§ 51)

$$S_s = \frac{A}{4\pi e}.$$

The condenser is charged to a difference of potentials $V - V'$, and is then discharged through a ballistic galvanometer (§ 354). We have

$$Q = \frac{1}{2}\frac{H\tau}{G\pi}\theta = S_m(V - V')_m.$$

Let us assume that the difference of potentials $V - V'$ is that between two points A and B of a wire of resistance R carrying a current C; this gives

$$(V - V')_m = RC = R\frac{H}{G'}\tan\alpha,$$

and consequently

$$S_m = \frac{1}{2R}\frac{G'}{G}\frac{\tau}{\pi}\frac{\theta}{\tan\alpha}.$$

The ratio of S_s to S_m is equal to v^2.

390. Numerical Value of v.—The values obtained for v by these various methods are all very nearly 3×10^{10} C.G.S. Accordingly the constant v is a velocity of 3×10^{10} centimetres per second. This velocity is exactly that of light, and thus the reasoning of § 337 is confirmed.

Maxwell made a noteworthy remark in reference to this. He assumes, and the hypothesis has been confirmed by Professor Rowland's experiments, that an infinitely long straight wire having a statical charge, m, per unit length, moving in its own direction with a velocity c, would be equal to a current of strength mc in electrostatic, and of strength $\frac{mc}{v}$ in electromagnetic measure.

Two such wires placed parallel at the distance d, and moving in the direction of their length with the same velocity, would act like two parallel currents in the same direction. The attraction exerted by either on unit length of the other is (§ 260)

$$\frac{2m^2c^2}{v^2}\cdot\frac{1}{d}.$$

At the same time, in consequence of their charges, each will repel unit length of the other with a force which a simple calculation shows is equal to

$$\frac{2m^2}{d}.$$

The attraction is accordingly equal to the repulsion, and the two wires are without action on each other if

$$c = v;$$

hence the velocity v is that with which each wire must move in the direction of its length in order that the two wires may have no action on each other.

CHAPTER XXXII.

CONSTANT-CURRENT MACHINES.

391. Constant-Current Machines.—Generator; Motor.—We shall take as type of these machines Faraday's disc (§§ 267, 309). A metal disc (Fig. 328) driven by any kind of motor is made to rotate about an axis in a uniform field. Two contacts or *brushes*, one pressing on the circumference and the other on the axle, connect the disc with an external circuit. A current is produced, and if the rotation is uniform, the electromotive force of the circuit is perfectly constant. If H is the component of the field parallel to the axis, A the area of the disc, n the number of turns in a second, and R the resistance of the circuit, the current is (§ 309)

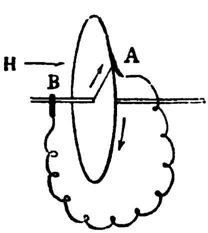

FIG. 328.

$$C = \frac{nAH}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The machine is reversible: that is to say, if driven by a motor it will produce a current; if connected with a battery it will produce work. For the same direction of the current the rotations are in opposite directions in the two cases, but in either of them the product CAH represents the work corresponding to one turn. In the first case, the machine consumes mechanical energy and produces electric energy, in this case it is said to act as a *generator;* in the second case it consumes electric and produces mechanical energy, and acts as a *motor.*

392. Magneto- and Dynamo-Electric Machines.—If the field in which the disc rotates is produced by fixed magnets, or by electromagnets excited by an extraneous source, so as to be independent of the working of the machine, this is said to be a *magneto-electric* machine. For a given speed, whether on open or closed circuit, it acts like a battery of constant electromotive force. Formula (1) shows, moreover, that this electromotive force is proportional to the speed.

But we may suppose the field in which the disc rotates to be maintained by the current which traverses the disc. It is sufficient for this purpose for the disc to be in circuit with the wire of the coil, or of the electromagnet, which produces the field, in other words, with the wire of the *field-magnet.* The intensity H of the field is, in that case, a function of the strength C of the current. The machine is then a *dynamo-electric* machine, or, as it is usually called, a *dynamo.* The whole of the current may traverse the coil of the field-magnet, in which case the machine is said to be *series-wound;* or a portion of the current may be shunted, giving a *shunt-wound* dynamo; or, lastly, we may have a combination of the two, in which case the machine is a *compound dynamo.* Whatever be the arrangement, provided the volume of the copper and the density of the current are the same, the same field is always produced (§ 271).

393. Dynamo without Soft Iron.—Two cases are to be distinguished according as the field-magnet of the dynamo does or does not contain soft iron.

If the field-magnet is a coil without a soft iron core, the intensity of the field is proportional to the strength of the current, and if P is a constant we may put

$$AH = PC.$$

Combining this with equation (1) we obtain the relation

$$n = \frac{R}{P} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

between the constant P, the resistance R, and the speed; the condition here expressed depends only on the construction of the machine, and must be satisfied if it is to act as a generator.

For the mechanical energy taken in per second from the motor is nPC^2, the energy expended in the same time in generating heat is RC^2. Assuming that there is no other work than generation of heat, we have

$$nPC^2 = RC^2,$$

that is to say, equation (2).

The condition which this equation expresses is that in every revolution the machine expends exactly the amount of energy put into it.

For a speed $n_1 < n$, the machine expends more than it receives. In this case it cannot maintain a current, and if a current is started it dies down again. For any velocity $n_2 > n$ the machine takes in, on the contrary, more energy than it expends as heat, and, if the resistance of the circuit remained constant, the strength of a current would increase until the wires fused and the machine was destroyed. Lastly, for the value n, the strength of current will increase until the work done during each turn is equal to that furnished by the motor, and will therefore only depend on the power of the motor. Such a machine could not be utilised in practice.

394. Dynamo with Soft Iron—Characteristic.—If the field-magnet has an iron core, the field is still a function of the current, but a more complicated one.

It increases with the magnetisation, at first almost proportionately to the strength of current, then less rapidly, and finally tends towards a limit when we get near saturation. If $\phi(C)$ is the magnetic flux cut during one turn of the disc when the current-strength is C, the function $\phi(C)$ represents the electromotive force for one turn per second; for a speed of n turns the electromotive force will be $n\phi(C)$. The product $C\phi(C)$ represents the positive or negative work of the electromagnetic forces for one turn of the disc for any speed.

All the properties of the machine are known if the function $\phi(C)$ is known. The curve obtained by taking C as abscissa and $\phi(C)$ as ordinate is called the *characteristic*.

395. Machine used as Motor.—Let us suppose the machine joined up with a battery of constant electromotive force E, and that its axle is acted on by a break, which keeps the work done per turn at the constant value W for any speed. As the electric measure of the work done by the machine for each turn is $C\phi(C)$ for a current of strength C, we must have $C\phi(C) = W$. Let R be the resistance of the circuit. If the battery cannot produce the current C in the resistance R, the machine will not turn; if it can produce a current of greater strength, the current will, nevertheless, still be equal to C; the machine will turn with a greater or less speed so as always to use up the energy furnished to it in excess of C^2R per unit of time. For a velocity of n' turns per second we shall have

$$EC = C^2R + n'C\phi(C),$$

or putting $E' = n'\phi(C)$,

$$C = \frac{E - E'}{R}.$$

The efficiency is

$$u = E'/E,$$

and the work done by the machine in each second

$$E'C = E' . \frac{E - E'}{R}.$$

This work, being expressed by the product of two factors the sum of which is constant, is a maximum when the two factors are equal, that is to say, when

$$2E' = E,$$

in other words, when the current-strength C is half that which the battery would produce in a circuit of resistance R if the machine were at rest. The corresponding efficiency is equal to 0.50. This is a result at which we have previously arrived (§ 132).

396. Machine acting as Generator.—Suppose the machine is worked by a motor, and the current is employed to do work such as the generation of heat, the production of chemical decomposition, or driving a motor. The problem is to determine the strength of current corresponding to a given speed, or conversely.

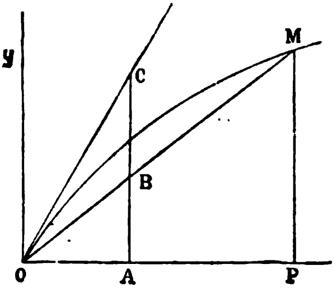

FIG. 329.

Let us suppose a simple circuit of the total resistance R, and the machine rotating with a speed of n turns per second: Ohm's law gives

$$C = \frac{n\phi(C)}{R},$$

from which follows

$$\frac{\phi(C)}{C} = \frac{R}{n}.$$

If OM is the characteristic (Fig. 329), and the abscissa OP represents C, the ordinate MP represents $\phi(C)$, and $\frac{\phi(C)}{C} = \frac{\text{MP}}{\text{OP}}$ is the tangent of the angle MOP. Hence to solve the problem, we need only take OA = 1, draw a perpendicular at A, and on this perpendicular take a length AB $= \frac{R}{n}$, and join the point B with the origin; the point where the right line OB cuts the curve determines the position of M, and therefore OP $= C$, the strength of the current.

It will be seen that the current will be constant if n is made to increase proportionately to R.

For a given speed, the current diminishes as R increases, the point M continually approaching the origin. This goes on until R/n becomes equal to AC/OA, OC being the tangent to the curve at the origin. If the resistance has a still greater value the machine ceases to work. The reason of this has been already given in § 393; it is that the machine expends more energy than it receives.

If the field is proportional to the current, as is the case when the field-magnet has no soft iron, the characteristic becomes simply a straight line passing through the origin; from which it follows that for a given speed there can be equilibrium only when the resistance of the circuit has a definite value, whatever the strength of the current.

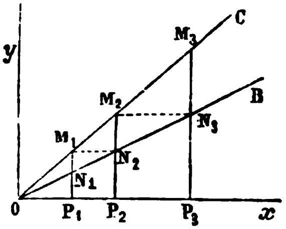

FIG. 330.

For let OC (Fig. 330) be the straight characteristic, and OB the line whose angular coefficient, $\frac{R}{n}$, gives the ratio which must exist between the electromotive force and the current, in order that the latter may be stable. If, at a given moment, the strength of the current were OP_1, the machine would give rise to an electromotive force, M_1P_1, greater than N_1P_1, necessary to maintain the existing current. This electromotive force would therefore produce a stronger current, OP_2, which in turn would develop an electromotive force M_2P_2, and so on; the current would thus continue to increase without limit. The result would be reversed if the line BO lay farther from Ox than CO.

If the electromagnet has any residual magnetism, or if it has a second wire traversed by a current independent of the machine, $\phi(C)$ is no longer zero when $C = 0$; in this case the characteristic does not pass through the origin, but is displaced parallel to itself by a quantity OO_1 (Fig. 331), so that for $C = 0$, we have $\phi(0) =$ OH.

The current is obtained by the same construction as before, but the machine always starts whatever the resistance.

397. Let us suppose that the circuit having resistance R contains a constant electromotive force E', independent of the current, that of a battery or of a voltameter, for instance. The electromotive force E' must be taken positive if it acts in the same direction as the machine, and negative if it is in the opposite direction. Let us consider the latter case; the equation

$$C = \frac{n\phi(C) - E'}{R} \quad . \quad . \quad (3)$$

gives

$$C\frac{R}{n} = \phi(C) - \frac{E'}{n}.$$

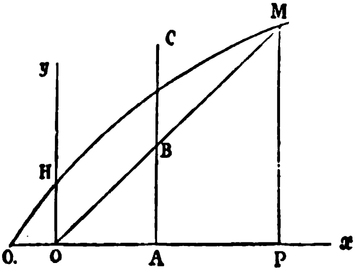

FIG. 331.

It will be seen (Fig. 332) how the construction must be modified: the line whose angular coefficient is $\frac{R}{n}$, instead of passing through the origin must be drawn through a point O′ of the axis of y, at a distance $OO' = \frac{E'}{n}$ upwards or downwards, according as E' is negative or positive.

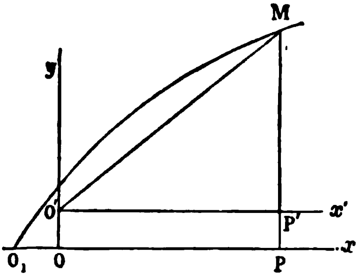

FIG. 332.

398. Let us suppose, lastly, that the machine has to drive a motor. Let $\phi(C)$ and $\phi'(C)$ be the characteristic functions for the two machines, n and n' their speeds. Both are traversed by the same current; and the current-strength, the efficiency, and the work given out by the motor are expressed by the equations

$$C = \frac{n\phi(C) - n'\phi'(C)}{R} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$u = \frac{n'\phi'(C)}{n\phi(C)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$W = n'C\phi'(C) \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The problem may be stated in several ways. Most frequently the prime mover drives the generator at a fixed speed, being able to supply energy to it up to a definite rate. The motor has to do an amount of work per revolution which may or may not be defined. In the former case the current is determined by the condition

$$W = C\phi'(C),$$

and the speed n' is given by equation (4). In the second case n' and C are arbitrary, and they are determined either by the condition of having a definite efficiency or by that of obtaining a maximum rate of work.

It is clear that if the motor is prevented from rotating the current has its maximum strength, but the work done and the efficiency are both nothing. If the motor were able to move without mechanical resistance, its velocity would increase until its electromotive force became equal to that of the generator; the current would then be nothing, and the efficiency unity, but only because both factors of the ratio would vanish (comp. § 132). Between these two extremes there may be all possible efficiencies from 0 to 1, and the conditions for a maximum rate of work may be satisfied. Although the value of W', when C is replaced by its value derived from equation (4), is represented by a product of two factors the sum of which is $n\phi(C)$, the maximum does not occur when the two factors are equal, for this sum is not constant, C being a function of n', and the condition of the maximum cannot be determined so long as $\phi(C)$ is not given.

If the two machines are identical, the formulæ reduce to

$$\left.\begin{aligned} C &= \frac{(n-n')\phi(C)}{R} \\ u &= \frac{n'}{n} \\ W' &= n'C\phi(C) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (7)$$

The first shows that, for a given current, the difference of speeds is proportional to the resistance of the circuit; the second that the efficiency is equal to the ratio of speeds; the third that the work transmitted is proportional to the speed of the motor. It is not correct to say, as is often done, that the maximum is when $n' = \frac{n}{2}$, and corresponds to the efficiency 0.50.

399. Transmission of Energy.—Let us suppose that n being given, we wish to obtain from the motor an efficiency u. Equation (5) gives n', equation (4) gives C, and substituting the values in (6) we obtain the equation

$$W' = n^2u(1 - u)\frac{\phi^2(C)}{R},$$

which shows that we may transmit the same quantity of energy with the same efficiency and the same speed of the generator and motor, provided that we make $\phi^2(C)/R =$ constant; that is to say, provided the characteristic function of the two machines is varied in proportion to the square root of the total resistance of the circuit. Suppose, for instance, that the resistance is multiplied by 25; if we make the electromotive force per turn in each machine five times as great as before, the strength of the current given by formula (4) will be one-fifth: the two machines will do the same work as before for the same number of turns, and the energy used up in overcoming resistance will remain as it was.

In order to make the electromotive force of the two machines five times as great, their construction and dimensions may be modified. But we may also, at each end of the circuit, couple on the same axle five machines connected in series and each of them identical with the original one.

400. The results obtained in this chapter apply to machines with a strictly constant current, like a Faraday's disc, moving in a uniform field, or one that is symmetrical about the axis. Unfortunately, machines of this type only give feeble currents. The very powerful currents obtained from ordinary machines result from much more complicated conditions which we shall investigate in the next chapter. They give continuous currents in this sense that their direction is constant, but the strength is not uniform. Moreover, the electromotive force for a given current is no longer proportional to the speed, and if $\phi(C)$ represents the electromotive force for a speed of one turn per second, the electromotive force for a speed of n turns per second is not exactly represented by $n\phi(C)$.

CHAPTER XXXIII.

CONTINUOUS-CURRENT MACHINES.

401. Gramme Machine—Essential Parts.—The principal part of the machine is a coil formed of a series of loops or sections, B, all coiled in the same direction, and completely encircling a soft iron ring, A (Fig. 333). Each loop is in connection with the next by means of a conducting *commutator-strip*, L, mounted on an insulating cylinder concentric with the ring, and rigidly connected with it, called the *collector*. The two ends of each loop or section of the coil are fixed to two consecutive strips, and each strip is soldered to the termination of the preceding section, and to the beginning of that which follows. The successive sections thus form a continuous coil constituting a closed circuit. If any two diametrically opposite strips were connected with the poles of a battery, the current would divide equally between the two halves of the ring, and the soft iron core would be magnetised so as to have one pole where the current enters, and the other where it leaves. Each section of the coil may be made up of one or of several turns of wire.

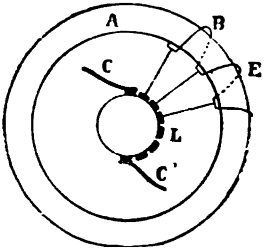

FIG. 333.

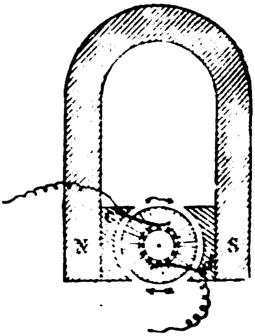

FIG. 334.

The ring is placed between the poles of a horse-shoe magnet or of an electromagnet, and is made to rotate uniformly (Fig. 334). Two brushes, *c d*, rest on the collector at the ends of a diameter, which is nearly at right angles to the line of the poles of the magnet, and collect the currents in the ring; these are the poles of the machine.

Without the soft iron ring the field between the branches of the magnet, or rather between the *pole-pieces* of soft iron which are attached to it, would be nearly uniform; the ring, placed in this field, acquires by induction poles at the two ends of a diameter which are of the opposite kind to those of the adjacent pole-pieces; it thus absorbs the lines of force proceeding from the pole-pieces, and conducts them through its mass, so that only a very small number of lines pass through the empty space inside the ring (Fig. 335). It follows from this that the coils moving in the field cut the magnetic flux only by the part of each which is on the outside of the ring. As far as the explanation of the action of the machine is concerned, it is immaterial whether the iron core is supposed stationary, and the whole set of coils supposed to slide round it; or whether the core itself turns carrying the coils with it; in the latter case the poles are displaced within the ring, but they remain fixed in space. In this case, which is that met with in practice, it is important, in order to prevent currents from being induced in the mass of the ring, or so-called Foucault currents (§ 318), that it should be formed of a bundle of iron wires, or thin plates placed close together; the wires or plates are insulated from each other, and their planes are perpendicular to

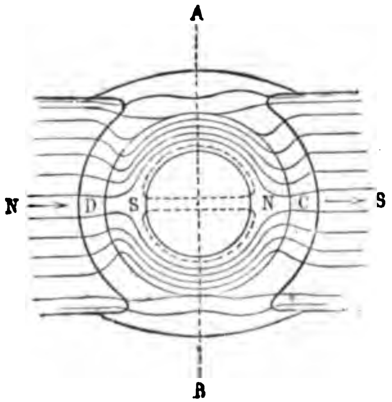

FIG. 335.

the axis of the ring, in other words, to the plane of the windings, and therefore to the direction in which the electro-magnetic induction acts. The ring is usually in the form of a hollow cylinder.

402. Action of the Machine.—Let us confine our attention in the first place to a single section of the coil while the machine makes one complete revolution. In order to account for the effects of induction produced in this section, we may consider either the variations of the magnetic flux through the area enclosed by it, or the flux cut at each instant by the various elements of the contour (§ 293). According to the first point of view, the plane of the coil at A and at B (Fig. 335) is perpendicular to the magnetic flux, but in passing by ACB from the first position to the second the coil turns through 180°. If Q_a be the total flux through the iron ring, counting both halves, the change of flux in the coil is simply equal to Q_a. An equal variation, but of opposite sign, takes place in the coil in the second part of the revolution as it passes from B to A through D.

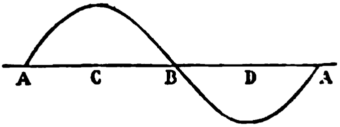

FIG. 336.

According to the second mode of looking at the matter, it is easy to see that no flux is cut except by that portion of a winding, which covers the ring on the outside. The electromotive force of induction is nothing at A, increases from A to C, then decreases from C to B. At B it passes through zero, changes sign, and, in the second half rotation, from B to A through D, it again passes through the same series of values, but with the signs reversed.

The direction of the electromotive force is thus opposite on the two sides of a vertical diameter perpendicular to the line of the poles. In the conditions for which the figure is drawn, the lines of the field go from left to right, and the motion of the coil is like that of the hands of a watch; the current then goes from front to back of the figure in the outside part of the winding during the motion BDA, and from back to front during the motion ACB. The electromotive force varies during a complete revolution in the way represented by a curve like ABA (Fig. 336), more or less resembling a curve of sines.

Let us now consider the condition at a given moment of the whole system of moving coils: all those on the left of the diameter AB are the seat of electromotive forces in the same direction, acting, for instance, in consequence of the mode of winding, so as to make the current traverse the coil from B to A; and since all the sections communicate with each other, these electromotive forces are added together, and tend to produce a current from B to A. All the sections of the coil on the right of the diameter AB are also the seat of electromotive forces in the same direction; relatively to each section of the coil, these electromotive forces act in the opposite direction to the former, but as the face of each section, which was previously turned upwards, is now turned downwards, the electromotive forces still tend to make the current go from B to A, and the total electromotive force in this half of the coil is manifestly equal to that in the first half.

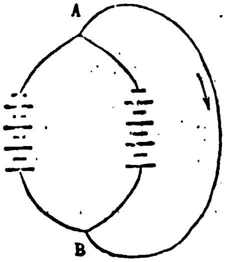

FIG. 337.

Since the two brushes connect the points A and B with the outer circuit, this will be traversed by a continuous current from A towards B, equal to the sum of the currents which traverse the two halves of the ring. The case is the same as that of a battery, the elements of which are arranged in two equal and parallel series (Fig. 337); to make the analogy complete, it must be assumed that the electromotive force of the elements decreases symmetrically in each series from the middle where it is a maximum to the ends where it is zero.

403. Variations of the Current.—The current is continuously in the same direction, but is not quite uniform. One brush is alternately in contact with a single commutator-strip and with two.

In the latter case, the corresponding section of the coil is short-circuited, and its contribution to the resultant electromotive force is withdrawn for the time being. Hence arise periodical variations both of electromotive force and of resistance, and therefore of the strength of the current. The curve which represents the current as a function of the time, instead of being a straight line parallel to the axis of the time, is indented, the length of each indentation corresponding to the time necessary for one commutator-strip to take the place of the preceding one.

The indentations are the smaller and the current the more uniform the greater the number of subdivisions of the commutator.

404. Displacement of the Brushes—Angle of Lead.—If the machine turns uniformly on open circuit, the facts are in exact agreement with the preceding discussion. A difference of potential is established between the two poles formed by the brushes, and this difference is a maximum when the line of the brushes is in the plane of symmetry of the field-magnets. Moreover, for a constant strength of field, the difference of potential is proportional to the speed of rotation of the ring.

If the circuit is closed other phenomena are produced; strong sparks pass between the brushes and the commutator-strips which they leave. These sparks would soon destroy the commutator, and, in order to prevent them, experiment shows that the line of the brushes must be displaced in the *direction of the motion.* A position, A′B′ (Fig. 338), is thus found, which varies with the strength of the current, for which the sparks are totally suppressed, or, at all events, reduced to a minimum. The angle which the line A′B′ of the brushes makes with the line of symmetry AB is called the *angle of lead.*

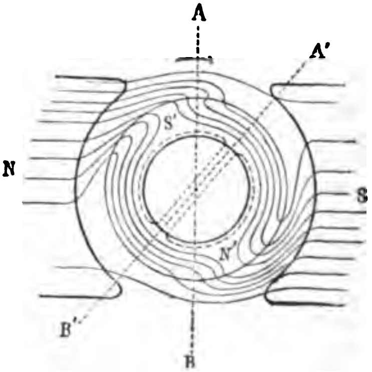

FIG. 338.

The machine is clearly reversible. If it works as a motor, for a current of the same strength and in the same direction, the ring will turn in the opposite direction, but the position of the brushes for which sparking ceases is still, as before, the position A′B′; accordingly, the diameter of contact must be turned through the same angle, but in *the opposite direction to the motion.*

Let us consider the last section of the coil to the left of the line A′B′ at the moment at which the commutator-strip connected with the leading end is in contact with the brush; it is traversed by the current $\frac{1}{2}C$; the next moment it is short-circuited, the brush being in contact simultaneously with the strips connected with both ends, and the current therefore ceases; then, as soon as the leading strip is set free, the section enters the second half of the rotation and is again traversed by the current $\frac{1}{2}C$, but in the opposite direction.

The self-induction of the coil opposes so much resistance to the sudden starting of the current, that the greater part of the flow passes as a spark across the air-space between the commutator-strip and the brush which it has just left. It is the same case as that of the experiment of Fig. 293, except that the quantities of electricity that come into account in the present case are usually much greater.

These discharges and their destructive effects would be avoided if the section, while on short circuit, were submitted to an inductive action capable of gradually developing in it beforehand a current equal to that which it will receive when the commutator-strip quits the brush. This is done by bringing the coil into a part of the field which can produce this induction. It is evident from this that the angle of lead must increase with the strength of the current.

405. Reaction of the Armature.—The necessary displacement of the brushes complicates the action of the machine. The principal effect is to modify the shape of the field and the magnitude of the electromotive force.

The field results from the magnetisation produced both by the current in the coils of the field magnets and by that in the ring. In the case represented in the figure and with the direction of winding assumed, the current in the ring would tend to produce a south pole at A, and a north pole at B, that is to say, a magnetisation perpendicular to that due to the field-magnets. If the line of the brushes remained in the plane of symmetry, the field would still be symmetrical, but if the line of brushes is brought to A′B′ (Fig. 338), there results a distortion of the field readily observable by means of iron-filings, an idea of which is given by the figure.

It is not accurate to say that the effective electromotive force is the sum of the electromotive forces which would be given by each of the magnetisations separately, but it is certain that the second acts in the opposite direction to the first. Moreover it is evident that owing to the displacement of the brushes, the poles developed at A′ and B′ must exert an influence on the pole-pieces, supposing them not independently magnetised, tending to turn the ring in the direction of the actual motion. The effect of this action is to diminish the work against electromagnetic forces, and as the electromotive force is equal to this work for each unit of current, the effective electromotive force is diminished by so much.

When the machine acts as a motor, the same causes have the effect of increasing the electromotive force.

406. Losses of Energy.—Whenever a section of the coil is short-circuited by a brush, it loses the intrinsic energy (§ 296) which it possessed, and which must be restored to it a moment after, without contributing to the action of the machine. The strength of the current being $\frac{1}{2}C$, if l is the coefficient of self-induction of the section, the loss of energy in each turn is $\frac{1}{8}lC^2$. As the effect occurs twice during each revolution for each section, if $2m$ is the number of sections, and n the number of revolutions per second, the loss for each revolution is $2ml\frac{C^2}{4}$, and in a second $2mnl\frac{C^2}{4}$, or $nL\frac{C^2}{4}$, L being the coefficient of self-induction of the whole ring. The loss is the same as would be caused by an increase of the resistance of the ring equal to nL.

Each portion of the ring during one revolution goes through a complete magnetic cycle, which represents a definite amount of work (§ 201). Lastly, the precautions taken to diminish the Foucault currents (§§ 318, 401) do not destroy them completely. These losses are usually greater than that which arises from the resistance of the ring, and it is to them chiefly that the heating of the machine is due.

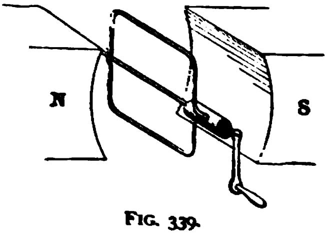

FIG. 339.

407. Siemens' Machine.—This only differs from Gramme's machine in the winding of the armature. A rectangular conducting frame (Fig. 339) turning uniformly between the poles of an electromagnet, may be regarded as an elementary constituent of such a machine. The two sides of the rectangle parallel to the axis are at each moment the seat of equal and opposite electromotive forces, but these evidently act so as to cause a current to circulate in the same direction round the rectangle. The electromotive force is a maximum as the rectangle passes the plane parallel to the field, it is zero and changes sign when the rectangle is perpendicular to this plane.

The ring of the Gramme machine is replaced by a drum of soft iron on the outside of which wire is wound forming a series of rectangular sections analogous to the preceding, and placed in equidistant planes passing through the axis. Each of these sections is formed of several windings, and all are connected by means of a Gramme commutator, the end of one section and the beginning of the next terminating in the same segment of the commutator. Thus, to describe the simplest mode of winding, the wire from the strip No. 1 goes directly to the circumference of the drum, follows the corresponding generating line of the cylinder, crosses the second base diametrically, returns along the opposite generating line, and after being carried round eight times in this way, is brought to the commutator strip No. 2, and so on for the other sections. Only half the whole number of strips are used for windings which completely cover the drum; the winding is continued in the same manner, using the second half of the commutator, and forming a second outer layer of wire, so that sections connected to strips which are diametrically opposite each other are exactly superposed.

Two brushes placed in a plane nearly perpendicular to the field receive a continuous current, but rather less uniform than that of the Gramme machine, the number of sections being generally less.

Thus the essential difference between the two machines is that in the Gramme ring the wire is coiled on the ring both outside and inside, and that on a Siemens drum it is only on the outside; the drum might therefore be a massive cylinder,

The two systems give pretty much the same results. Each has its inconveniences and its advantages. With Siemens' mode of winding the inactive portion of the armature wire is shorter; the magnetisation of the core is less, and therefore the reaction of the armature is also less. But on the base of the drum there are portions of the wire in close contact with each other which are at very different potentials, and this makes the insulation more precarious. Besides, the sections are more difficult to fix upon the drum, and one cannot be uncoiled without uncoiling all, while in the ring the sections are completely separate, and one can be easily replaced without interfering with the rest.

408. Various Types of Machines.—The machines of Gramme and of Siemens have served as the types of a great number of dynamos which only differ from them in the details of their construction.

They are manufactured in every variety of size and strength, according to the power required and the conditions under which this power is to be used. Machines are made which are capable of absorbing hundreds of kilowatts. A current-strength of 1000 amperes may be attained, and electromotive forces of 2000 or 2500 volts are attainable.

The metals employed are always wrought iron and cast iron. The former has far greater permeability, but it is difficult to work, and its use greatly increases the cost of construction. Cast iron is much cheaper and more easily worked, but these advantages are partly counterbalanced by the greater weight necessary to produce the same magnetic conductance. The two kinds of metal are often associated, soft iron being used for the armature and cast iron for the pole-pieces.

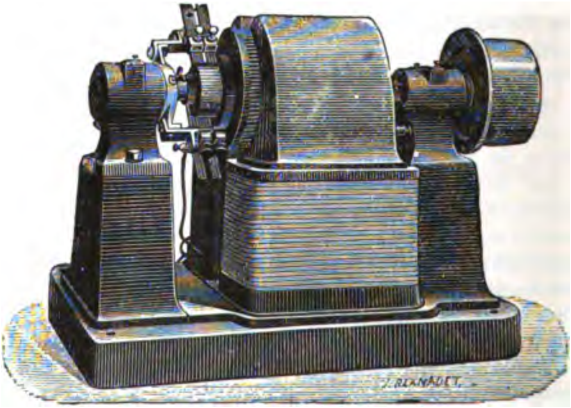

FIG. 340.

The various types differ chiefly as regards the shape of the field-magnets; this is most frequently a horse-shoe electromagnet.

Fig. 340 represents one of the simplest and best forms of Gramme machine. The ring turns between the pole-pieces of a horse-shoe magnet. The cores are of cast iron, and form a single casting with the framework.

In Edison's machines, which are drum machines, the yoke of the electromagnet is uppermost, and the drum turns at the bottom near the base. To prevent any of the lines of force being lost in the base, this is covered with a zinc plate.

Sometimes also the field-magnet is formed of two horse-shoes with similar poles near each other, as in Fig. 341. This has the advantage of greater symmetry. Machines are also constructed with several poles. Fig. 342 represents one with four poles; others have as many as six or eight poles. The number of brushes may be the same as that of the poles; but by taking a number of sections divisible by the number of the poles, and connecting sections of the same order with each other, the number of brushes may always be reduced to two.

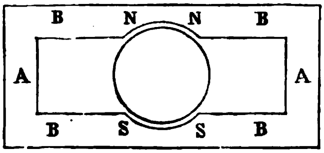

FIG. 341.

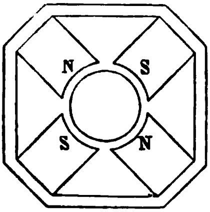

FIG. 342.

The greater the speed of the armature, the greater is the electromotive force for a given length of wire. The linear velocity of the circumference should not exceed 15 to 20 metres in a second; unless a machine is specially constructed with a view to high speed, a greater speed than this would be likely to injure it. One advantage of multipolar machines is to give great electromotive force with a comparatively small speed of rotation.

Good machines have an efficiency of over 90 per cent. They are all reversible, and may be used either as generators or as motors. A well-arranged machine ought to give no sparks at the brushes.

409. Calculation of a Machine.—The properties of a dynamo-machine like that of Gramme, considered as a whole, may be regarded as resulting from the relative motion of two closed circuits, each of which is the seat of a special kind of flux: the magnetic circuit, the seat of the flux of magnetic induction, and the electric circuit, the seat of the electric current, these two fluxes being connected by the equation $4\pi nC = \frac{1}{\mu}\frac{l}{S}Q$ (§ 277).

The magnetic circuit is made up of the field-magnets, the soft iron core of the ring or of the drum, frequently called the ***armature***, and by the space between the pole-pieces and the armature, a space partly filled by air, partly by copper, which is sometimes called the *interferricum*.

The flux of magnetic induction is not the same in all parts of the circuit; part escapes at the ends of the pole-pieces, and a further portion at the entry into the armature. It is the aim in construction to render these losses as small as possible; they may be taken as being a constant fraction of the total flux for a given machine.

This fraction can be readily determined experimentally after the machine is made, by coiling round each part one or two turns of wire connected with a ballistic galvanometer, and reversing the inducing current (§ 307).

Let Q_a be the total flux of induction through the armature, divided equally between the two halves in the case of a ring-armature, Q_b the flux in the interferricum, Q_m that in the field-magnets: representing by p and q factors greater than unity, which may be determined in the manner just mentioned, we may write

$$Q_b = pQ_a$$

$$Q_m = qQ_b = pqQ_a.$$

It is evident that if Q_a is known we know also the electromotive force. At every revolution there is a change of flux through each turn of wire equal to Q_a; if there are $2m$ turns of wire, and if n is the number of revolutions per second, we have

$$E = 2mnQ_a.$$

Let R with a corresponding suffix represent the factor $\frac{1}{\mu}\frac{l}{S}$ for each part of the circuit; in other words, the magnetic reluctance of this part (§ 276). The value of R varies with the magnetic induction; the value of μ, which is required for the calculation, is deduced in each case from the curves of magnetisation. If we apply formula (2) of § 277, remembering that, in the armature, there are as many turns of wire traversed in one direction as in the opposite direction, and that therefore the corresponding term in the expression for the magnetomotive force will vanish, we obtain

$$Q_a(\mathrm{R}_a + p\mathrm{R}_b + pq\mathrm{R}_m) = 4\pi NC,$$

N being the number of turns of wire on the field-magnet, and C the strength of the current which traverses them.

Assuming a series of values for Q_a, the corresponding values of C may be calculated, and a curve may be constructed taking C as abscissa and Q_a as ordinate. In this way a kind of *characteristic* curve is obtained for the machine, which may be called its magnetic characteristic. This is not the same as the characteristic already spoken of (§ 394), and is independent of the speed.

In actual machines the aim is to get the greatest possible value of Q_a in the armature—values, for instance, amounting to as much as 15,000 to 20,000 units per square centimetre (§ 202). In the field-magnets, on the contrary, the aim is not to exceed semi-saturation. The section of the field-magnets relatively to that of the armature is calculated from this consideration, and from the values taken for the coefficients p and q.

CHAPTER XXXIV.

ALTERNATE-CURRENT MACHINES.

410. General Properties.—The type of all machines generating alternating currents is the coil rotating about a diameter in a uniform field (Fig. 343). The current is sinusoidal and has a retardation of phase ϕ relatively to the electromotive force defined by the equation

$$\tan 2\pi\phi = \frac{2\pi L}{RT};$$

the maximum value of the current is

$$C_0 = \frac{E_0 \cos 2\pi\phi}{R},$$

E_0 being the maximum value of the electromotive force (§ 321, equations (5) and (6)).

The work is equal to the product of the effective electromotive force into the effective strength of the current, multiplied by the cosine of the difference of phase (§ 324),

$$P = \frac{E_0 C_0}{2} \cos 2\pi\phi.$$

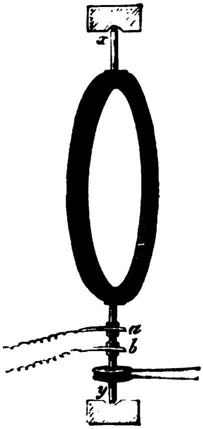

FIG. 343.

It may be added, further, that if, in Fig. 344, OB represents the maximum value E_0 of the electromotive force, and the angle BOA the lag $2\pi\phi$, OA represents the maximum of the effective electromotive force, and OC the maximum electromotive force of self-induction (§ 322).

The effective strength of current is $\frac{\text{OA}}{R\sqrt{2}}$, while the work per second, or power, is $\frac{\text{OA}}{R\sqrt{2}} \times \frac{\text{OB}}{\sqrt{2}} \cos \text{BOA}$, or $\frac{\text{OA} \, . \, \text{OA}}{2R}$. The phase of the electromotive force of self-induction being always 90° behind the current, the cosine of the difference of phase vanishes, and the corresponding work is nothing. Hence self-induction produces no loss of energy; its only effect is to limit the power of the machine.

411. Coupled Machines.—Let us imagine two machines coupled together, two coils, for instance, mounted on the same axis, and making an angle of a with each other, the wires being joined so as to form a single circuit. Adopting the same mode of representation, the two lines OC and OD (Fig. 345) making the angle a, representing the given electromotive forces, the maximum value of the resultant electromotive force is the diagonal OB of the parallelogram, and the construction is completed, as before. As the coefficients of self-induction are simply added together, the difference of phase relatively to the effective electromotive force or the current, that is to say, the angle BOA is independent of a.

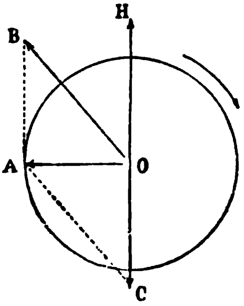

FIG. 344.

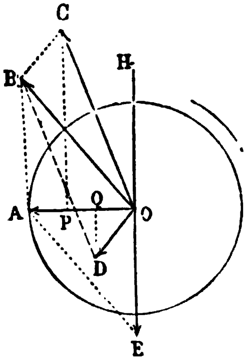

FIG. 345.

The work expended per second in the form of heat in the circuit is $\frac{\mathrm{OA}}{R\sqrt{2}} \cdot \frac{\mathrm{OA}}{\sqrt{2}}$, or $\frac{\mathrm{OA}}{2R} \cdot \mathrm{OA}$; the contribution of the first machine is $\frac{\mathrm{OA}}{2R}\,\mathrm{OP}$, and that of the second $\frac{\mathrm{OA}}{2R}\,\mathrm{OQ}$. These quantities of work are respectively proportional to the lines

OA, OP, OQ,

and it is apparent from the figure that the first is in all cases the algebraical sum of the other two.

If the two projections fall on the same side of the point O, the amounts of work represented by them are of the same sign, and the two machines act as generators absorbing work. If they fall on opposite sides (Fig. 346), the first machine absorbs work and acts as a generator, the second produces work and acts as a motor. The work that must be done upon the combined system, and is dissipated in the form of heat, is the difference between the amounts of work taken in by the first machine and given out by the second.

412. Coupled Alternators continued.—Instead of two machines mounted on the same axle, let us consider two machines each driven by a special motor and not connected with each other otherwise than electrically. The motors are supposed to be regulated so as to drive the machines at very nearly the same speed. If the machines were independent, the smallest difference of speed would make them pass successively through every possible difference of phase; there would be no stable condition, unless they were able to act on each other in such a manner as to maintain their synchronism and their difference of phase.

Let us take the simplest case, that of two identical machines; the conclusion arrived at will be none the less generally applicable. It follows evidently from the construction (Fig. 345) that the *leading* machine always makes, with the effective intensity, a greater angle than the *following* machine. It follows from this that the latter always does more work than the former; moreover, all causes of disturbance have the effect of increasing the lead of the leading machine and the lag of the follower; they therefore tend to increase the difference of phase until the electromotive forces are in opposition, and the current is at its minimum, which minimum is zero when the two machines are identical.

Two alternate-current machines placed *in series* in the same circuit cannot, therefore, be made to work in the same *direction*; they can, however, if joined *in parallel*. For in the closed circuit containing the two machines, at two points of which the external circuit is connected, the two machines will act in opposition; if the external circuit has a relatively high resistance, which is always the case in practice, its influence on the *régime* may be neglected, and therefore the machines work under conditions of stable equilibrium. Hitherto, however, no general agreement has been arrived at, so far as the machines are concerned, as to the qualities which best suit coupled working.

These conditions are also those met with when one of the machines is used as generator and the other as motor. Equilibrium corresponds to that difference of phase for which the motor gives out the work required from it. The efficiency is equal to (Fig. 346)

$$\frac{OQ}{OP},$$

and the coefficient of loss in the circuit to

$$\frac{OA}{OP}.$$

413. Various Types of Machines.—There are very many kinds of alternate-current machines: in a sense, such machines may be said to be the most natural. The currents in Gramme's and in Siemens' machines are only continuous because the commutator combines in the proper way currents which are really alternate. Whatever be the nature of the machine, the current can always be rectified in the external circuit by means of a suitable commutator; but the use of commutators always causes a loss of energy in the form of sparks which rapidly destroy the collectors.

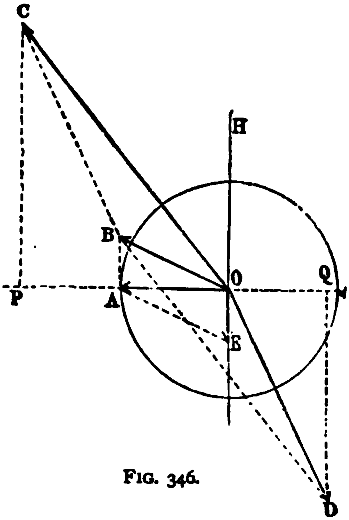

FIG. 346.

Alternate reversal of the current causes no inconvenience in many applications, especially for electric lighting.

The field-magnets of alternators are usually excited by an independent continuous-current dynamo, which is called the *exciter*.

The armature has usually no iron core. When such a core is used, it ought to be very finely laminated, so as to avoid Foucault currents. Nor should the magnetisation be pushed too far, otherwise there is an unnecessary loss of energy in each magnetic cycle.

The presence of soft iron causes the value of the coefficient of self-induction to vary with the strength of the current, and thus greatly complicates the theory of the machine.

One characteristic of alternate-current machines is that they can be easily made to produce very high potentials. This is a point of great importance in connection with the transfer of energy to a distance, which has received an immense development since the introduction of transformers (§ 417).

For equal differences of potential, alternating currents seem to require more care on the part of those who work with them than continuous currents.

As regards efficiency, alternating machines do not seem to be inferior to continuous-current machines, and they are not found to heat more when at work.

The frequency of ordinary machines varies from 40 to 140. In the case of very high frequency, regard must be had to the want of uniform distribution of the current in the conductor (§ 327).

The work taken in or given out during any very short interval is given by an elementary portion of the area of a curve analogous to curve B of Fig. 289 (§ 326), the ordinates of which are the products of the ordinates of two sine-curves, one representing the electromotive force, and the other the current. This work is sometimes work absorbed, and sometimes work given out. The lag being always less than 90°, the work taken in predominates, but it decreases in proportion as the difference of phase increases.

An alternating machine consequently acts like a fly-wheel in relation to the motor, sometimes absorbing and sometimes giving back work. The periodic variations of the driving couple produce vibrations which give rise to a humming sound more or less loud.

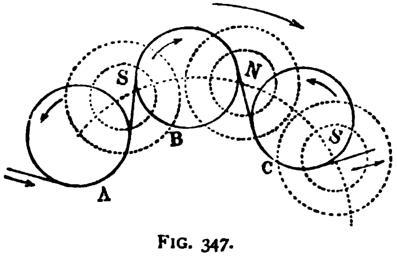

Fig. 347.

As examples of alternating machines, we will describe those of Siemens and of Gramme.

414. Siemens' Machine.—The inductor consists of two sets of electromagnets, each of them arranged with their axes parallel and at equal distances round the circumference of a circle, and presenting alternately opposite poles in the same direction (Fig. 347, SNS). The two circles of magnets are set facing each other a small distance apart, the axis of each magnet on one circle coinciding with that of one on the other circle which has the same direction of magnetisation as itself, so that opposite poles are presented towards each other. The space comprised between the two circles of magnets forms a magnetic field, the maxima of which, alternately in opposite directions, coincide with the axes of the successive electromagnetic coils. The armature consists of a corona of flat coils, without soft iron cores, equal in number to the electromagnets. These coils are wound alternately in opposite directions, and move uniformly in the inducing field. The variation of magnetic flux through the movable coils vanishes when their axes coincide with those of the fixed coils, and at this moment the direction of the current is reversed without the effects of self-induction. A complete period corresponds to the interval which separates two poles of the same name. If there are eight coils in each circle, the number of periods is four for one revolution. The current is almost exactly simple harmonic.

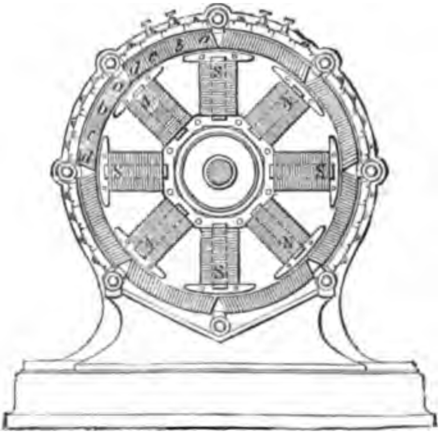

FIG. 348.

415. Gramme's Machine.—In this machine the armature is fixed and the field-magnets revolve (Fig. 348). The armature consists of a cylindrical drum, formed like that of a continuous-current Gramme machine (§ 401), of a core of soft iron wire wound with insulated copper wire parallel to the generating lines of the cylinder. These windings are divided into eight sections coiled alternately in opposite directions. The inductor is formed of eight electromagnets with alternate poles, which turn inside the drum. These electromagnets carry with them the magnetic flux, which penetrates the core and traverses the different windings; at each instant the current has the same value in all the sections. The ends of the coils being fixed, they may be combined in any way that is required.

416. Alternating Motors. — Most alternators can, like the preceding, be used either as generators or as motors. In this latter case the field-magnets are excited by an independent continuous-current machine, and an alternating current is sent through the armature coils. The armature can move indifferently in either direction. Nevertheless, as the number of dead points is considerable, the load can only be put on after the armature has been made to rotate by hand or otherwise with something like the velocity which it ought to have. The reactions finally determine the synchronism of the motor and of the alternating generator that drives it.

There are, however, several motors constructed on a different principle, in which the alternating current of the generator is sent into the field-magnets, and the armature, forming a closed circuit of itself, has no connection with other parts.

The motor of E. Thomson depends upon the effects described in § 326. The field-magnet is an electro-magnet with alternating poles, and the armature is a drum, divided into several sectors, which can turn between the poles. The alternating current is allowed to circulate at first in the armature as well as in the field-magnet, until the armature has acquired a sufficient speed; the current is then cut off from the armature, which is short-circuited, and continues to revolve in virtue of the reactions set up between the currents induced in it and the field.

Tesla's motor is a form of Arago's experiment (§ 316), in which a revolving magnet draws round after it a conducting disc.

The disc is replaced by a drum, that can rotate about its geometrical axis, and is divided into four sectors, each of which forms a closed circuit. The drum is concentric, with a soft iron ring, similar to that of a Gramme alternator, but having only four coils, those at opposite ends of the same diameter forming part of the same circuit. The two systems of coils, AA′ and BB′, are traversed by currents that alternate harmonically in the same period, with a difference of phase of 90°, as, for example, the currents from two parts of a single alternator.

As is well known, two equal simple harmonic vibrations of the same period, at right angles to each other, differing in phase by a quarter of a vibration (or by 90°), compound to a uniform rotation in a period equal to that of either component. As magnetic fields are compounded geometrically, the two alternating electromagnets give a field of constant strength, the direction of which revolves uniformly, and they consequently drag the armature round in the same direction with an amount of lag that depends on the work done by the armature. As in this arrangement brushes are dispensed with, there is no sparking.

417. Transformers.—Instead of directly using the alternating current, it may be used to excite a current of the same kind in a neighbouring circuit. The primary current corresponding to certain numbers, E and C, of volts and amperes, the secondary current would correspond to other numbers E' and C'. If the transformation is effected without loss of energy, we have

$$EC = E'C',$$

and by a suitable arrangement of apparatus the two factors of the product may be modified at will.

Such apparatus are called *transformers*. They seem destined to play an extremely important part in the transmission of electrical energy to a distance. An example will illustrate the practical aspect of the question.

Suppose that it is desired to transmit 500 amperes at an electromotive force of 100 volts, or 50,000 watts, to a distance of 1000 metres with a loss of 10 per cent.

For direct transmission, the conductor, 2000 metres in length, must have a resistance of only 0.02 ohm; this would imply a section of 16 square centimetres; the weight of the conductor would be 28,000 kilogrammes, and its price about £3360.

If the same amount of energy were transmitted by a current of 50 amperes with a potential difference of 1000 volts, and then transformed at the farther station into a secondary current of 500 amperes at 100 volts, the resistance of the connecting wire might be 2 ohms, its section 0.16 square centimetre, its weight 280 kilogrammes, and its price about £34.

418. Various Types of Transformers.—In any transformer there are three circuits to be considered: the primary circuit, the secondary circuit, and the magnetic circuit. It is most advantageous to use a closed magnetic circuit. The types which appear to give the best results are represented in Figs. 349 and

350. The arrangement of the first is like that of a Gramme ring; the primary circuit AB and the secondary *ab* are coiled simultaneously on a soft iron core, which usually consists of a coil of wire. In the second the two electrical circuits AB and *ab* are wound together, so as to form the core, and the iron wire is coiled on the outside. In the former case the *core*, and in the latter the *envelope*, is of soft iron; in the former the magnetisation is longitudinal, and in the second transverse (§ 281). Both systems appear to give equally good results.

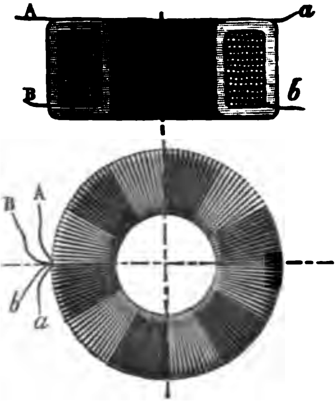

FIG. 349.

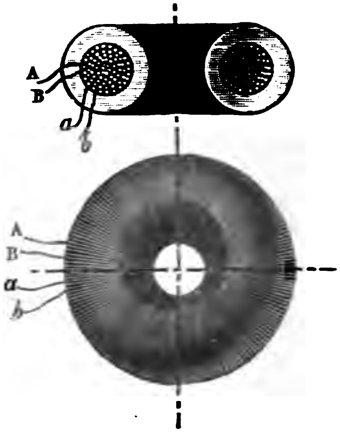

FIG. 350.

419. Theory of the Transformer.—Let us suppose that at the two terminals of the primary circuit AB a difference of potential is established which varies harmonically. If, as in § 325, we neglect variations of μ and the effects of hysteresis, the two electrical circuits and the magnetic one will each be the seat of a flux which varies harmonically in the same period, but with different phases.

We will represent by the same letters, but with suffixes 1 and 2, corresponding quantities connected with the two circuits, primary and secondary; thus, n_1 and n_2 will be the number of turns of wire, R_1 and R_2 the resistances, and c_1 and c_2 the simultaneous current-strengths.

The maximum strengths of current C_1 and C_2, which are directly deduced from the effective currents, may be regarded as data furnished by experiment.

The variation of the flux of induction is the same for each turn of the primary and of the secondary circuit; if we represent by Q the maximum value of the flux of induction, the maximum values of the electromotive forces developed in the two circuits are (§ 320)

$$E_1 = n_1 \frac{2\pi}{T} Q, \quad \text{and} \quad E_2 = n_2 \frac{2\pi}{T} Q;$$

they are therefore in the same ratio as the numbers of the turns of wire, and both differ in phase by 90° as compared with Q. If we further assume that the secondary circuit is without self-induction, there is no difference of phase between the electromotive force and the current which it produces, and we have simply

$$E_2 = C_2 R_2;$$

E_2, E_1, and Q are thus known.

Moreover, if R is the magnetic reluctance (§ 276)

$$\mathrm{R}q = 4\pi(n_1 c_1 + n_2 c_2);$$

this equation, in which $q = Q \sin 2\pi \frac{t}{T}$ and $c_2 = C_2 \cos 2\pi \frac{t}{T}$, enables us to calculate $n_1 c_1$. We have

$$n_1 c_1 = n_1 C_1 \sin 2\pi \left(\frac{t}{T} + \phi\right)$$

where $2\pi\phi$ is the acute angle of a right-angled triangle OAB (Fig. 351), the hypothenuse OB of which is $n_1 C_1$ and the opposite side AB is $n_2 C_2$. It remains to find the intensity and the phase of the electromotive force acting at the ends of the primary wire. If the values at a given moment are denoted by small letters, we have

$$e = R_1 c_1 + e_1.$$

With the mode of construction already used, it will be seen that the electromotive force required is the projection of the diagonal E of a parallelogram, the sides of which are $C_1 R_1$ and E_1. To construct this parallelogram, let us take two rectangular axes, and draw the triangle mentioned above. The direction OA corresponds to the magnetic induction Q, the direction OB to the current C_1; let us represent the electromotive forces of induction by the

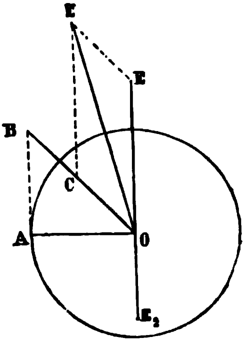

FIG. 351.

lines OE_1 and OE_2 perpendicular to OA; on OB take a length OC, equal to C_1R, and complete the parallelogram; the diagonal OE is the maximum value of the difference of potential between the ends A and B of the primary wire, and the angle $2\pi(\phi + \psi)$ which it makes with OA is the angle of *lead* relatively to the magnetisation.

In ordinary transformers the angle $2\pi(\phi + \psi)$ is very near 90°; hence the angle $2\pi\psi$ is the complement of $2\pi\phi$. Observing that $\sin 2\pi\phi = \frac{n_2 C_2}{n_1 C_1}$, we have approximately

$$E = E_1 + R_1 C_1 \cos 2\pi\psi = C_2\left(\frac{n_1}{n_2} R_2 + \frac{n_2}{n_1} R_1\right).$$

From this equation we obtain

$$C_2 = \frac{\frac{n_2}{n_1} E}{R_2 + \left(\frac{n_2}{n_1}\right)^2 R_1},$$

or, observing that the factor $\left(\frac{n_2}{n_1}\right)^2$ is very small,

$$C_2 = \frac{\frac{n_2}{n_1} E}{R_2};$$

an expression which shows that the secondary current is very nearly that which would be produced in the secondary circuit by an electromotive force which was a fraction $\frac{n_2}{n_1}$ of that acting at the ends of the primary wire.

The resistance R_2 is made up of two parts: the resistance of the external circuit, and that of the wire *ab*; as this does not exceed a few hundredths of an ohm, it follows that the difference of potentials at the terminals of the secondary circuit is sensibly

$$\frac{n_2}{n_1} E.$$

The work per second done in the secondary circuit is

$$W_2 = \tfrac{1}{2} E_2 C_2 = \tfrac{1}{2} R_2 C_2^2;$$

the energy supplied at the terminals of the primary circuit is

$$W_1 = \tfrac{1}{2} E C_1 \cos 2\pi\psi = \tfrac{1}{2} E \frac{n_2}{n_1} C_2;$$

the efficiency is therefore

$$u = \frac{W'_2}{W_1} = \frac{E_2}{\frac{n_2}{n_1}E} = \frac{R_2}{\frac{n_2}{n_1}\left(\frac{n_2}{n_1}R_1 + \frac{n_1}{n_2}R_2\right)}$$

$$= \frac{1}{1 + \left(\frac{n_2}{n_1}\right)^2 \frac{R_1}{R_2}}.$$

This differs but little from unity. Experiment shows that the practical efficiency is greatest when the transformers work at full load. The magnetisation thus oscillates within narrow limits, and the losses due to hysteresis are small.

420. Ruhmkorff's Coil.—Ruhmkorff's coil is a true transformer, in which, contrary to what occurs in modern transformers, the aim is, by means of a strong current generated by a source of small electromotive force, to obtain, in the secondary wire, a considerable electromotive force capable of giving long sparks and of charging Leyden batteries,—capable, in short, of reproducing all the effects which are ordinarily obtained with electrostatic machines.

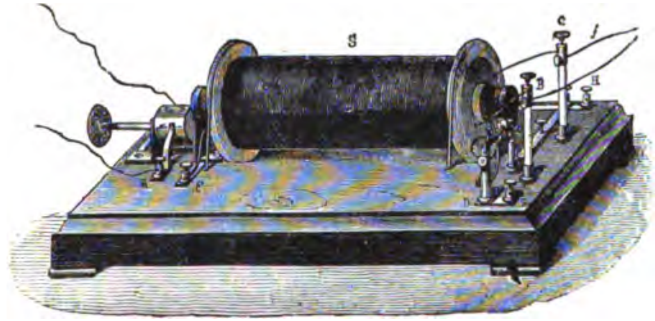

FIG. 352.

The apparatus represented in Fig. 352 consists of a cylindrical core of soft iron formed of a bundle of parallel wires; on this core is wound the primary coil formed of stout insulated wire, and outside this is coiled the secondary wire, consisting of a great number of turns of fine wire connected with the two terminals B and C. This secondary wire is very carefully insulated so as to avoid internal discharges. Instead of coiling it in successive layers parallel to the axis, it is wound in separate sections perpendicular to the axis, and these are separated from each other by insulating partitions. In this way the potential increases from one end of the wire to the other without there being anywhere too great a difference between two adjacent layers.

The primary current is given by a battery, and the induction results from the alternate making and breaking of the circuit. This arrangement, which would not be suitable in the case of a closed magnetic circuit in consequence of the retention of magnetisation (§ 196), may be used in the case of a short cylinder, and it is even preferable to the use of a current varying harmonically, inasmuch as variations of the current are more sudden.

421. Make and Break.—The make and break is often effected by means of a piece of soft iron, O, placed below the end M of the soft iron core (Fig. 353). The hammer, OD, and its anvil, with the corresponding pieces, *e*, *a*, H, form part of the primary circuit. When the current passes, the core is magnetised, the hammer is raised, and the circuit broken; the hammer in falling again closes the circuit, and so forth.

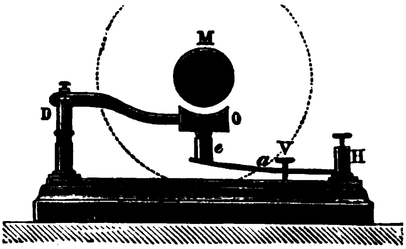

FIG. 353.

The break is made less sudden by the spark which passes between the hammer and the anvil. In order to make the break more rapid, Foucault used a platinum point vibrating vertically up and down and dipping into mercury (Fig. 354). The spark is considerably reduced by covering the mercury with a layer of alcohol. The vibrating motion is kept up by a separate battery, and electromagnetic contact breaker.

422. Fizeau's Condenser.—The power of the coil is increased by connecting the ends of the primary wire with the two coatings of a condenser. The effect is a maximum when the condenser has a certain definite size.

When the circuit is broken the greater part of the energy of the primary current, instead of being expended in producing a spark at the contact-breaker, charges the condenser; this is then discharged in the form of an oscillating current through the primary wire.

At the first oscillation the current is in the opposite direction to the battery current; the effect is almost the same as if the battery current had been reversed instead of being broken, so that the variation of the magnetic flux through the secondary wire is virtually doubled.

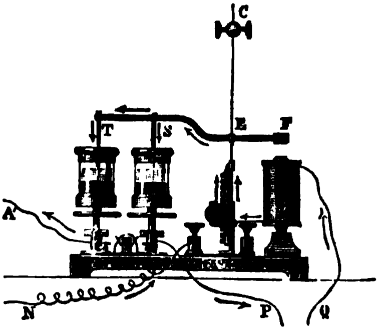

Fig. 354.

It will be understood that the effect is greatest when the periods of oscillation are the same for both the primary and secondary circuits.

423. Direct and Inverse Currents.—If the two poles of the apparatus are connected by a conductor, this is traversed by equal currents alternately in opposite directions, which neutralise each other as regards action on the galvanometer.

If the wire is cut and the ends are separated, the current continues to pass, forming loud sparks, which sometimes attain a great length. The large coils made by Ruhmkorff, which are about 60 centimetres in length, and have 120 kilometres of secondary wire, are capable of giving sparks 45 centimetres long.

These sparks are always somewhat slender. They become denser and less frequent if the ends of the secondary wire are connected with the coatings of a condenser. In this way the capacity of the wire is increased. With large coils a single Leyden jar would be incapable of withstanding the great difference of potential between the two poles, and accordingly two or more jars are used, arranged in cascade.

When the two ends of the wires attached to the terminals of the secondary coil are drawn gradually apart, it may be observed that in general the direct current passes more readily than the inverse one, and when the distance is sufficiently great the former alone passes. The coil gives then an intermittent current always in the same direction, which can be used to charge a Leyden battery.

When the poles are connected by a tube containing a rarefied gas, a Geissler's tube, for instance, beautiful stratifications are obtained. The investigation of these luminous phenomena by means of a rotating mirror shows, moreover, that the discharge is not a continuous phenomenon, but is due to a series of oscillations alternately in opposite directions.

This is also the case when the ends of the coil are separated so far that no spark can pass: by measuring the difference of potentials at the ends of the secondary wire, after successive very short intervals of time, it is found that, at each break of the primary current, the secondary wire is the seat of isochronous oscillations, which are rapidly damped out, as shown in Fig. 292 (§ 330).

424. Yield of the Coil.—The yield, as in the case of electrostatic machines, may be measured by the time necessary to charge a battery to a given potential. The two coatings of the insulated battery are connected on the one hand with the arms of a spark-micrometer, with its knobs at a distance, D, and on the other with the two terminals of the coil, a break, $d < D$, being interposed in one of the connecting wires, so as only to allow the direct current to pass, and to prevent the battery from being discharged at each alternation.

At each spark of the coil the battery receives the same quantity of electricity; after a certain time it is discharged through the micrometer. This phenomenon is periodic, and one spark of the battery corresponds to a constant number of sparks of the coil. Experiment shows that the quantity of electricity corresponding to one spark of the coil gradually diminishes when the rapidity of the breaks is increased beyond a certain limit.

CHAPTER XXXV.

ELECTRIC LIGHTING.

425. Incandescent Lighting.—Electric energy is used for lighting purposes chiefly in two ways, namely, in *incandescent* lamps and in *arc lights.*

The former method consists in raising a conducting thread to so high a temperature that it becomes luminous. Platinum was first tried for this purpose, but it rapidly disintegrates. The best substance is a thin filament of carbon placed in a vacuum so as to prevent its being burnt. In Edison's lamp, for instance (Fig. 355), the carbon is a filament of bamboo calcined at a high temperature, and contained in a glass bulb which is exhausted by means of a mercury pump; it is fixed at the ends to two platinum wires fused into the glass, which are the electrodes.

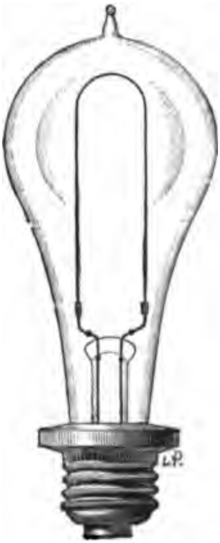

FIG. 355.

Before the bulb is finally sealed the filament is kept incandescent for some time in an atmosphere of a hydrocarbon. Carbon resulting from the decomposition of the hydrocarbon by the great heat is deposited on the filament, which thereby becomes more compact and regular.

The rarefaction is pushed to one or two hundredths of a millimetre of mercury. At a lower pressure the filament rapidly disintegrates, and the bulb becomes covered with a layer of carbon; at a greater pressure, 2 or 3 millimetres, for example, the bulb becomes heated.

The illumination* rapidly increases with the strength of the current; but the life of the lamp is thereby shortened.

The conditions of maximum economy are those in which the gain in luminosity and the loss in the life of the lamp, each estimated by its money value, exactly compensate each other.

Equilibrium of temperature is attained when the rate at which heat is lost by radiation is equal to the rate at which it is generated in the carbon filament, the conditions of this generation being expressed by Joule's law. The ratio of the light emitted to the energy expended is a function of the temperature only, and does not depend on the shape of the carbon filaments, provided they have the same emissive power.

By measuring the quantity of heat imparted by the lamp to a calorimeter with opaque sides, which absorbs the whole of the radiation, and then to one which transmits the light, it is found that the light is about 5 per cent. of the total energy; in the case of an ordinary candle the ratio is scarcely 3 in 1000.

An Edison lamp of 15 candle-power works with a current of 0.8 ampere, and a difference of potential of 100 volts. Such a lamp usually lasts about 1000 hours.

The energy consumed in this case per second is $100 \times 0.8 = 80$ joules. But 80 joules per second, or 80 watts, is nearly one-ninth of a horse-power; hence 1 horse-power radiated from such lamps would amount to about 140 candle-power. The resistance of the lamp when hot is $\frac{100}{0.8} = 125$ ohms.

The lamps are generally arranged in parallel circuit. Suppose there are n lamps of the same kind: let r be the resistance of one lamp, and c the current through each. On the other hand, let E be the electromotive force of the generator and R the resistance of the circuit, exclusive of the lamps; the resistance ρ of the whole of the n lamps in parallel is $\frac{r}{n}$, and we have

$$E = (R+\rho)nc = (nR + r)c;$$

the useful work is nrc^2, and the efficiency $\frac{r}{nR+r}$.

---

* The English official standard of illumination is a spermaceti *candle*, ⅞-inch in diameter, six to the pound, burning 120 grains per hour. The French standard is the Carcel lamp, burning 42 grammes of oil in an hour: this is equivalent to about 8.9 English candles.

Suppose it is desired to work lamps of the preceding type by means of a battery. Let us assume that the only appreciable external resistance is that of the battery; in order to obtain the maximum rate of doing work (§ 131), we must take $nR' = r = 125$ ohms. For Bunsen's elements the preceding formula gives

$$1.8x = 250 \times 0.8 = 200,$$

x being the number of elements. From this we deduce $x = 111.1$. Hence there must be 111 elements in series, whatever be the number of lamps to be maintained. The resistance alone must vary with the number of lamps, and will be equal to

$$\frac{125}{111n} = \frac{1.126}{n} \text{ ohms.}$$

Lamps of the Edison type require a high electromotive force and a weak current. As an example of a different type, the Bernstein lamp may be mentioned. This is formed of a straight rod of carbon, and requires a current of 10 amperes at a potential difference of 7 volts, which gives 0.7 ohm for the resistance of a single lamp when hot. The lamps are arranged in series. A calculation similar to the foregoing shows that to work them with Bunsen's elements, arranged to give the maximum of external energy, eight elements must be taken per lamp, each having one-eighth the resistance of a single lamp.

The use of Bunsen's elements is only mentioned by way of example. In practice the lamps are always fed by dynamos or accumulators. Take the case of an installation of 1000 lamps. With high-resistance Edison lamps arranged in parallel circuit, the dynamo must give 800 amperes with an electromotive force of 100 volts. With Bernstein lamps in series, 10 amperes would be required with 7000 volts. As no machine could in practice work under these latter conditions, the lamps might be arranged in five parallel series with 200 lamps in each. A current of 50 amperes at 1400 volts would in that case be sufficient. In this calculation no allowance is made for the resistance of the conductors.

426. Voltaic Arc.—Davy having attached two rods of carbon to the poles of a battery of 2000 elements, found that, on gently drawing them apart after being in contact, a flame formed between the two points, to which he gave the name of the *voltaic arc*.

The phenomenon is so brilliant that it can only be looked at through a dark glass ; a still more convenient way of observing it is to project the image of the arc and the two carbons on a screen by means of a lens. It is then seen that the arc has far less brilliance than the points of the carbons themselves (Fig. 356); that it consists of two distinct parts—one the arc, properly so called, which is blue, and the other, which has the ordinary appearance of a flame, and is reddish; that the positive carbon is brighter than the negative, and that a greater length of it is luminous, indicating that its temperature is higher; that the positive carbon is hollowed out in the form of a crater, while the negative one is sharpened to a point; finally, that the positive wears away more rapidly than the negative carbon. In a vacuum the effects are the same, apart from the action of air on the carbons, and it is clearly seen that matter is carried from the positive to the negative carbon.

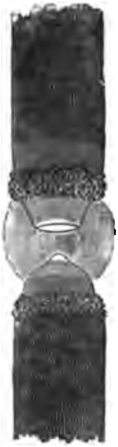

FIG. 356.

The temperature of the arc is very high, and difficultly fusible substances, such as platinum, melt easily in it. Hence a permanent arc can only be produced with rods of carbon. For illuminating purposes it is best to have the positive carbon, which is the brightest, above, as the light is thereby better diffused. If the heat is to be used for melting or volatilising a substance, the lower carbon is made the positive one, and its extremity is hollowed out.

The light emitted is very rich in highly refrangible rays, and appears bluish as compared with sunlight. In the spectroscope the carbons give a continuous spectrum extending far towards the violet; the arc itself, like all incandescent gases, gives a fluted spectrum showing the rays of carbon, and those of any metals which may be present in the carbons.

Calorimetrical experiments made in a transparent and in an opaque vessel show that the fraction of energy converted into luminous rays is about one-tenth of the total energy expended in the arc.

The arc is acted upon by a magnet like ordinary movable conductors (§ 266).

In air the arc produces ozone, and in an atmosphere of hydrogen it gives acetylene.

427. Electromotive Force of the Arc.—Measured either by an electrometer or by a high-resistance galvanometer, the difference of potential between the two carbons is found to be never lower than 30 volts. It varies from 30 to 70 volts. Experiment shows that this difference or fall of potential, which of course takes place in the direction of the current, consists of two parts. One of these is fixed, and is independent of the strength of the current and of the distance of the carbons, acting therefore like a true electromotive force; the other varies with the strength of the current and the distance of the carbons. According to recent experiments by Mrs. Ayrton, the connection between the observed difference of potentials, E, the length of the arc, L, and the strength of the current may be expressed by an equation of the form

$$E_C = p + qL,$$

when the strength of the current is constant; and by an equation of the form

$$E_L = s + \frac{t}{C},$$

when the length of the arc is constant; p, q, s, and t being numerical quantities determined by experiment. These expressions are included in the following more general equation

$$E = a + bL + \frac{e + fL}{C},$$

when a, b, e, f are again constants.

When the length of the arc was measured in millimetres, the current in amperes, and difference of potentials in volts, the values obtained with solid carbons agreed with the formula

$$E \doteq 38.9 + 2.07\,L + \frac{11.7 + 10.5\,L}{C}.$$

This formula indicates an electromotive force at the carbons of about 52.6 volts as being required to maintain an arc of 4 mm. with a current of 10 amperes.

428. Work Expended in the Arc.—The difference of potentials multiplied by the strength of the current gives the rate of expenditure of energy in the arc. For the rate of expenditure in watts, Mrs. Ayrton's experiments give

$$W = 11.7 + 10.5\,L + (38.9 + 2.07\,L)\,C,$$

or 526 watts for an arc such as that referred to in § 427.

A brilliant arc of about 900 candle-power requires a current of 15 amperes and a potential difference of 50 volts at the carbons—that is to say, 750 watts, or almost exactly one horse-power. The brightness of the illumination increases much more rapidly than the energy expended; and therefore large lamps are relatively more economical than small ones.

429. Use of Alternating Currents.—With alternating currents the light is as steady as with continuous ones, although it gives rise to a peculiar humming sound, the pitch of which depends on the number of reversals of the current per second. The carbons wear away at equal rates, and both become pointed. By means of a kind of phenakistoscope their condition may be examined, and even photographed, at various phases of the period. The arc disappears when the current is zero, but it is spontaneously renewed in the incandescent gas. The brightness of the two carbons varies periodically, and passes through maxima and minima; further, each of the two carbons becomes in turn the most luminous when it is positive. These variations can be easily observed, even when they are reproduced a hundred times in a second.

If the difference of potential is continuously observed, it is found that it does not vary harmonically in the same way as the strength of the current. Its absolute value remains almost constant, but the sign changes very rapidly the moment the current is reversed, a result agreeing well with the existence of an inverse electromotive force.

430. Carbons.—Davy used wood charcoal. Foucault replaced this by the carbon deposited in the interior of gas retorts and known as gas-graphite; it is harder, is a better conductor, and does not waste so rapidly. Carbons prepared artificially are used now; they are purer, more homogeneous, and more regular in shape. They are prepared from a paste of powdered coke, lampblack, and a very thick syrup of gum and sugar. The whole is well mixed, pressed, passed through a draw-plate, dried, and hardened at a high temperature. The carbons are baked repeatedly, being each time dipped in boiling syrup. What are called cored carbons are now frequently used; the centre of these carbons is of a different composition from the rest. They take a better shape when in use, and give a more regular arc, like that in Fig. 356. It is very important that the carbons should be pure; in particular, they ought to be free from silica.

Carbons are often coated with copper or nickel; by this means their conductivity is increased, and they last rather better.

431. Regulators of the Electric Light.—The prime requirement of any system of lighting is the constancy of the light. In order to obtain this constancy with the electric arc, notwithstanding the wasting of the carbons and accidental variations in the current, *regulators* are used. We shall not attempt more than to give the principle of the chief arrangements which are used.

Two mechanisms are obviously necessary; one, which brings the carbons near each other when the current diminishes, and makes them touch each other when it stops, since they do not relight unless they are in contact; and another which, as soon as the current is re-established, pulls the carbon apart to the proper distance, so as to *strike the arc*, and separates them still further when the current tends to increase. To keep the light steady, the two carbons must move at the same rate, if the current is an alternating one; while the positive carbon must move at twice the rate of the negative one, if the current is continuous. Otherwise it is sufficient if one of the carbons is movable. In most cases, the upper carbon, which should be the positive one, tends to sink from its own weight. The function of the mechanism is to regulate its descent and to raise it by a suitable amount if it comes into contact with the negative carbon. The mechanism, of whatever nature, is controlled by a coil through which the whole or part of the current passes. Two plans are in use; in one the coil is traversed by the whole of the current, and the regulation depends upon and determines the strength of the current; in the other the coil is wound with a very fine wire, forming a shunt in respect of the two carbons. The regulation in this case depends upon the strength of the current in the coil, but as this in its turn depends on the difference of potential at the ends of the shunt, the regulation causes a fixed difference of potential in the arc. As the former system affects the whole current, it can only be applied where there is a single lamp in a circuit; when there are several lamps in series in the same circuit, one regulator must not be affected by the action of another regulator in its vicinity; the second plan is adapted for this case.

432. Cost of Electric Lighting.—Experiment shows that one horse-power supplied in the form of electrical energy at the terminals of a lamp gives about 900 candle-power with a continuous arc, 450 with an alternating arc, and 180 with incandescent lamps.

Taking 90 per cent. as the efficiency of the dynamo, and 10 per cent. for the loss in the leads, one horse-power at the lamp represents 1.25 horse-power in the engine. With the very best engines this represents a consumption of between two and three pounds of coal in the hour.

Of 1000 units of thermal energy due to the combustion of the coal, the steam-engine utilises 100; of this the lamp receives 80, and converts 8 into light in the case of an arc, and 4 in that of an incandescent lamp.

These numbers are far smaller in the case of gas. A simple calculation shows that the quantity of light produced by burning a given volume of gas directly is far less than if the same volume is used in a gas-engine employed to drive a dynamo.

433. Electrical Furnace.—Another application of the heating effects produced by electricity is to the fusion of minerals and metals by the voltaic arc. The temperatures which can be obtained by combustion are necessarily limited by dissociation, but this does not apply to the voltaic arc. The question of temperature, however, is not the only one that comes into account; in many cases the reducing properties of the negative electrode appear to play an important part.

Sir William Siemens showed that electric energy might be economically employed to melt iron and steel. A crucible of graphite about 20 centimetres in diameter is used, surrounded by charcoal powder. A carbon rod, like those used for electric lighting, passes through the bottom of the crucible and forms the positive electrode. The negative electrode is kept at a proper distance above it by a kind of regulator. With a current of 36 amperes and a motive power of 4 horse-power, a kilogramme of steel is melted in fifteen minutes. It may be taken that the heating and fusion of a kilogramme of steel requires 450 kilogramme-degrees of heat, or $450 \times 4180 = 1,881,000$ joules of work. Now, 4 horse-power in fifteen minutes give $4 \times 745 \times 15 \times 60 = 2,682,000$ joules; so that the efficiency is about 70 per cent.

434. Electrical Welding.—Of all known modes of heating, the electric arc is that by which the greatest quantity of heat can be concentrated in a given point. This fact is made use of industrially for fusing together two pieces of the same metal—iron, for example—without using a second metal as solder. In order to join two pieces of sheet-iron, they are placed one over the other with their edges together; they are then connected with the positive and negative pole of a battery of 80 to 90 volts, and the carbon, held in a suitable handle, is moved along the edge; the arc which passes between the carbon and the iron melts the iron and solders the two pieces firmly together.

Another entirely different method, for joining two rods end to end, consists in pressing the ends, suitably prepared, against each other, and then passing a current which fuses the parts in contact. The current, which only needs to act a short time, should have great strength with low electromotive force. It is obtained by means of a transformer. The apparatus is arranged, for instance, so that a mean current of 20 amperes and 600 volts in the primary gives 1 volt and 12,000 amperes in the secondary circuit.

CHAPTER XXXVI.

GALVANIC DEPOSITION.

435. Applications of Electrolysis.—This chapter gives a brief account of the methods in which the electric current is utilised to liberate the metal contained in the solution of a salt; either in order to deposit a thin coating of the metal on any object, or to take a cast in the metal, or for the extraction or purification of the metal in the solution.

The only metals which meet with important industrial applications in one or the other of these respects are nickel, silver, gold, and copper. The methods must obviously be different for different metals. We have to consider—(1.) the nature and composition of the bath; (2.) the electromotive force required for the operation, and the density of the current, that is, the strength of the current per unit surface of the electrode; (3.) the treatment of the object both before and after its immersion in the bath.

436. Electrodes.—The object to be coated forms the negative electrode or *kathode;* it must have a conducting surface, and if the material is not itself a conductor, a conducting surface is produced by a coating of graphite or of silver sulphide, which is obtained by dipping the object in a solution of silver nitrate, and then exposing it to the action of sulphuretted hydrogen. The preparation of the surface is one of the most important conditions for the success of the process.

The positive electrode or the *anode* is ordinarily a plate of the same metal as that in solution; it loses as much metal as is deposited on the kathode, and the average strength of the solution remains constant. The strength of the solution is kept uniform throughout by stirring it.

When the metal of the two electrodes is the same as that of the bath, the fall of potential is the same at both electrodes, and the corresponding consumption of energy at the two electrodes is the same but of opposite signs. It follows that the work of electrolysis, properly so called, is zero, or practically so; the whole energy expended is reduced to the work of overcoming the resistance.

437. Chemical Work of an Ampere.—Starting from the fact that a coulomb liberates 0.00001038 of the equivalent of the metal expressed in grammes (§ 153), we obtain the following table:—

		Metal deposited by a current of one ampere in			Number of coulombs for 1 gramme.
	Eq.	1 second. grm.	1 minute. grm.	1 hour. grm.	
Silver .	107.7	0.001118	0.06708	4.025	894.5
Copper .	31.6	0.000328	0.01968	1.184	3050
Nickel .	29.3	0.000304	0.01824	1.094	3289

438. Electrical Deposition of Nickel.—For this purpose a 10 per cent. solution of the double sulphate of nickel and ammonium is used. The solution should be nearly neutral—rather acid, however, than alkaline; it is important that all the substances used—salt, water, and metal—should be perfectly pure. It is desirable, especially in coating iron, to begin with a rather strong current, so as to coat the object rapidly, and thus protect it from the action of the bath; the density of the current is then diminished. It is easy to deposit nickel on iron to the extent of 2 grammes per square decimetre; this represents a layer $\frac{1}{40}$ of a millimetre in thickness, and on account of the hardness and solidity of the metal this is quite sufficient. The more highly the surface to be coated is polished, the more brilliant is the coating of nickel; great pains should therefore be taken with the preparation of the surface. The object, suitably polished, is placed in a bath of boiling caustic potash, then carefully washed and placed in a bath of potassium cyanide, and after being again washed is placed in the nickel bath. When taken out it is washed, heated, and dried rapidly in sawdust.

The bath becomes alkaline by use and turbid, and the deposit of nickel acquires a yellow tint; citric acid is then added to the bath until it slightly reddens litmus paper.

439. Electrosilvering.—This is a very important industry.

The bath is a solution of double cyanide of silver and potassium, its composition being—

Potassium cyanide	500 grammes.
Silver cyanide	250 „
Distilled water	10 litres.

The pieces to be silvered, which are usually of metal, undergo the following four operations in succession before being immersed in the bath :—

1. The removal of grease by immersion in a 10 per cent. boiling solution of caustic potash, followed by washing in hot water.

2. Treatment with acid in a 10 per cent. bath of sulphuric acid, and subsequent washing in cold water.

3. Scouring in a bath formed of a mixture of nitric and sulphuric acids and salt, followed by washing with a large volume of water.

4. Amalgamation by immersion for a few seconds in a bath formed of 10 litres of water and 100 grammes of mercuric oxide, dissolved by the requisite quantity of sulphuric acid. The object is once more washed with pure water, and is finally transferred to the bath.

The dynamo employed should give 2 to 3 volts, and a current of at most 50 amperes per square metre. The operation lasts three or four hours. The deposition of 300 grammes of silver per square metre gives a good coating. On being taken out of the bath the objects are immersed in a solution of potassium cyanide, washed with boiling water, and dried in sawdust.

It is important that the soluble electrodes be of pure silver. The coatings are more brilliant and more adhesive when old baths are used than with new ones.

440. Electrogilding.—Gilding is also effected with baths of double cyanide of gold and potassium. The coating is always extremely thin, but nevertheless looks well, and is very durable. It is obtained in a few minutes with a current which should not exceed 10 amperes per square metre, and an electromotive force of 1 volt. The mode of preparing the objects is the same as for silver. The tint of the deposit of gold is varied by adding to the gold bath solutions of copper or of silver; the former gives a reddish and the latter a greenish tint to the gold.

441. Electrical Deposition of Copper — Electrotyping. — Copper is the metal most readily deposited under the action of the current, and it is the one which gave rise to the first use of the process (A.D. 1839). The bath most frequently used is a slightly acid solution of copper sulphate. Four methods may be employed. The object itself may be taken as the positive plate of a Daniell's element, which is then closed on short circuit; the copper is deposited on the positive plate, and crystals of copper sulphate are kept in the solution, so as to keep it nearly saturated.

Otherwise, an external electromotor is used, which may be either a battery or a dynamo. In the former case the best solution is an almost saturated one of copper sulphate, to which 12 grammes of sulphuric acid per litre are added; in the second case 825 grammes of sulphuric acid are added to 10 litres of water, and the water thus acidulated is saturated with copper sulphate.

To begin with, the current must be very weak, and its strength should be gradually increased. After the first, the density of the current may vary within wide limits; for instance, from 20 to 200 amperes per square metre.

A mould is made of the object to be reproduced—for example, a medal, an engraved wooden block, or a statuette; the mould may be either of gutta-percha, gelatine, or plaster of paris soaked in stearine, so as to make it impervious; the surface is rendered conducting, and the mould is then used as a negative electrode.

In this way medals, statuettes, &c., may be reproduced. One of the most important applications is that of *electrotyping*—that is, the reproduction of plates engraved on wood or on metal. The *cliché* thus obtained is used to print with, and the original plate preserved to procure fresh ones if necessary.

Clichés which are to be used for printing are hardened by the operation of *steeling;* the cliché is cleaned, then brushed with caustic potash, and immersed, along with a plate of pure iron forming the positive electrode, in a bath of iron. The bath in question is made by passing the current between two plates of pure iron through a solution of 12 parts of ammonium carbonate in 75 parts of water. When the bath has taken up a sufficient quantity of iron, the cliché is substituted for the negative plate. The electromotive force should be 4 volts. The cliché is dipped several times, the operation lasting for a few minutes; it is then carefully washed in boiling water, then in cold water, and is lastly rubbed with benzine.

An equally important industry is that of coppering objects of cast-iron, such as statues, candelabra, &c. The difficulty is to get a solution which does not attack iron. Such an one is obtained by dissolving copper sulphate in a solution of cupro-potassic tartrate made alkaline by caustic soda. The excess of soda makes the copper tartrate soluble, and prevents it from attacking iron. An external current may be used, or the objects may be connected with zinc plates in porous vessels containing soda placed inside the bath itself.

442. Refining of Copper.—The conductivity of copper is greater the greater its purity. In order to avoid loss of energy in dynamos and in conducting circuits, copper of high conductivity should be used. Ordinary copper is not sufficiently pure, and at the present time electrolytic copper is almost exclusively used for these purposes. The ore is treated by the ordinary methods, so as to obtain plates containing at least 95 per cent. of copper. These plates are used as positive electrodes in a bath of copper sulphate, and the pure metal is transferred by the current to thin sheets of copper, which form the negative electrode. Impurities, among which are gold and silver, fall as mud to the bottom of the bath, and are afterwards treated separately.

In this way plates of 1 square metre, and 1 or 2 centimetres thick, may be obtained, of great purity, very malleable, and of high conductivity.

As the electrolysis takes place between electrodes which are practically identical, the opposing electromotive force is almost zero, and the work to be done is reduced to that against the resistance of the bath. If W is the available energy, N the total number of troughs arranged in p series of n troughs, r the resistance of one trough, ρ that of the rest of the circuit, and R the total resistance; we have

$$W = RC^2$$

$$P = N\varpi \frac{C}{p} = n\varpi C;$$

ϖ being the weight of metal reduced by one coulomb, and P the total weight obtained in a second. From this we get

$$W = \left(\frac{\rho}{n^2} + \frac{r}{N}\right)\frac{P^2}{\varpi^2},$$

which shows that, for a given amount of energy expended, the weight of metal deposited will be greatest when n is as great as possible, and therefore when $n = N$, that is to say, when all the troughs are joined in series; and moreover that this weight will increase indefinitely with the number of troughs. But the formula shows also that to double the quantity of metal, the number of troughs must be quadrupled. There is thus a limit beyond which the expense arising from the increase of plant would not be compensated by the increased output.

443. Electrometallurgy.—By this name is understood the whole of the electrical methods suited for the preparation and the extraction of metals.

Electrolysis has been tried for the production of metals in two ways: the *wet way* and the *dry way*.

Davy in 1807 first obtained the alkaline metals and those of the alkaline earths by means of electricity (§ 152). He used mercury as negative electrode, with which the metal, when liberated, at once amalgamated; by then heating the resulting amalgam in an inert gas the mercury was driven off, and the metal left behind.

Bunsen obtained larger quantities of the metals by electrolysing the fused salts, particularly the chlorides, by means of two carbon electrodes. When the metal is lighter than the fused mass, it rises to the surface and is burnt, and to prevent this notches are cut in the negative electrode, in which the metal lodges. The experiment is always a difficult one, and it only yields small quantities of metal. It is only used for such metals as calcium or barium, which have not yet been obtained bv the usual methods.

It has been attempted to extend these methods to the metals in common use. The methods by the wet way have two drawbacks: the necessity of using considerable electromotive forces which render the operations very costly, and the secondary reactions. In the case of copper the method seems to have been successfully applied. The ore, which is a double sulphide of copper and iron, is run into plates, which are then used as the negative electrodes in a bath containing copper sulphate.

Aluminium has been extracted in the dry way, either combined with copper as aluminium-bronze or in the free state.

Mr. Cowles obtains directly aluminium-bronze containing 15 to 20 per cent. of aluminium and quite free from iron. The ore used is a corundum or crystallised alumina, which is powdered and then mixed with bruised charcoal, and with twice its weight of very pure granulated copper, such as that of Lake Superior.

The whole is placed between two electrodes formed of large charcoal rods, such as are used for lighting, which are surrounded by a packing of charcoal previously soaked with milk of lime in order to prevent its being changed into graphite under the action of heat. Aluminium unites very energetically with copper. The operation is effected by means of a dynamo giving 5000 amperes at an electromotive force of 60 volts, representing therefore 300,000 watts; 15 kilos of corundum with 70 kilos of copper are worked in one operation, and give about 82 kilogrammes of alloy. Each kilogramme of aluminium in the alloy represents the work of 44 horse-power during an hour.

M. Minet obtains free aluminium by electrolysing fused cryolite, mixed with 60 per cent. of sodium chloride, between two carbon electrodes. The mixture is contained in a rectangular iron trough connected with the negative electrode ; the trough thus transmits about 5 per cent. of the current, and so becomes coated with a thin layer of aluminium, which protects the iron. The aluminium liberated falls in drops into a crucible placed underneath the negative electrode. The aluminium thus obtained contains only 2 to 5 thousandths of iron.

CHAPTER XXXVII.

ELECTRIC TELEGRAPH.

444. General Conditions.—The applications which form the subject of the preceding chapters utilise the energy of the electric current, and the question of greatest importance is that of efficiency. Electric telegraphy belongs to that category of applications which only require a current giving a very small amount of useful work—in most cases only as much as suffices to call into action, by a sort of detent, some other mechanical energy, such as that of a weight or of a spring. The characteristic properties on which this class of applications depends are the facility with which the time and place at which electrical effects occur can be controlled, and the rapidity with which an electrical action at one point is followed by a resulting effect at a distant point.

445. Electrical Communication between Two Points.—Practice has shown that in order to put two stations, A and B, into electrical connection only one wire is needed, provided that at the station A that pole of the battery which is not joined up with the line wire, and at the station B the end of the wire itself, are connected with large copper plates buried in the ground. The action is as if the earth itself played the part of a *return* wire. This arrangement has a twofold advantage; in the first place it is more economical, as the wire is the most costly part of the installation, and in the second place there is only half the resistance in the circuit, since the resistance of a good earth-connection is almost inappreciable.

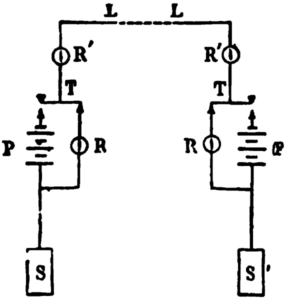

FIG. 357.

The same wire can be used for sending in either direction.

Each station must have its battery P, its transmitter T, and its receiver R (Fig. 357): S and S are the copper earth-plates. The receiver may be placed either at R or at R′. In the second case the two receivers always work at the same time, which enables the signals sent to be verified at the sending station.

446. Duplex Method.—It may even be arranged that the two stations communicate simultaneously by the same wire. One of the arrangements for this purpose is that represented in Fig. 358. The receiver is placed on the bridge of a Wheatstone's balance, the two sides a and a' of which are equal, while the two other sides are the line and a resistance ρ put to earth. If the two resistances are equal, it is readily seen that a current from A does not pass through the receiver at A as the bridge is in equilibrium, but that it acts on the receiver at B, and conversely. Three cases are possible. In the first case a single key is depressed, which is the case just described. Secondly, the two keys are depressed simultaneously; the two receivers then give the same signal under the influence of the same current if the two batteries have their opposite poles to the line, and under the influence of a local current if they are arranged in the same direction—that is, so that the same poles are joined to the line, their electromotive forces acting then in opposition to each other. Lastly, one of the keys is depressed, while the other is free and is not connected either with the battery or the earth: the current then gives the desired signal at the receiving station, but it acts also on the local receiver, though only for so short a time that no great inconvenience is caused; this difficulty may, moreover, be obviated by a somewhat more complicated arrangement, which we need not now describe.

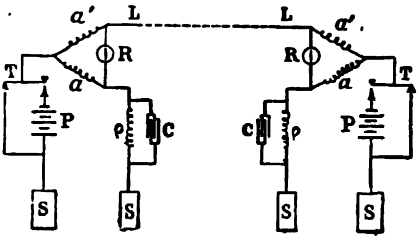

FIG. 358.

By means of condensers C, joined to the resistance ρ, the capacities of the real line and the artificial line may be balanced in the same way as their resistances are.

447. Lines with Condensers.—It is often desirable to isolate the line wire completely by interposing condensers C at the ends

(Fig. 359). When the key is depressed the local battery charges the first coating of the condenser, and the potential of the second becoming raised or lowered to a corresponding degree, sends to the line a flux of the same kind as that which the battery would have given. This flux charges the first coating of the second condenser, which by the same action causes a flux, always of the same sign, through the instrument of the receiving station. The advantage of this arrangement is that it protects the line against the action of earth-currents, which sometimes interfere with the working.

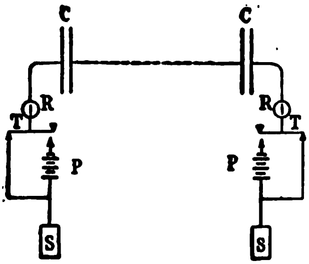

FIG. 359.

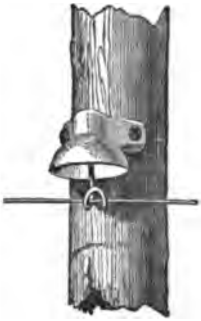

FIG. 360.

448. Air Lines.

448. Air Lines.—The wire ordinarily used for air lines is galvanised iron 4 millimetres in diameter. Its resistance is 10 ohms, and its electrostatic capacity 0.01 microfarad per kilometre. The use of copper had been given up, owing to its small tenacity and its high price; but it is coming into use again in the form of a special kind of bronze, of which wires can be made which are almost as strong as iron, and have almost as high a conductivity as pure copper.

As the wire is surrounded by a medium which is absolutely devoid of conductivity, it is sufficient if it is insulated at the points of support. Porcelain is generally used for insulators (Fig. 360). There is a certain amount of leakage at the supports, which varies with the state of the atmosphere: an estimate of its magnitude at any time can be obtained by observing the extent to which the apparent resistance of the line is decreased.

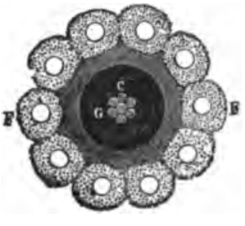

FIG. 361.

449. Submarine and Subterranean Wires.—In lines laid in the ground or under water, where the surrounding medium is a conductor, an insulating envelope is needed. The conductor or *core* consists of a strand of copper wires CC (Fig. 361), surrounded by several layers of gutta percha, then by a layer of jute, and lastly by a protecting sheath of steel wire covered with tarred hemp. The whole thus forms a condenser having a considerable capacity. The cable from Ireland to Newfoundland has a capacity per kilometre of 0.22 microfarad, with a resistance of 1.62 ohm.

450. Signalling Instruments—Alarum.—We shall only describe a few of the ordinary pieces of apparatus, and shall restrict ourselves to the indication of their most important features.

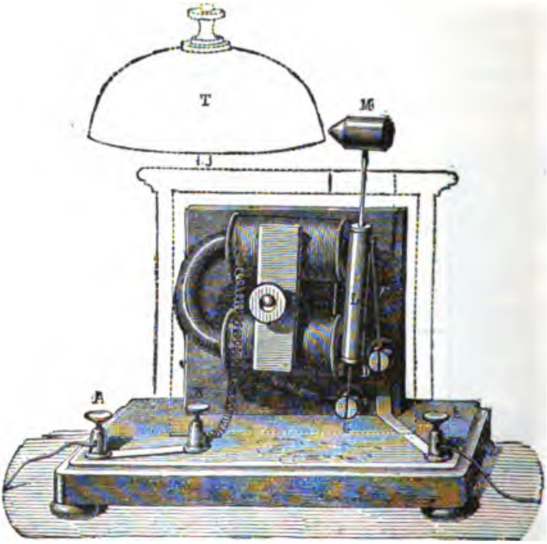

FIG. 362.

A piece of soft iron L (Fig. 362), is held in front of an electromagnet by means of a spring, D. When at rest it touches a spring, *r*. The circuit comprising the wire of the electromagnet is thus closed between the terminals A and E, and when a current passes the electromagnet is thereby excited and attracts the iron, L, so that it now does not touch the spring, *r*; the current is thereby broken, the iron, L, again touches *r*, and so forth. This arrangement is known as an *alarum*, and can be constructed so as to give as many as 1000 vibrations in a second.

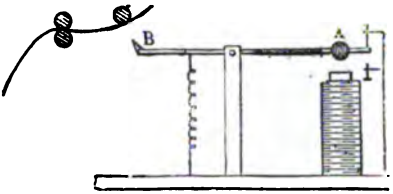

FIG. 363.

451. Morse's Apparatus. — The characteristic part of a Morse instrument is a horizontal lever, AB (Fig. 363). At the end A is a piece of soft iron which is attracted by an electromagnet, forming part of the line circuit, when the current passes; the end B then strikes against a paper strip or tape, which is payed out by a clockwork motion

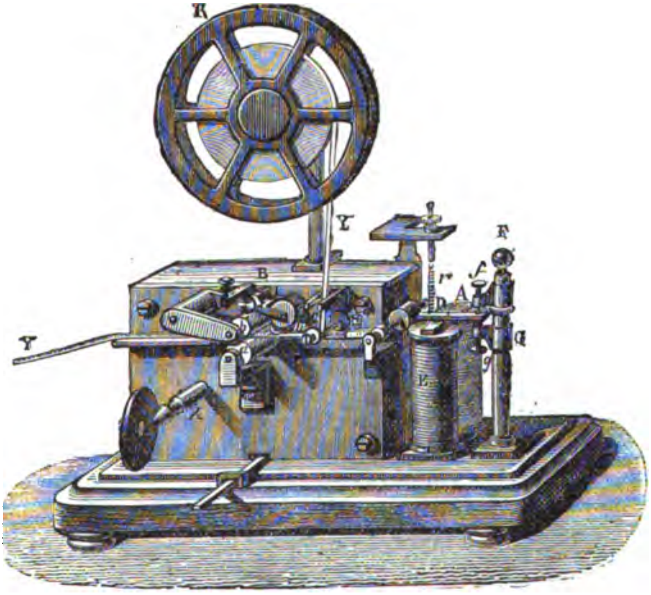

FIG. 364.

and makes either a dot or a dash according to the duration of the contact. The complete apparatus is represented by Fig. 364. The case, B, contains the clockwork which drives the cylinder *a*.

This forms with the cylinder *b* a pair of rollers which pull the tape through the instrument. The end of the lever makes the paper strike against a thin rotating disc, the circumference of which is kept fed with ink. The sending instrument is a simple contact-key, by depressing which the line is brought into the circuit of the battery (Fig. 365). The various letters of the alphabet are represented by combinations of dots and dashes, the number of signals varying from one to four for each letter, the more frequently occurring letters having the simplest signs (Fig. 366).

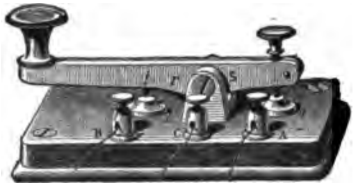

FIG. 365.

.—	—...	—.—.	—..	.	..——	——.	....	..
A	B	C	D	E	F	G	H	I
.———	—.—	.—..	——	—.	———	.——.	——.—	.—.
J	K	L	M	N	O	P	Q	R
...	—	..—	...—	.——	—..—	—.——	——..	
S	T	U	V	W	X	Y	Z	

FIG. 366.

452. Hughes's Printing Telegraph.—The main element of the receiving instrument is a small wheel, the *type-wheel*, the circumference of which carries the twenty-six letters of the alphabet standing out in relief, a space being left between Z and A. The letters are kept constantly inked with a fatty ink.

The wheel revolves uniformly about its axis at the rate of one or two turns a second. Every time the electromagnet acts, the travelling strip of telegraph-paper is sharply pressed against the lowest part of the circumference of the wheel and thus receives the impression of whatever letter occupies this position at the moment, but the motion is so rapid as not to interfere with the rotation of the wheel. On falling back from the type, the paper moves forward through a distance equal to the breadth of a letter, so that the next letter is printed alongside the last one and not on the top of it. Accordingly, in order to print a word, it is only needful to make the electromagnet act at the proper moments, so as to catch the successive letters of the word as they pass.

The sending instrument consists of a horizontal slab having twenty-six holes, corresponding to the letters of the alphabet, arranged round the circumference of a circle. These holes allow small vertical rods to pass freely through them, any one of which can be raised at pleasure a little above the surface of the slab by pressing the proper key of a key-board. A horizontal arm revolves uniformly above the circle of holes, carrying an appropriate contact-maker at its outer end, and whenever this encounters a rod projecting above the rest, a current is sent into the line and acts upon the electromagnet of the receiving instrument. Consequently, if the revolving arm and the type-wheel have exactly the same period of rotation, and, allowing for the time taken in transmission, the same phase, letters will be printed at the receiving end corresponding to whatever keys may be successively pressed at the sending end and in the same order.

In order to test whether the sender and receiver are properly adjusted to work together, the same key, corresponding to the letter A, for example, is struck several times in succession. If the same letter is printed by the receiving instrument an equal number of times, the adjustments are right ; but if, instead of the letter A, the receiver prints, say, M, N, O, P, Q, . . ., the speed of the type-wheel is too great, and exceeds that of the sender by 1/26. In this case the speed is altered until the receiver prints the same letter, R, for instance, every time. All that is now required is to alter the phase so as to make the receiver give the letter A. By a simple mechanical contrivance, these two adjustments can be made without interfering with each other.

As in this arrangement only a single battery-contact is required for each letter, messages can be sent by it more quickly than by the Morse instrument.

453. Rapidity of Signalling.—We have so far not taken into account the propagation of electricity, and have tacitly assumed that at a given instant all sections of the circuit are traversed by the same quantity of electricity. This assumption is no longer admissible in the case of very long circuits, especially of those having a large capacity like the Transatlantic cables. When one end of the wire is connected with the pole of a battery at constant potential, the current is not immediately set up through the whole extent of the line. It does not move in the wire like a cannon-ball, but rather like a mass of water issuing from a reservoir to fill a channel communicating with lateral basins, which have to be filled at the same time as the channel itself. The *wave front* advances with increasing inclination from the vertical, and between the first arrival of the water and the final establishment of the ultimate level an interval elapses which is longer the more distant the section considered is from the origin. Thus, on the Transatlantic cables from Ireland to Newfoundland, there is no trace of a current in Newfoundland until 0.2 of a second after connection has been made with the battery in Ireland; at the end of 0.4 second the current has only 0.07 of its final strength; it has one-half at the end of a second, and only after 3 seconds can it be considered that the full strength is attained. Hence in the propagation of electrical phenomena there is no definite velocity, as for light and sound. The apparent velocity, which it has sometimes been attempted to measure, on the assumption that the propagation is uniform, by determining the time necessary for producing a definite effect at a certain distance, depends on the nature, the form, and the position of the wire, and also on the sensitiveness of the apparatus employed for observing the effect.

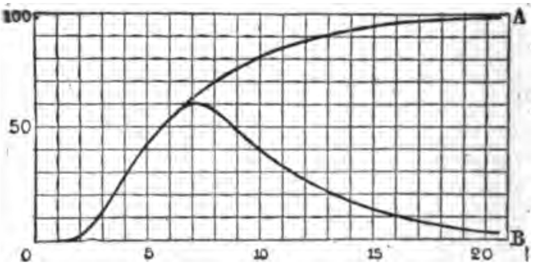

FIG. 367.

454. Lord Kelvin's Theory.— Lord Kelvin has given the mathematical theory of these phenomena, and has shown that if, for a given wire, and at a given distance from the origin, a curve A (Fig. 367) is constructed, taking as abscissæ the intervals of time that have elapsed since connecting the end of the wire with the battery, and for ordinates the corresponding strengths of the current at the given distance, the final strength being taken as unity, this same curve may be used for all cables and all distances, provided the interval represented by unit-length along the axis of time is suitably chosen. Let a be this interval: if ρ is the resistance in ohms and γ the capacity in microfarads of unit-length of the cable, the value of a for a point M at a distance l from the origin O is given by the formula

$$a = 233 \times 10^{-10} \gamma \rho l^2.$$

Let R be the resistance of the cable, and Γ its capacity, from O to M: we have

$$R = l\rho, \quad \Gamma = l\gamma, \quad \text{and } \gamma\rho l^2 = \Gamma R;$$

consequently

$$a = 233 \times 10^{-10} \Gamma R.$$

It is to be observed that the value of a is independent of the unit of length taken. The product ΓR represents a time (§ 328); it may be regarded as the constant characteristic of the cable.

The formula shows further that the value of a increases proportionally to the square of the distance l.

For the Transatlantic cable already mentioned we have, taking the kilometre as unit of length, $\gamma = 0.22$ farad, $\rho = 1.62$ ohm, $l = 5000$; from this we deduce

$$a = 0.2 \text{ seconds.}$$

In ordinary iron telegraph wires 4 millimetres in diameter $\gamma = 0.01$ farad at most; $\rho = 10$ ohm. For the line from Paris to Bordeaux, which is nearly 600 kilometres long, we have

$$a = 0.0008 \text{ second,}$$

that is, 0.001 as a round number—or a time 200 times as small as with the submarine cable.

The examination of the curve of *arrival* shows that the current may be regarded as having attained its normal value after a time equal to $20a$: if it is broken at the origin after the lapse of this time, it will still require $20a$ to vanish. If then two successive currents are to attain their maximum value, and produce signals which do not encroach on each other, they must not follow each other at smaller intervals than $40a$.

455. Use of Alternate Contacts.—By means of the arrival-curve the effect produced by a rapid succession of currents either in the same or in opposite directions may be estimated. Suppose, for instance, that the current is broken at the sending end of the wire, after having been made for a time na; in order to find the effect at the receiving station, it is only needful to draw a curve identical with the first, but shifted parallel to itself through a distance equal to na, then to construct a fresh curve, B, the ordinates of which are the differences of those of the two former curves. Suppose now that, after a fresh interval $n'a$, a negative current of the same strength is started at the sending end. The original curve must be displaced parallel to itself by the amount $(n + n')a$, and the differences obtained by subtracting its ordinates from those of B taken for a new curve, and so on. In this way dentated curves more or less regular are obtained, the ordinates of which are sometimes positive and sometimes negative, but in which the effect of each positive or negative contact is well marked, although the actual current at any instant depends on the previous contacts, as many as thirty or forty, for example, producing an effect if they follow each other at intervals of a.

456. Use of the Mirror Galvanometer.—The effect of these rapid contacts, some positive and the others negative, succeeding each other at regular intervals, is very apparent in a mirror galvanometer suitably damped. Every positive contact moves the spot of light suddenly to the right, for example, and every negative contact to the left; and this no matter whether, as the result of preceding impulses, the instantaneous position is at the zero, or to the right or left. The Morse alphabet (Fig. 366) is used, a dot being represented by a movement towards the right and a dash by a movement towards the left.

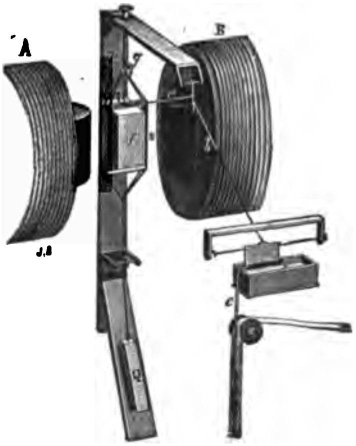

FIG. 368.

457. Siphon Recorder.—For submarine signalling, Lord Kelvin has substituted for the mirror galvanometer a writing instrument, which works as rapidly as the former, and is known as the *siphon recorder*. This apparatus, on which the Deprez d'Arsonval galvanometer (Fig. 314) is based, consists of a very light and mobile rectangular coil, S (Fig. 368), placed in the field of a strong electromagnet, AB; a piece of soft iron, *f*, occupies the space in the middle of the coil, and increases the strength of the field in the space occupied by the wire. The coil has a bifilar suspension;

it moves either way according to the direction of the current, which enters at *pq*. The motions are rapid and very well marked, being nearly dead-beat in consequence of the very great damping. The motion of the coil either to the right or the left is communicated to a fine capillary glass siphon, one end of which dips in a reservoir of ink, while the end of the longer limb is close to a paper tape, payed out at a regular speed by clockwork. The ink is electrified by a small electrical machine, not represented in the figure, and, owing to electrical repulsion, a continuous jet is sent against the paper, forming on it a straight line if the current does not pass, but giving deflections either to the right or the left according to the direction of the current. Fig. 369 represents the alphabet, and will give an idea of the manner in which the apparatus works.

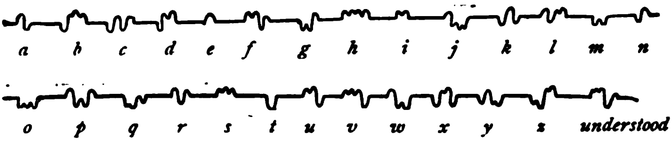

FIG. 369.

458. Phonograph.—Everyday experience proves that sonorous vibrations transmitted through the air are capable of being communicated to solid matter, and that this, in turn, can give back again to the air the vibrations that have been imparted to it. It is thus that speech can be transmitted clearly and unchanged through a wall or other solid obstacle.

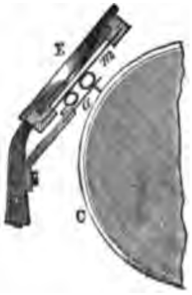

FIG. 370.

If a solid body is in the form of a plate of sufficient thinness, the vibrations produce very perceptible movements of the surface. This is clearly shown by Edison's phonograph (Fig. 370): *m* is the vibrating plate, and a style, *a*, attached to it impresses every movement of it on a moving sheet of tin-foil, or better, on a revolving wax cylinder, C. If afterwards the point of the style is made to retrace the impression it has made, the vibrating plate is caused to repeat all its previous movements and so to give back to the air vibrations identical, in all but intensity. with those which produced these movements.

459. Bell's Telephone.—In Bell's telephone (Fig. 371) a thin plate of iron, M, is fixed in a frame immediately in front of a bar-magnet, A, the end of which is surrounded by a coil, B, of fine wire. Any motion of this thin plate modifies the magnetisation of the bar, and causes a variation of the magnetic flux through the coil.

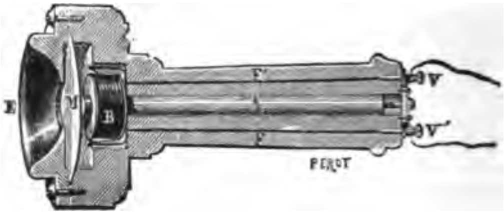

FIG. 371.

The resulting induced currents pass into the coil of a second apparatus identical with the first, and cause corresponding changes in the magnetisation of the second magnet, and motions in the iron plate which are identical with those of the first, except as regards strength.

The system is perfectly symmetrical, and therefore reversible. The two apparatus, the transmitter and the receiver, are in reality two identical alternating current machines, the former acting as generator and the latter as motor; the period continually varies from one moment to another, but at each instant is identical for the two.

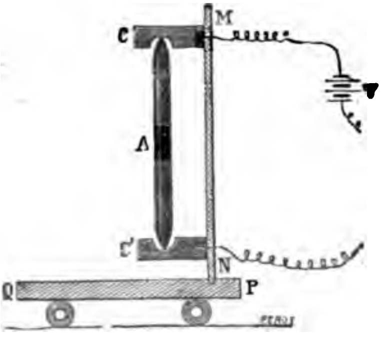

FIG. 372.

460. Microphone.—The currents which work the telephone are extremely feeble, not exceeding a few hundred-thousandths of an ampere. The action has been greatly improved by using the current of a battery instead of induced currents. This is effected by means of Hughes's microphone.

A pointed rod of carbon A (Fig. 372) is in contact with two pieces of carbon C and C', fixed to an upright piece of wood. The carbon forms part of a circuit made up of the battery V, the line wire, and the telephone receiver. With a permanent current the telephone does not speak, but any movement of the carbon changes the resistance of the circuit, and, modifying the strength of the current, entails a displacement of the plate of the telephone. Experiment shows that, when a person speaks before the instrument, the vibrations communicated to it are transmitted to the plate of the telephone so as to reproduce the spoken words with greater or less distinctness.

Theoretical considerations show that the effect of a periodical variation in the resistance is equivalent to a periodic variation of the electromotive force, the resistance remaining constant. They also show that if in a complex sound the ratios of the amplitudes and the differences of phase of the constituents are to be retained, and therefore the clearness and quality of the original sound, the variation of the telephonic currents must be a very small fraction of their mean strength.

The form of the original microphone has been variously modified. It is desirable to increase the number of contacts, so as to avoid scratching noises. The Ader microphone consists of ten carbon rods, laid in two sets of five each on three cross-pieces, also of carbon, fixed to the same piece of wood. Good results are also obtained by using small fragments or filaments of carbon. One of the best microphones consists essentially of a thin plate of carbon resting on a packing of the filaments of incandescent lamps.

461. Transmission by the Telephone. — In the telephone, which is an apparatus producing very rapidly alternating currents, a considerable influence of the effects of induction and capacity is to be expected. Applying the theory which explains so well the phenomena of transmission in cables, we arrive at this result, that in the case of a vibratory motion the electrical effect is propagated in the wire at a uniform rate, with a velocity dependent on the duration of the period. Now it is known that the complex motions corresponding to articulate speech may be regarded as arising at each instant from the superposition of a certain number of harmonic vibrations of different periods. If the corresponding electrical effects were propagated with different velocities, they would undergo a kind of dispersion in the wire, and would not be superposed on arrival. The quality and the articulation would be profoundly modified. Experiments show that this is not the case. This is probably due to the very rapidity of the vibrations, which brings into play the effects of self-induction, which does not affect the relatively very slow vibrations employed in telegraphic transmission, and acts in the contrary direction to the capacity (§§ 327, 328).

However this may be, experiment shows that in suitable conditions of installation, conversation may be directly kept up at great distances, between Paris and Marseilles, for instance, a distance of upwards of 500 miles. The conditions are to use copper instead of iron, and not to use the earth for the return circuit, but two wires wound round each other like a double screw, so as to eliminate the effects of external inductive actions.

The line may even be used simultaneously for telephonic and telegraphic transmission. At each end the receivers of the telephone and of the telegraph are connected in parallel circuit, and in order to prevent the telegraph currents from acting on the telephone, and the telephone currents from being partially lost in the telegraph circuit, electromagnets are introduced into the latter, and condensers into the telephone circuit: the effect of this is to increase the self-induction of the one and the capacity of the other. The telegraph signals are thus retarded, flattened out, as it were, so that the movements they impart to the membrane of the telephone give no sound. As regards the telephone currents, the quasi-resistance due to the self-induction causes them to pass entirely in their own circuit.

CHAPTER XXXVIII.

ATMOSPHERIC ELECTRICITY.

462. Potential at a Point in the Air.—Experiment shows that in an open space the potential at a point in the air is always different from that of the earth. Two methods may be used to determine the value of this potential.

Let a small insulated sphere, of radius r, be placed at the point in question and connected for a moment with the earth by means of a very fine wire. If V is the value of the potential at this point due to external masses, the potential of the earth being as usual taken as zero, the sphere will acquire a charge, m, of electricity such that the potential of the sphere becomes zero, like that of the earth. The potential at the centre being $V+\frac{m}{r}$, we have $V+\frac{m}{r}=0$. If the sphere is then placed in a Faraday cylinder (§ 15), we may measure its charge m, and so obtain

$$V=-\frac{m}{r}.$$

But the simplest method consists in putting at the place in question a point which forms part of an insulated conductor (§ 20). Assuming that the point is *perfect*, equilibrium cannot exist so long as the point, and therewith the conductor of which it forms part, is at a different potential from that of the air near the point.

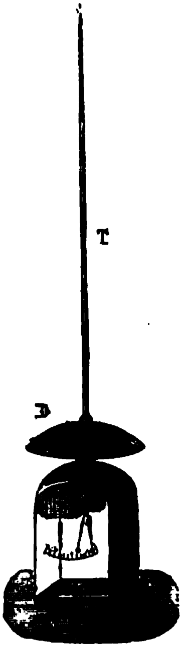

FIG. 373.

Saussure used a small electroscope provided with a point (Fig. 373). If the case is at the potential of the ground, the divergence of the leaves varies with the potential of the air at the end of the point; but the action of the point is too imperfect to allow us to consider that equilibrium is attained.

We have seen how a water-dropping apparatus (§ 81) enables us to realise the effect of a perfect point, and how it assumes the potential of the air at the point where the drops separate from the conductor. The dropping apparatus (Fig. 374) is preferable to a lighted wick, owing to a small difference of potential produced by the combustion, which may amount to half a volt.

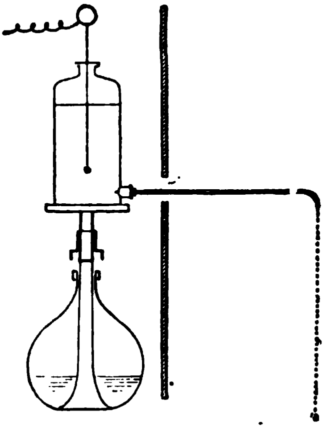

FIG. 374.

The insulated reservoir is connected with an electrometer, the deflection of which measures the potential. By using a mirror and allowing the image to fall on a uniformly moving band of photographic paper, a continuous registration of the indications of the instrument is obtained.

It is thus found that in *fine* weather the potential anywhere in the open air is always *positive;* that its value increases with the height of the point above the ground, and almost in direct proportion; but that at the same place rapid and often large variations occur.

The results vary so much that it is difficult to give numerical statements. Above an open plain, for example, the change of potential with height is often between 10 and 1000 volts per metre, but it is sometimes far more.

If, instead of insulating the point, or the arrangement which acts as a point, we connect it with the earth, statical equilibrium cannot be established, and a continuous flow of electricity traverses the conductor. The flow is manifestly equal to the amount given out by the point. It increases with the difference of potential, but it cannot be used to measure this difference, the quantity given off being always so extremely small.

If there is a small break in the conducting wire, the difference of potential at the break may be great enough to produce a succession of sparks. Sparks are sometimes produced in this way between the needle and the quadrants of the electrometer.

463. Distribution of Potential.—At a given instant, the surfaces where the potential has constant and equidistant values above an open plain are equidistant horizontal planes. If the surface of the ground is irregular, the nearest equipotential surfaces follow its undulations, and approach each other over the elevated parts, and the more so the higher and more abrupt these are. Around a house, all parts of which may be regarded as connected with the earth, and therefore at zero potential, the equipotential surfaces are vertical near the walls and follow the contours of the roof, being closer to each other over the ridge; on the other hand, they separate widely from each other in a court surrounded by high walls, or in a street. As we rise, the effects of the inequalities disappear, and it may be assumed that beyond a certain height the equipotential surfaces are horizontal planes.

As the equipotential surfaces tend to become parallel to the surface of the ground the nearer they are to it, the electric force at each point is perpendicular to the surface. And since, in fair weather, the value of the potential increases with the distance, it follows that the electric force is directed towards the earth; its intensity at each point is inversely proportional to the distance between two consecutive equipotential surfaces (§ 33).

464. Negative Electrification of the Earth.—The phenomena observed near the ground are thus the same as they would be near a negatively electrified conductor in equilibrium. It follows from Coulomb's theorem (§ 43), that if F is the value of the force near a conductor, the electric density σ at the corresponding point of the surface is given by the equation

$$\sigma = \frac{F}{4\pi}.$$

Let us assume that the equipotential surfaces are horizontal, and that the potential increases by 1 volt per centimetre—that is, in electrostatic C.G.S. units, by $\frac{10^8}{3 \times 10^{10}} = \frac{1}{300}$; we deduce from this for the value of F in dynes, $F = \frac{1}{300}$, and for the density, or charge per square centimetre, $\sigma = \frac{1}{1200\pi}$. Expressed in practical units, that is to say in coulombs, this is $\frac{10}{1200\pi \times 3 \times 10^{10}} = \frac{10}{36\pi 10^{12}}$ $= 10^{-13}$ nearly.

The electrostatic pressure—or, in other words, the force which would tend to lift a surface of 1 square centimetre placed on the ground—would be

$$2\pi\sigma^2 = \frac{10^{-4}}{72\pi} = 4.4 \times 10^{-7},$$

that is to say, half a millionth of a dyne, a force wholly insufficient to raise the lightest body.

This charge, small as it is, might however be made evident by a method like that of the proof plane: a disc like that of an electrophorus applied to the ground, and then removed by an insulating handle, would show a negative charge when tested by a delicate electroscope.

Let r be the radius of the disc; it will take a charge $q = \pi r^2\sigma$, and since the capacity of a circular disc of radius r is $\frac{2r}{\pi}$, this charge will raise it to the potential $\frac{1}{2}\pi^2 r\sigma$. If we assume $\sigma = \frac{1}{1200\pi}$, and remember that an electrostatic unit of potential is equal to 300 volts, the expression for the potential in volts will be $\frac{1}{8}\pi r$. If the radius were 25 centimetres, this would give a potential of nearly 10 volts.

The measurement of the charge of the disc would give the value of σ, and therefore of F. We deduce from it for the value of the potential at a height h from the ground (§ 32),

$$V = Fh.$$

The potential of the air is not, however, always positive, and therefore that of the ground is not always negative; in cloudy weather, especially during rain, and sometimes, though very rarely, with cloudless sky, the potential of the air is negative, and therefore that of the soil positive. This, however, must be regarded as quite exceptional, and there is reason to believe that if at a given moment the sky were clear over the whole surface of the globe, the globe itself would be wholly negative.

465. Position of the Acting Masses.—The measurement of the potential at a point of the air near the ground teaches us nothing as to the situation of the acting electrical masses. An example will make this intelligible. Suppose that a water-dropping collector is placed in a closed room, and that a sphere charged with positive electricity is then introduced; the electrometer will at once show positive potential. Again, instead of bringing in an electrified body, let the positive charge of a Leyden jar, whose outer coating is connected with the inside of the room, escape by a point, the air of the room will be positively charged, and the electrometer will again indicate a positive electrification, equal, it may be, to that in the former case, although the acting masses are distributed in a totally different manner. As a matter of fact, any observations made near the ground give nothing more than the electrical condition of the ground itself; they tell us, as we have seen, the density of the surface-layer, but furnish no information as to whether this layer is the result of an independent charge, or is due to an external influence—that of the air, for example, which is positively electrified.

It may be observed, however, that if the electrification belongs exclusively to the earth, the potential above an extended plane ought to vary strictly as the distance—in other words, the electric force should be constant. On the other hand, if the mass of the air is itself electrified, the force must vary with the height, and should diminish if the air is positive, and increase if it is negative. We have, however, no data as to the law of the variation of the force.

Experiment seems to show that the mass of air is positive; two water-dropping apparatus set up, one in the open and the other under a wire cage, with large meshes completely closed and in communication with the ground, give in general proportional indications. External charges have no effect on the second apparatus; the potential is due solely to the electricity of the air within the cage, and this may be regarded as being at each instant only a specimen of the outer air.

If the air has really an independent charge, the incessant variations of potential at a given point are explicable as due to the displacement of masses of air more or less electrified, and we might infer the distance of those masses from the extent of the surface of the ground over which the variations of potential at the same moment are proportional.

466. Origin of Atmospheric Electricity.—The question naturally suggests itself: What is the origin of the electricity of the air and the ground? A very seductive hypothesis is that which ascribes the electricity to the evaporation of water; the vapour being supposed to carry positive electricity with it, leaving negative electricity in the water and on the ground. Unfortunately none of the experiments made with a view of confirming this hypothesis have given it any conclusive support; on the contrary, the fact that rain is usually negatively electrified appears to contradict it.

The origin of atmospheric electricity has also been sought for in the induction currents which the motion of the earth might develop in the upper regions of the atmosphere, supposing these to be conductors (§ 73). Let us imagine a conducting arc, AB (Fig. 375), of any given shape, connecting the pole and the equator, to be either fixed, or at any rate to have a smaller angular velocity than the earth. The magnetic flux, cut by the arc as the earth turns from west to east, develops therein an inductive electromotive force, the direction of which is from the equator to the pole. If the arc were in contact with the globe by sliding contacts at N and E, and the circuit closed, the induction would produce a continuous current; if the supposed conducting arc were insulated, there would be an accumulation of positive electricity at the pole and of negative at the equator, and to this accumulation might be ascribed the polar auroras on the one hand, and on the other the daily thunderstorms in the equatorial regions.

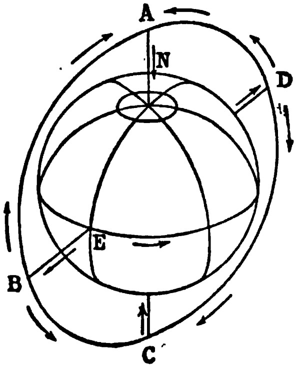

FIG. 375.

467. Polar Aurora.—In the polar regions the aurora borealis is visible, we may say, every night in winter. Its forms are excessively varied, and we shall not attempt to describe them. It is certain that the aurora is an electrical phenomenon. It is a discharge in rarefied air quite analogous to that produced in Geissler's tubes (§ 72). The discharge, or, in other words, the flow of positive electricity, appears to be towards the surface from the upper regions of the air. The phenomenon is produced at very various distances; some have been observed at as small a height as a mile and a quarter, and others at more than ninety miles. The upper summits, moreover, are often seen to be bounded by a yellow glow. The light of the aurora is due, like that of the tubes, to gaseous substances made incandescent by the discharge. In addition to the rays ordinarily seen in tubes from which the air has been exhausted, spectrum analysis shows a special ray between the yellow and the green ($\lambda = 5570$), which has not yet been identified in laboratory experiments. M. Lemström has made experiments on a large scale on the production of the aurora; he constructed a network of copper wire in the polar regions extending over an elevated area of 900 square metres, and having a number of points directed towards the sky; he thus succeeded in producing the characteristic luminous phenomena, including the peculiar ray of the aurora.

468. Earth Currents.—All electric potentials are in practice referred to that of the earth taken as zero; if there were electric equilibrium, the potential of the earth would be the same at all points of the surface; but experiment shows that this is rarely the case. Telegraph lines, the two ends of which are connected to earth, are generally traversed by currents of very irregular strength and direction, which are sometimes so powerful as to stop the working of the lines. Such interruptions are remedied in certain cases by using condensers (§ 446). Experiments show further that the currents are the same whether the wire is in the air or in the ground; that they have the same phases on two lines which are in the same direction but of different lengths, and that the electromotive force is proportional to the distance of the extreme points. The simultaneous investigation of the currents on two lines in different directions, one parallel and the other perpendicular to the meridian, show that the maximum is more or less north and south.

A comparison of the earth-current curves with the simultaneous curves of magnetic variations reveals an intimate relation between the two orders of phenomena. There is an obvious correspondence between the variations of current-strength parallel to the meridian and those of magnetic declination on the one hand, and on the other between the variations of the earth-currents perpendicular to the meridian and those of the horizontal component. There is, however, this remarkable circumstance, that the phases do not coincide, but that the maxima of the one correspond to the minima of the other. This can only be explained by assuming that the same current which exerts an electromagnetic action on magnetic needles acts by induction on the lines. According to this, earth-currents must be looked upon as only secondary currents, and a comparison of the directions of these currents with the deflections of the needle shows that the primary currents, the direct cause of both sets of phenomena, have their seat in the upper regions of the atmosphere.

469. Thunderstorms.—To Franklin the credit is due of having demonstrated by the use of insulated pointed conductors that thunderstorms are purely electrical phenomena. Thunderclouds are electrified conductors, some of them charged positively and others negatively ; lightning is a spark passing between two clouds, or between a cloud and the earth.

Passing from these general statements to details there is great uncertainty. Conjectures only can be made as to the constitution of thunderclouds and their origin, or as to the conditions and character of their discharge.

It is easy to understand that a cloud which is formed in contact with the earth may take with it negative electricity ; also that the condensation of a mass of vapour in the midst of a mass of positively electrified air may produce a positive cloud ; and lastly, that a cloud already charged may produce an inductive charge of the opposite kind upon another cloud that is for the moment in connection with the earth. We may even imagine actions like those of Holtz's machine, and conceive of electrification being produced by the relative displacement of two layers of clouds, one of which acts like the armature of such a machine. But can these causes account for the duration of some storms, the multitude of lightning discharges by which they are accompanied, and for the enormous quantity of electricity they put in play?

Are thunderclouds themselves to be regarded as conductors charged with electricity only on the surface, or as agglomerations of isolated masses, each having its own charge? The latter opinion appears the more probable.

Up to a certain point we can form some idea of the charge. Let us assume that the surface of the cloud is parallel to the earth ; the density is the same on the two faces opposite each other (§ 42). This density is 10^{-13} coulombs if the potential increases one volt per centimetre (§ 464) ; if we assume that in a violent storm it increases 1000 volts per centimetre, which is certainly exaggerated, the charge will be 10^{-10} per square centimetre, and therefore one coulomb per square kilometre. This is certainly an outside estimate.

470. Various Kinds of Lightning.—In his celebrated "*Notice sur le Tonnerre*," Arago discriminates between three classes of lightning, *zigzag* or *forked*, *sheet*, and *globe* lightning. The discharges of the first class consist of well-defined lines of fire, which, excepting as to dimensions, are perfectly similar to artificial electric sparks ; they are accompanied by a more or less prolonged noise which is called *thunder*. Those of the second class are the flashes of light which suddenly illuminate clouds without being accompanied by any noise; they seem due to ordinary discharges concealed from the observer, or rather to partial discharges between the clouds. Globe lightning consists of balls of fire moving slowly through the air, and then exploding suddenly; if globe lightning is not a mere optical delusion, it is a phenomenon which in the present state of our knowledge is wholly inexplicable. The only known phenomena which at all resemble it are the luminous globules obtained by Planté by putting the poles of a battery of 3000 to 4000 volts in contact with the surface of a liquid.

Photographs of lightning flashes taken in recent years show that discharges of the first class have a much more complicated structure than appears at first sight. The principal line of light is always accompanied by numerous ramifications, which seems to prove that, from the electrical point of view, the cloud does not act as a continuous conductor.

471. Effects of the Discharge.—The effects are those of an electrical discharge, but on a larger scale. The lightning discharge heats conductors, sometimes to the melting, or even the volatilising point. It smashes and scatters badly conducting substances, kills or paralyses living beings. Trees struck by lightning are generally torn to pieces as though by a sudden explosion of steam. In its path it frequently presents the most inexplicable appearances.

In order to get some idea of the energy at work, let us take the case of a cloud, such as that considered in § 464, at a height of 1000 metres; its potential will exceed that of the earth by 10^8 volts. As the charge is one coulomb per square kilometre, the complete discharge will represent an amount of work equal to $\frac{1}{2} . 10^8$ joules, or very nearly 5×10^6 kilogramme-metres (§ 39); this represents the work done by a mass of 5000 kilogrammes falling from the height of the cloud.

472. Duration of the Lightning Discharge.—An interesting point to consider is the duration of the lightning discharge. Arago indicated the principle on which the observations may be made. Let the light from a discharge illuminate a wheel formed of a number of white radial lines on a black ground rotating with uniform velocity. Let the number of lines be 100, and the velocity 100 turns per second. The time required for one line to take the place of its neighbour will be the ten-thousandth part of a second. If the duration of the flash is equal or superior to the ten-thousandth of a second, the wheel, owing to the persistence of luminous impressions, will appear as a uniformly illuminated disc; but if the duration is one-half, one-third, or one-quarter of a ten-thousandth of a second, the wheel will appear formed of alternately bright and dark sections, the bright sections being equal to one-half, one-third, or one-quarter of the dark sections, If, as stated by Arago, the lines appear as sharp as if the wheel were stationary, the duration of the lightning is not an appreciable fraction of a ten-thousandth of a second. Everything seems to show that it is less than the hundred-thousandth of a second.

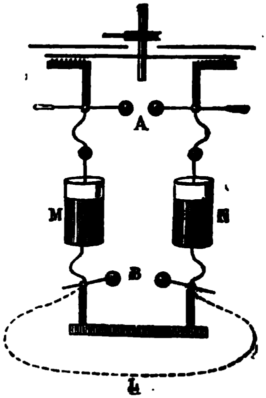

FIG. 376.

By photographing a lightning discharge upon a rapidly revolving plate, more definite results might be obtained than by means of Arago's wheel. Probably this process would show whether the discharge is continuous or oscillatory.

473. Two Kinds of Discharge.—We have already seen that discharges may be produced under two very different conditions. Let us return to the experiment of § 331. At A (Fig. 376), the difference of potential increases gradually, and when it has reached the value corresponding to the sparking distance, the discharge takes place; at B the difference of potentials is produced suddenly, and the spark passes at once without previous preparation. The effects of self-induction are more intense, the spark is longer and the lateral discharges more violent. We shall distinguish the two cases by the letters *a* and *b*. In order to show the application to the case of thunder-clouds, Professor Lodge arranged experiments in the following manner:—A and B (Figs. 377-379) are two insulated conducting plates, between which the discharge of a Leyden jar passes. The wires marked + and - connect the jar to the machine, and enable it to be charged. Fig. 377 corresponds to the case *a*; Figs. 378 and 379 to case *b*. Bodies of very various shapes are placed at varying heights between the two plates,

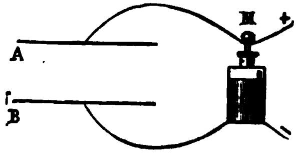

FIG. 377.

a dome, a knob, a point, or a flame. In case *a*, the point and the flame produce a silent discharge, and *protect* the other bodies; in case *b*, they are all struck indiscriminately, and by sparks of the same kind; it seems immaterial also whether the object is placed directly on the lower plate or is insulated.

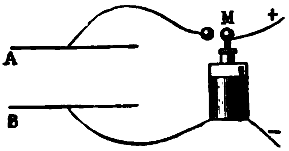

FIG. 378.

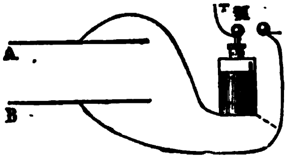

FIG. 379.

Do these two cases occur in nature? There seems little doubt that they do, especially if we consider the clouds as made up of isolated masses.

In the experiments described, the discharge is oscillatory. Are the conditions which permit oscillation (§ 329) realised in the case of a thunder-cloud? This is more doubtful; but it may be observed that the effects which we have in view are not so much connected with the oscillatory character of the discharge as with the extreme rapidity of the variations of potential.

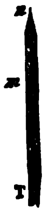

FIG. 381.

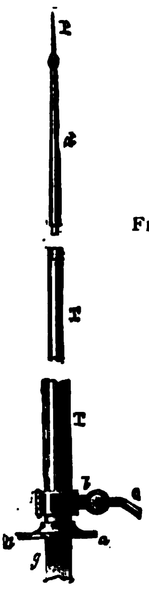

FIG. 380.

474. Lightning Conductors.—Shortly after the discovery of the properties of points, Franklin conceived the idea of utilising them for protecting buildings against the effects of lightning discharges. Franklin's lightning conductor consists of a long metal rod pointed at the top, placed at the highest part of the building to be protected, and connected with the earth by continuous good conductors. As now made, lightning conductors have a fine point of platinum or a stouter one of copper (Figs. 380 and 381). Franklin ascribes to the lightning conductor a twofold effect. Under the influence of the cloud the point allows the electricity of the opposite kind to escape, and this, carried by the particles of air, silently neutralises the electricity of the clouds. This is the *pre-*

ventive effect. But instead of supposing that the electricity which issues by the point neutralises that of the cloud, we may also attribute the preventive action to the electrification of the air by the point; in this way there may be formed an electrified cloud, which tends to produce at all points below it a potential of opposite sign to that of the thundercloud, and therefore neutralises the influence of the latter. If such a cloud of electrified air remained floating above the conductor, equilibrium would be rapidly attained, and the point itself would cease to act.

If, notwithstanding the point, a discharge takes place, it strikes the rod in preference to the other parts of the building which are within the radius of its protection, and the conductor leads the electricity to earth without any damage. This is the *preservative effect.*

The conductor is usually an iron rod, 1.5 centimetres in diameter, experience having shown that lightning has never damaged a rod having this section; its lower end is connected with water or earth that is always damp. All large pieces of metal belonging to the building, whether inside or outside, should be connected with it. It is sometimes assumed, but quite arbitrarily, that a conductor protects a circular area, the radius of which is equal to twice the height of the conductor.

In the erection of lightning conductors, and in instructions drawn up for this purpose, the case *a* has alone been taken into consideration, and even within these restricted conditions it does not appear that sufficient allowance has ever been made for the effects of self-induction. The duration of the discharge is so short that the effects of self-induction greatly preponderate over those of resistance. A flat band of metal is better than a solid circular rod. The relative advantages of copper and iron have frequently been discussed. Iron appears preferable if magnetic permeability does not come into play. The experience of more than a century shows that Franklin's lightning conductor is an efficient protection in the great majority of cases, but it would be an exaggeration to say that it has never failed except by some defect in the construction.

475. Theoretical Conditions.—If we inquire what conditions theory prescribes for the protection of a structure and all that it contains against damage by lightning discharge, we arrive at the conclusion that the structure should be a closed conductor, such as a room, the walls, flooring, and ceiling of which are of iron.

Whatever forces may be exerted from outside, the potential in the state of equilibrium will be constant for all points of the surface and the bodies which it encloses; there would be no trace of electricity on any of them. When equilibrium is disturbed, as by a lightning discharge, it is to be supposed that the sides would form a protecting screen for all bodies in the interior, and that sparks like those of Hertz would not be obtained. In any case, these sparks would not be dangerous, and, except perhaps in the case of a gunpowder magazine, there would be no need to consider them.

For an external body connected with the ground no sparks are possible in the case of equilibrium, but in the case of a sudden discharge, there might be dangerous sparks.

We have already seen that, in order to produce a conducting surface at constant potential, it is not necessary that the metal should be continuous; the conditions are realised by a network of even larger meshes. The experiment in § 335 shows that even when equilibrium is disturbed a network has the same effect as a continuous surface. The essential condition is that no conductor shall project into the interior, which might have a potential different from that of the envelope, and might therefore form a kind of electrode capable of giving sparks. Such, for instance, would be gas or water pipes; conductors of this kind should be connected with the conducting enclosure at the point where they enter.

476. Practical Conclusion.—The general result at which we ultimately arrive is that the most certain way to protect a building from the effects of lightning is to encase it in a metal network in perfect conducting communication with the earth. The network may be made of galvanised telegraph wire. The conductors should follow the ridge, the corners, the angles, the chimneys, &c., and loops or sharp angles should be avoided. Gas and water-pipes should be connected with the network on their entrance, and inside the building the pipes should be in metallic connection wherever they are near each other. The external network should be connected with the ground by the greatest possible number of points. All masses of metal on the outside—roofing, eaves, rain-troughs, &c. —should form part of the network. Connection should be made with the underground water-bearing stratum by means of large plates dipping in a pit dug in this stratum, and so deep that even when the water is at its lowest the plate is partially immersed.

Large masses of metal inside the building should be connected with each other, but instead of joining them to the outer network, it is better that they should have an independent connection with the earth. The building thus protected might be struck by lightning, but there would be nothing to fear from the effects.

A few strokes of lightning, probably harmless in any case, might perhaps be avoided by providing the conductors with points. From this point of view, it should be attempted to surround the building with an electrified atmosphere, and this would be attained more easily the more numerous are the points and the more widely they are distributed. We are thus led to the plan of lightning conductors devised by Melsens.

The most important point, and that most frequently neglected, especially in the older installations, is that of connection with the earth. A system of lightning conductors badly connected with the earth is not only useless but dangerous.

TABLES.

RESISTANCE OF METALS AND ALLOYS.

	Resistance in ohms of a length of 10 metres.		Coefficient of variation for 1°.
	1 sq. mm. section.[1]	1 mm. diameter.	a.[2]
Silver, annealed	0.1492	0.1900	0.00380
„ hard-drawn	0.1620	0.2062	...
Copper, annealed	0.1584	0.2017	0.00388
„ hard-drawn	0.1620	0.2063	...
Gold, annealed	0.2041	0.2599	0.00365
„ hard-drawn	0.2077	0.2645	...
Aluminium	0.2889	0.3679	0.00390
Platinum	0.8982	1.1435	0.00247
Iron	0.9638	1.227	0.00463
Nickel	1.236	1.573	...
Mercury	9.407[3]	11.978	0.000887
2 gold + 1 silver	1.078	1.372	0.00065
9 platinum + 1 iridium	2.163	2.754	0.000133
2 platinum + 1 silver	2.419	3.080	0.00025
German silver	2.076	2.643	0.00040

---

[1] By shifting the decimal point in this column four places to the right, the values of the *specific resistances* in C.G.S. units are obtained.

[2] $R_t = R_0(1 + at)$.

[3] Length of 1 ohm = 106.3 cm.

RESISTANCE OF AQUEOUS SOLUTIONS.

1 part Crystalised Salt dissolved in	Specific Resistance in C.G.S. Units.			
	At 10° C.		Sulphuric Acid at 16°.	
	Copper Sulphate.	Zinc Sulphate.	Density.	Resistance.
40 parts water	16.44×10^{10}	18.29×10^{10}	1.10	0.845×10^{10}
20 " "	9.87 "	11.11 "	1.20	0.666 "
10 " "	5.90 "	6.38 "	1.25	0.624 "
5 " "	3.81 "	4.21 "	1.30	0.662 "
2.6 " "	2.93 " (saturated)	...	1.50	1.72 "
0.752 " "	...	3.37 " (saturated)	1.70	4.23 "

ELECTROMOTIVE FORCES OF GALVANIC CELLS.

Volta . . .	Zinc Water Copper	about 1
Leclanché . .	Amalgamated zinc Solution of sal-ammoniac Binoxide of manganese and carbon	1.46
Poggendorff .	Amalgamated zinc 12 potassium bichromate, 25 sulphuric acid, 100 water Carbon	2.01
Daniell . . .	Amalgamated zinc 1 part sulphuric acid, 10 parts water Saturated solution of copper sulphate Copper	1.12
Grove . . .	Amalgamated zinc 1 sulphuric acid, 12 water Nitric acid Platinum	1.88
Latimer Clark (standard)	Amalgamated zinc Saturated solution of zinc sulphate Paste of mercurous sulphate Mercury	1.435

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